The Standar Model of Particle Physics Lecture II

Radiative corrections, renormalization, and physical observables

Laura Reina

Maria Laach School, Bautzen, September 2011

Outline of Lectures II

- Radiative corrections and Renormalization:
  - $\longrightarrow$ renormalization, general structure;
  - $\longrightarrow$ SM case: main results.

- Radiative corrections and physical observables:
  - $\longrightarrow$ precision tests: testing consistency and constraining unknowns
    - $\hookrightarrow$ lepton collider dominated (LEP)
  - $\longrightarrow$ obtain best estimate of SM processes for searches of new physics
    - $\hookrightarrow$ hadron collider dominated (Tevatron, LHC)

$\downarrow$

This last point will be expanded in Lectures III and IV

Systematic of renormalization, in a nutshell

Simplest case: scalar $(g\phi^4)$ theory $\to \mathcal{L}(\phi_0, \partial_\mu\phi_0, m_0, g_0)$

$$\Downarrow$$

Calculating scattering amplitudes $\langle f|\phi\ldots\phi|i\rangle$ via perturbative approach introduce divergencies beyond the tree level:

$\longrightarrow$ ultraviolet (UV): in the $p^2 \to \infty$ region of momentum loop integrals, ex.:

$$\int \frac{d^4p}{p^2(p^2-m^2)} \overset{p\to\infty}{\approx} \int \frac{dp}{p} \quad \to \quad \text{log.divergence}$$

$\longrightarrow$ infrared (IR): in both loop and real corrections, due to soft $(p^2 \to 0)$ or collinear $(p \cdot p_i \to 0)$ radiation/loop-momenta;

$$\int \frac{d^4p}{p^2(p^2+2p\cdot q_1)(p^2+2p\cdot q_2)} \overset{p\to 0}{\approx} \int \frac{dp}{p} \quad \to \quad \text{log.divergence}$$

Actions taken: at a given perturbative order,

⟶ regularize and extract UV and IR singularities, most common:

$$\int d^4p \rightarrow \mu^{4-d} \int d^dp \quad \text{dimensional regularization}$$

- ▷ divergencies extracted as poles in $(4-d)$.
- ▷ $\mu \rightarrow$ (renormalization) scale parameter associated to regularization procedure

⟶ cancel UV singularities by switching from "bare" to "renormalized" parameters/fields, fixed by suitable renormalization conditions,

$$\mathcal{L}(\phi_0, \partial_\mu \phi_0, m_0, g_0) \longrightarrow \mathcal{L}(\phi, \partial_\mu \phi, m, g)$$

where

$$m_0^2 = m^2 + \delta m^2 \quad \phi_0 = \sqrt{Z_\phi}\phi \quad g_0 = g + \delta g$$

and m, ϕ and g (alternatively δm^2, $Z_\phi = 1 + \delta Z_\phi$, δg) are defined by fixing the renormalized proper vertices (or vertex functions) of the theory.

⟶ cancel IR singularities in the sum of virtual+real corrections (only hard radiation can be resolved).

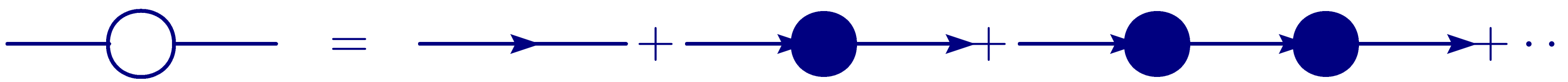
UV systematics: consider the proper vertices of the theory $\Gamma^{\phi\cdots\phi}$

$\Gamma^{\phi\cdots\phi} \rightarrow$ one-particle irreducible (1PI) diagrams with n external legs.

2-point proper vertex : $\Gamma^{\phi\phi}$

$$\Gamma^{\phi_0\phi_0} = i(p^2 - m_0^2) + i\Sigma(p^2)$$

Σ=self-energy=sum of 1PI (one-particle-irreducible) diagrams (all orders).
Notice relation with all orders propagator:



$$= \frac{i}{p^2 - m_0^2} + \frac{i}{p^2 - m_0^2} i\Sigma(p^2) \frac{i}{p^2 - m_0^2} + \cdots$$

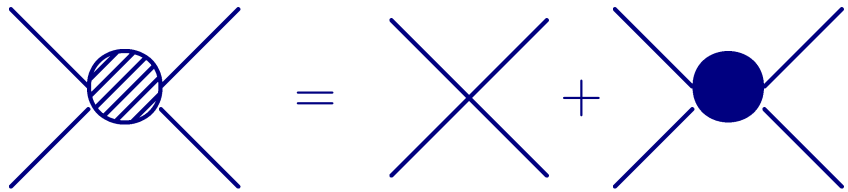
$$= \frac{i}{p^2 - m_0^2 + \Sigma(p^2)} = (\Gamma^{\phi_0\phi_0})^{-1}$$

