PHY 5669 : Quantum Field Theory B, Spring 2017

January 26^{th}, 2017
Assignment # 2
(due Thursday February 9^{th}, 2017)

1. Using the path integral formalism, calculate the ground state to ground state transition amplitude for the case of a harmonic oscillator with hamiltonian,

$$H(p,q) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2 \ ,$$

in the presence of an external force f, $\langle 0|0\rangle_f$. Starting from the definition of $\langle 0|0\rangle_f$ as,

$$\langle 0|0\rangle_f = \int \mathcal{D}p\mathcal{D}q \exp\left\{ i\int_{-\infty}^{+\infty} dt\,[p\dot{q} - (1-i\epsilon)H(p,q) + fq]\right\} \ ,$$

you should be able to show that

$$\langle 0|0\rangle_f = \exp\left[-\frac{i}{2}\int_{-\infty}^{+\infty} \frac{dE}{2\pi}\frac{\tilde{f}(E)\tilde{f}(-E)}{E^2-\omega^2+i\epsilon}\right] \ ,$$

where $\tilde{f}$ denotes the Fourier transform of f. Show that $\langle 0|0\rangle_f$ can also be written in terms of time-domain variables as

$$\langle 0|0\rangle_f = \exp\left[-\frac{1}{2}\int_{-\infty}^{+\infty} dt\,dt' f(t)G(t-t')f(t')\right] \ ,$$

where

$$G(t-t') = \int_{-\infty}^{+\infty}\frac{dE}{2\pi}\frac{i\,e^{-iE(t-t')}}{E^2-\omega^2+i\epsilon}$$

is the Green's function of the harmonic-oscillator equation of motion, i.e.

$$\left(\frac{\partial^2}{\partial t^2} + \omega^2\right) G(t-t') = -i\delta(t-t') \ .$$

You can also calculate $G(t-t')$ explicitly (by performing the integral above).

Use $\langle 0|0\rangle_f$ as your generating functional to calculate $\langle 0|Tq(t_1)q(t_2)|0\rangle$, $\langle 0|Tq(t_1)q(t_2)q(t_3)q)t_4)|0\rangle$, and more generally $\langle 0|Tq(t_1)\cdots q(t_{2n})|0\rangle$.

2. Using the functional integral formalism calculate explicitly the two- and four-point functions for the case of a free complex scalar field. You should also explicitly justify the form of the corresponding generating functional. Write the form of the generic non-zero N-point function and justify your answer.

3. Problem 9.1 of Peskin and Schroeder's book.