PHY 5669 : Quantum Field Theory B, Spring 2016

January 12^{th}, 2016
Assignment # 1
(due Thursday January 28^{th}, 2016)

Following the example in Chs. 6 and 7 of Peskin and Schroeder's book, consider the scattering process

$$e^-(p) \to e^-(p') + X \ ,$$

with $p' - p = q$ ($q^2 \neq 0$) and $p^2 = (p')^2 = m_e^2$, and work through the calculation of its cross section including the first order of QED radiative corrections, or $O(\alpha)$ corrections (for $\alpha = e^2/(4\pi)$). Here are some important steps that will allow us to then complete the discussion when we reconvene.

1) Show that the $O(\alpha)$ corrections to the cross section consist of both *virtual* (or one-loop) and *real* (or one-photon emission) corrections. Identify them explicitly in terms of Feynman diagrams.

2) One-loop QED corrections contains both ultraviolet (UV) and infrared (IR) divergences.

2.a) Using dimensional regularization, calculate the UV divergences and show that they can be reabsorbed by a redefinition of the fields, the electron mass (m_e), and the electric charge (e), by specifying well-defined renormalization conditions. Show how the renormalization procedure can be systematically implemented by defining a set of *counterterms*, and how the finite parts of such counterterms depend on the chosen renormalization conditions. Define the counterterms Feynman rules and show how you can use them in your calculation.

2.b) Verify that $\delta Z_1 = \delta Z_2$ at $O(\alpha)$, where $Z_1 = 1 + \delta Z_1$ and $Z_2 = 1 + \delta Z_2$ are the vertex and electron-field renormalization constants.

2.c) Show that the one-loop corrections also contain IR divergences, that arise from the integration over Feynman parameters. Show that there are two kinds of divergences: *soft* divergences that arise when the photon energy vanishes (and would be regulated by a non zero photon mass), and *collinear* divergences that arise when $m_e \to 0$ (and are regulated by the electron mass). Extract them using dimensional regularization and show how the renormalization conditions forces some of the counterterms to also contain infrared divergences. Calculate the IR part of the virtual cross section, i.e. the part of the $O(\alpha)$ virtual cross section that contains IR divergences. Notice that IR divergences do not cancel, i.e. they are not cured by the renormalization procedure.

3) Calculate the cross section for one-extra-photon emission ($e^- \to e^-\gamma + \gamma(k)$) and show that in the limit of $k^0 = |\mathbf{k}| \to 0$ (*soft* limit) this cross section contains IR divergences that exactly cancel the IR divergences of the same order virtual cross section (assume $m_e \neq 0$, i.e. no collinear divergences).