In terms of bare parameters, $\Gamma^{\phi_0\phi_0}$ is UV divergent: fix δm^2 and δZ_ϕ by requiring, e.g., that $\Gamma^{\phi\phi} = Z_\phi^{-1}\Gamma^{\phi_0\phi_0}$ has a pole at the physical mass with unit residue, which gives:

$$\Sigma(m^2) = \delta m^2 \quad \text{and} \quad \Sigma'(m^2) = -\delta Z_\phi$$

4-point proper vertex: $\Gamma^{\phi\phi\phi\phi}$

$$\Gamma^{\phi_0\phi_0\phi_0\phi_0} \quad = \quad i g_0 \quad + i\Gamma(p_1, p_2, p_3)$$



In terms of bare parameters, $\Gamma^{\phi_0\phi_0\phi_0\phi_0}$ is UV divergent: fix δg by requiring, e.g., that g corresponds to the coupling measured in a specific kinematic realization, which gives:

$$\delta g = 2g\delta Z_\phi - \Gamma(p_1^{exp}, p_2^{exp}, p_3^{exp})$$

where we also use that $\Gamma^{\phi\phi\phi\phi} = Z_\phi^2 \Gamma^{\phi_0\phi_0\phi_0\phi_0}$.

All other n-point proper vertices, $\Gamma^{\phi\cdots\phi}$: are obtainable using $\Gamma^{\phi\phi}$ and $\Gamma^{\phi\phi\phi\phi}$ as building blocks, and finite when expressed in terms of m and g.
Notice: parameters are now scale-dependent, $m(q^2)$, $g(q^2)$.

Any physical observables calculated in terms of m and g is finite and well defined, although affected by a systematic (perturbative) uncertainty .

Standard Model renormalization: main results

The SM Lagrangian is made of renormalizable field structures,

$$\begin{aligned}\mathcal{L}_{SM} &= \mathcal{L}_{QCD} + \mathcal{L}_{EW} \\ &= \mathcal{L}_{EW}^{\rm ferm} + \mathcal{L}_{EW}^{\rm gauge} + \mathcal{L}_{EW}^{SSB} + \mathcal{L}_{EW}^{Yukawa}\end{aligned}$$

where,

$$\begin{aligned}\mathcal{L}_{QCD} &\rightarrow \bar{\psi}(\not{\partial} - m)\psi\,,\ \bar{\psi}\not{A}\psi\,,\ \frac{1}{4}G^{a,\mu\nu}G^{a}_{\mu\nu} \\ \mathcal{L}_{EW}^{\rm ferm} &\rightarrow \bar{\psi}_L(\not{\partial})\psi_L\,,\ \bar{\psi}_L\not{V}\psi_L \\ \mathcal{L}_{EW}^{\rm gauge} &\rightarrow \frac{1}{4}F^{a,\mu\nu}F^{a}_{\mu\nu}\,,\ \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ \mathcal{L}_{EW}^{SSB} &\rightarrow \partial^{\mu}\phi\partial_{\mu}\phi,\ \mu^2\phi^2,\ \phi^4 \\ \mathcal{L}_{EW}^{Yukawa} &\rightarrow \bar{\psi}_L H \psi_R\end{aligned}$$

The systematic procedure outlined in these lectures will apply with extra constraints imposed by the presence of a partially spontaneously broken gauge symmetry.

The set of fundamental parameters of the SM Lagrangian is:

$$g_{s,0}\ ,\ g_0\ ,\ g'_0\ ,\ \mu_0\ ,\ \lambda_0\ ,\ y_{f,0}\ ,\ V_0^{ij}$$

here taken as bare parameters. Thanks to relations induced by the symmetries of the theory, e.g.

$$e = g\sin\theta_W = g'\cos\theta_W \quad\rightarrow\quad e = \frac{gg'}{\sqrt{g^2+g'^2}}$$

$$M_W = \frac{gv}{2}\ ,\quad M_Z = \frac{v\sqrt{g^2+g'^2}}{2} \quad\rightarrow\quad \frac{M_W}{M_Z} = \frac{g}{\sqrt{g^2+g'^2}} = \frac{e}{g'} = \cos\theta_W$$

we can trade them for other or "better" sets of input parameters, for example:

$$g_{s,0}\ ,\ e_0\ ,\ M_{W,0}\ ,\ M_{Z,0}\ ,\ M_{H,0}\ ,\ m_{f,0}\ ,\ V_0^{ij}$$

and switch to the corresponding set of renormalized or physical parameters upon imposing suitable renormalization conditions.

$\Rightarrow$ Relations like $M_W/M_Z = \cos\theta_W$ will automatically be finite but corrections depend on input parameters (e.g. m_t, M_H) $\rightarrow$ natural relations. Need to specify renormalization scheme and use consistency.

Definitions and renormalization conditions

QCD → in the absence of a mass scale, use $\overline{\rm MS}$ scheme or minimal subtraction scheme, i.e. subtract just pole parts of each divergent proper vertex.

EW → use procedure illustrated in this lecture for a scalar $g\phi^4$ toy model → on-shell subtraction scheme.

- mass/coupling renormalization:

$$M_{W,0}^2 = M_W^2 + \delta M_W^2\,,\ \ldots\,,\ m_{f,0} = m_f + \delta m_f\,,\ V_0^{ij} = V^{ij} + \delta V^{ij}$$

- field renormalization:

$$W_0^{\pm} = \sqrt{Z_W} W^{\pm}\,,\ \begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{ZZ}} & \sqrt{Z_{ZA}} \\ \sqrt{Z_{AZ}} & \sqrt{Z_{AA}} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix} \ldots$$

where, the following renormalization conditions are traditionally adopted:

$$\delta M_W^2 = \mathrm{Re}[\Sigma_{\rm T}^{\rm W}(\mathrm{M_W^2})]\ ,\ \ \delta Z_W = -\mathrm{Re}[\Sigma_T^{W\prime}(M_W^2)]\,,\ \ldots$$

and similar ones for other vector+scalar and field renormalization constants ⇒ the bulk of corrections are in the self-energies!

Flavor sector: need to carefully account for the rotation to mass eigenstates (beyond the scope of these lectures).

Finally, the QED electric charge renormalization condition is adopted: e defined as measure in the Thomson limit ($k \to 0$ scattering of photons, on-shell electrons)

$$\alpha(0) = \frac{e^2}{2\pi} \approx \frac{1}{137}$$

Once expressed in terms of the renormalized parameters and fields, any physical observable is finite and can be calculated at the proper perturbative order in QCD+EW and compared with experimental results.

$$\Downarrow$$

Electroweak precision fits

Electroweak precision fits

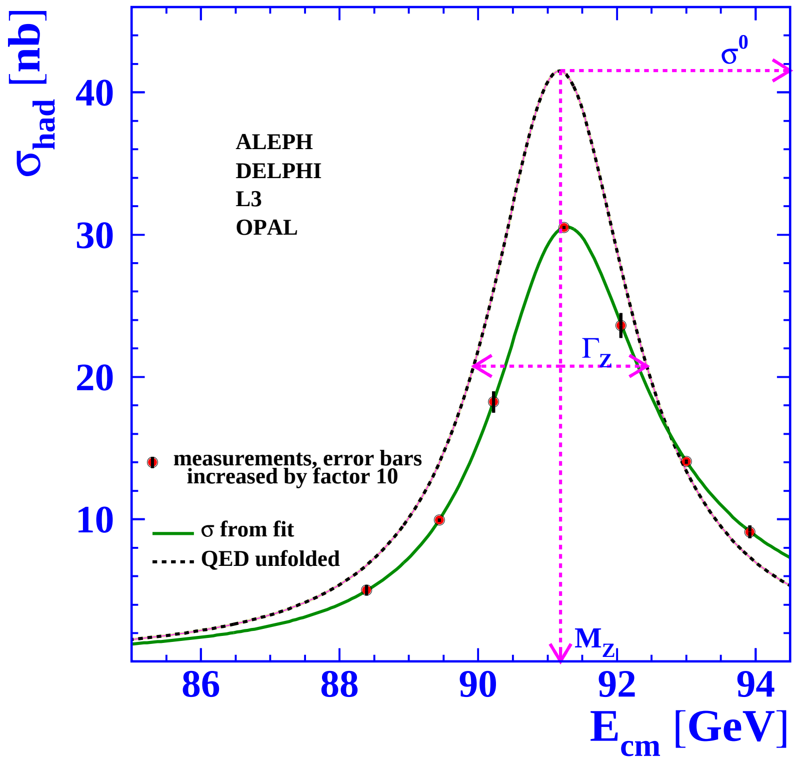
An incredible amount of measurements of electroweak observables have been collected over the past many decades:

- $Sp\bar{p}S$ at CERN (1981-1990) 300×300 GeV $p\bar{p}$ collider: discovery of W and Z bosons!
- LEP I at CERN (1989-1995) at energies around $\sqrt{s} = M_Z$: $e^+e^- \to Z \to f\bar{f}$.
- LEP II at CERN (1996-2000) at energies around $\sqrt{s} = 200 - 208$ GeV: $e^+e^- \to WW \to 4f$.
- SLC at SLAC (1989-1998) at energies up to 100 GeV, polarized beams.
- Tevatron at Fermilab, RUN I+II (1987-2011) 0.98×0.98 TeV $p\bar{p}$ collider: top-quark discovery! Precision measurement of M_W and m_t.
- LHC at CERN, now running at 3.5×3.5 TeV, will go up to 7×7 TeV: will rediscovery the SM and more!

Measurement of M_Z and Γ_Z at LEP I

At the Z pole $e^+e^- \to f\bar{f}$ $(f \neq e)$ dominated by Z exchange:

$$\frac{d\sigma_Z^f}{d\Omega} = \frac{9}{4} \frac{s\Gamma_{ee}\Gamma_{f\bar{f}}/M_Z^2}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} \left[(1 + \cos^2\theta)(1 - P_e A_e) + 2\cos\theta A_f(-P_e + A_e)\right]$$



P_e: polarization of electron beam

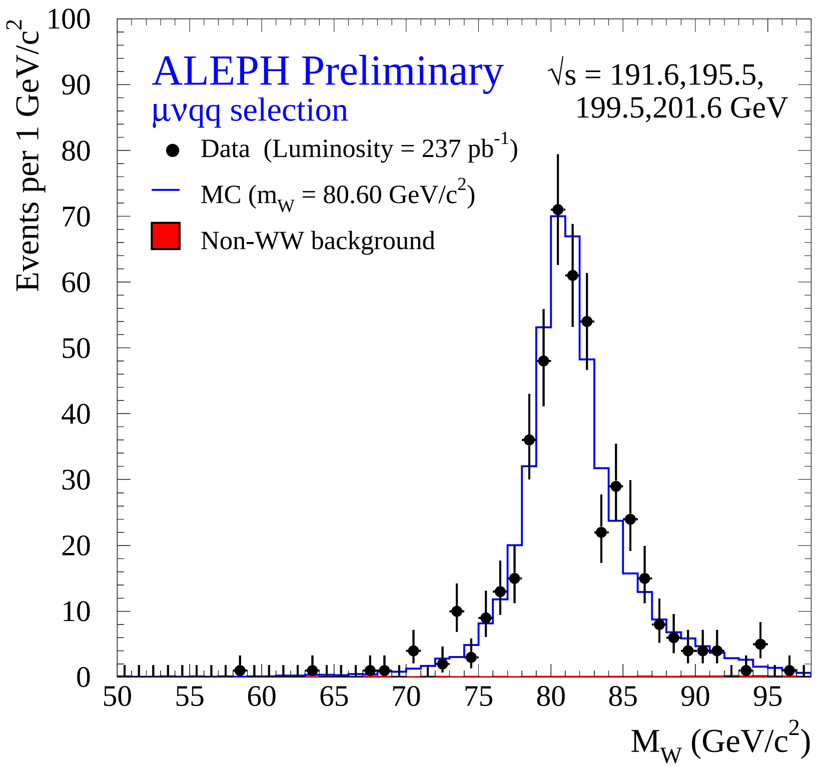
$\Gamma_{ee}, \Gamma_{f\bar{f}}$: partial widths for $Z \to e^+e^- f\bar{f}$

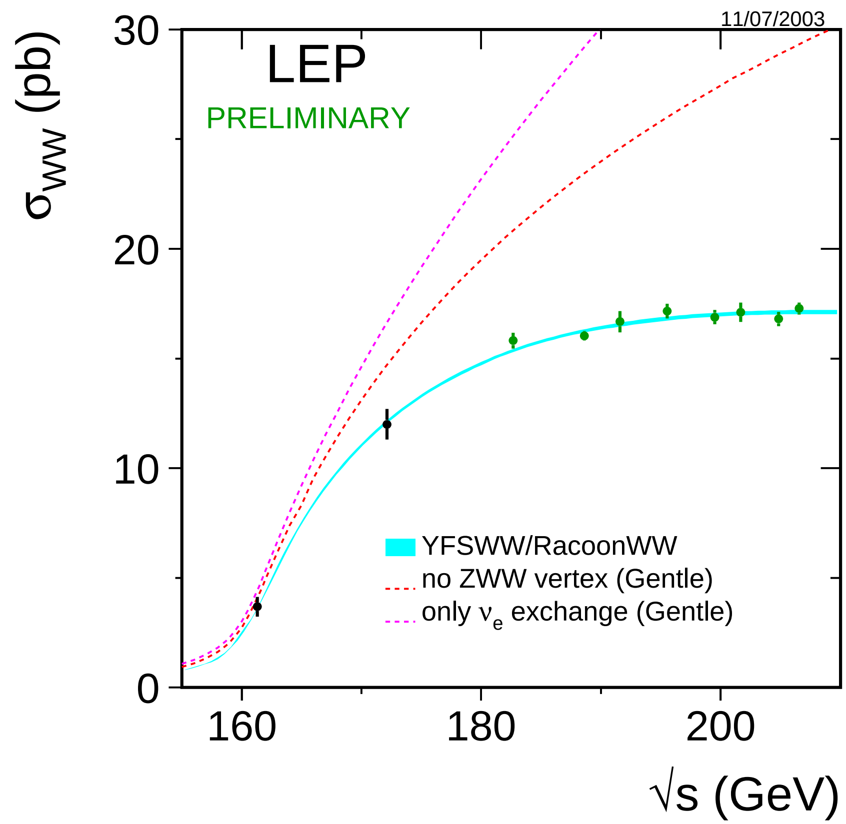
A_f: L-R coupling constant asymmetry

$$A_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2} = \frac{2g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}$$

Scanning at the Z pick and fitting to σ_Z^f yields measurements of M_Z, Γ_Z and $\sigma_{\text{had}} = 12\pi\Gamma_{ee}\Gamma_{\text{had}}/(M_Z^2\Gamma_Z^2)$.

Measurement of M_W at LEP II

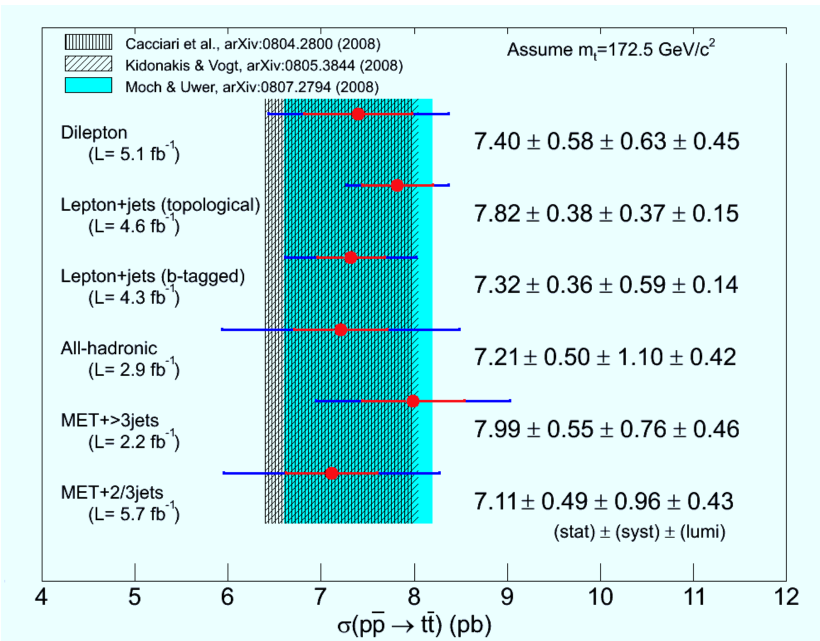


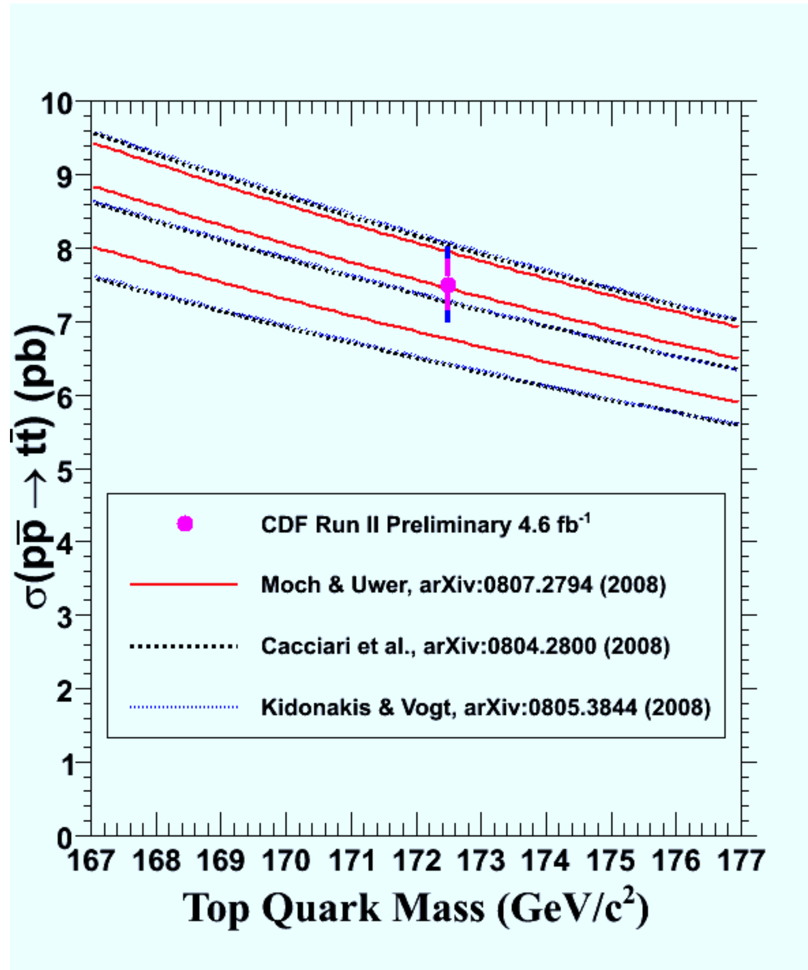


From W invariant-mass reconstruction: t-channel ν-exchange, s-channel γ- and Z-exchange $\rightarrow$ test of $3V$-gauge coupling.

From rise of the WW cross section near threshold (statistically limited) but till another test of non-abelian interactions.

Measurement of m_t at the Tevatron





New measurement from LHC should improve precision.

Strategy

Having a variety of measurement for different observables, test the SM by comparing theory and experiment.

- Pick a set of input parameters, typical choice:

$$\alpha_s\,,\,\alpha\,,\,G_F\,,\,M_Z\,,\,M_H\,,\,m_t\,,\,m_f,\ldots$$

- Compute theoretical predictions, including radiative corrections, in a given renormalization scheme treating the best measured parameters as inputs (α, G_F, fermion masses except m_t and m_c), i.e. as fixed parameters.

- Perform a best fit to the electroweak data, defined by a χ^2 test

$$\chi^2(\alpha, G_F,\ldots) = \sum_i \frac{(\hat{O}_i^{\rm exp} - O_i^{\rm th}(\alpha, G_F,\ldots))^2}{(\Delta\hat{O}_I^{\rm exp})^2}$$

  This results in a best fit of the non-fixed or floating parameters. Compare best-fit values to measurements if available (ex.: M_W, m_t, α_s, not M_H!)

- For the best-fit values of <u>all</u> input parameters, quote the SM theoretical prediction for each observable and compare with the experimental measurements. "Tensions" may signal new physics ...

Fine structure constant, α

Measured at low energies,

$$\alpha \equiv \frac{e^2(0)}{4\pi} = \frac{e_0^2}{4\pi(1-\Delta\alpha(0))} = \frac{1}{137.03599890(50)}$$

then evolved to M_Z:

$$\alpha_e(M_Z) = \frac{\alpha}{1-\Delta\alpha(M_Z)}$$

$\Delta\alpha \rightarrow$ QED and (2-loop) QCD contributions ($\Delta\alpha_{had}^{(5)}$).

Uncertainties from: h.o. perturbative and nonperturbative corrections, light quark masses (mainly m_c), lack of data below 1.8 GeV, slight disagreement in extraction of $\Delta\alpha_{had}^{(5)}$.

Fermi constant, G_F (or G_μ) From muon lifetime:

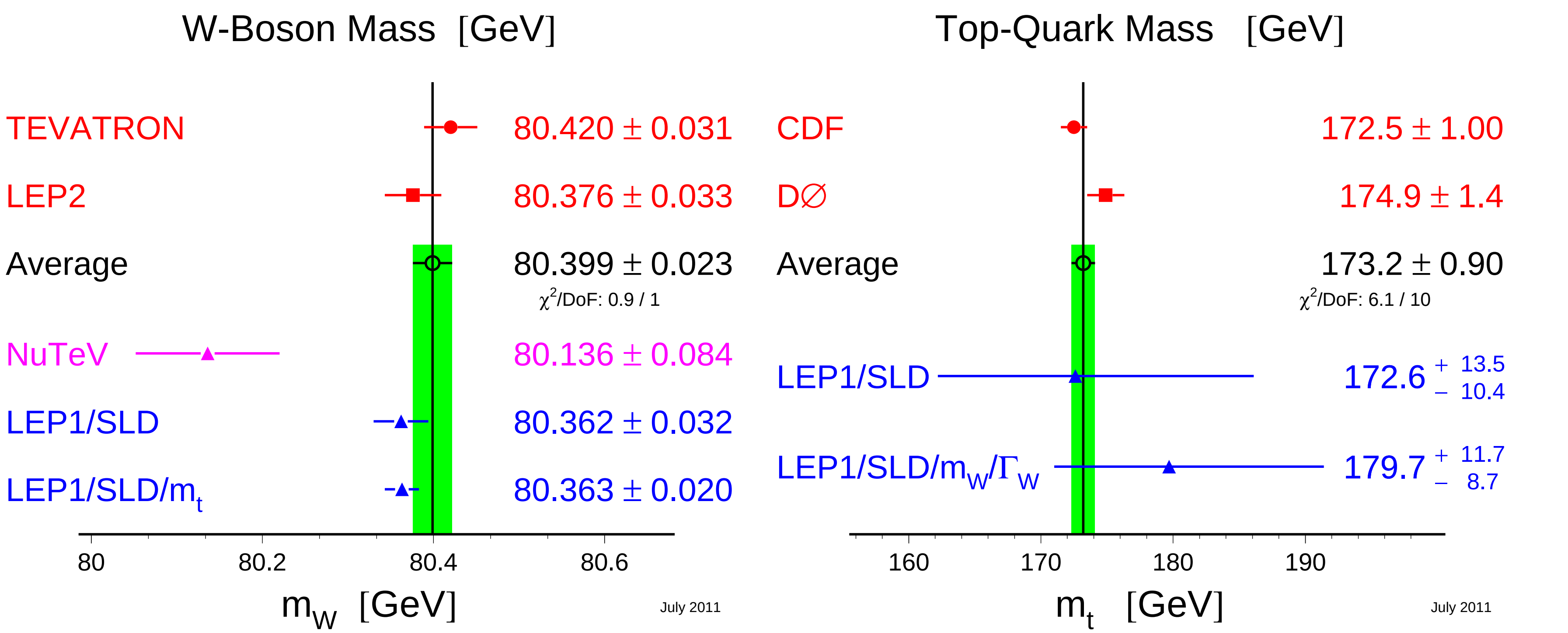
$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1+\frac{3}{5}m_\mu^2 M_W^2\right)\left[1+\left(\frac{25}{8}-\frac{\pi^2}{2}\right)\frac{\alpha(m_\mu)}{\pi} + O\left(\alpha^2\right)\right]$$

Measure:

$$G_F = 1.16637(1)\times 10^{-5}\,\mathrm{GeV}^{-2}$$

uncertainty: almost all from residual experimental error.

Example of best fit of floating parameters



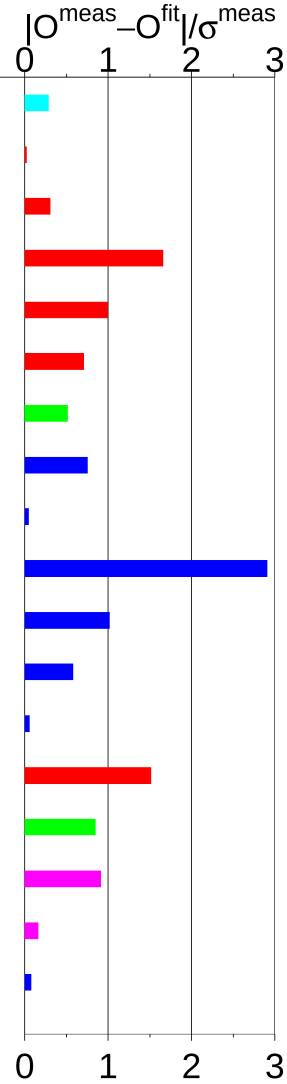
All following plots from:

The LHC Electroweak Working Group

(http://lepewwg.web.cern.ch/LEPEWWG/)

Summary of various "pulls": theory vs experiment

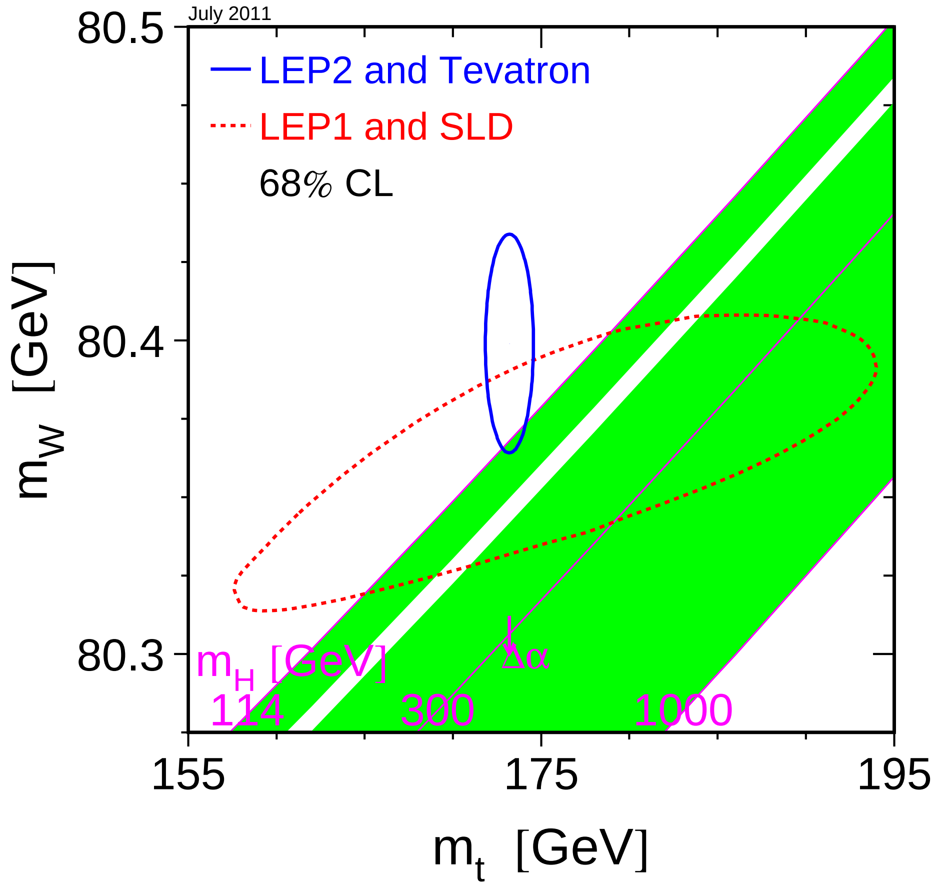
	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01646
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1482
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1039
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0743
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1482
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.399 ± 0.023	80.378
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.27



July 2011

SM Higgs-boson mass range: constrained by EW precision fits

Increasing precision will continue to provide an invaluable tool to test the consistency of the SM and its extensions.



$$m_W = 80.399 \pm 0.023 \text{ GeV}$$

$$m_t = 173.2 \pm 0.90 \text{ GeV}$$

$$\Downarrow$$

$$M_H = 92^{+24}_{-26} \text{ GeV}$$

$$M_H < 161\,(185) \text{ GeV}$$

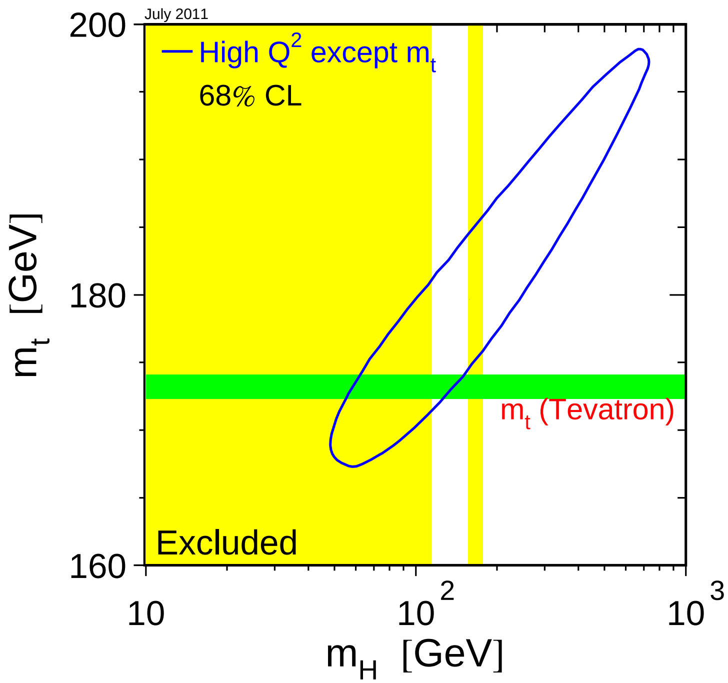
plus exclusion limits (95% c.l.):

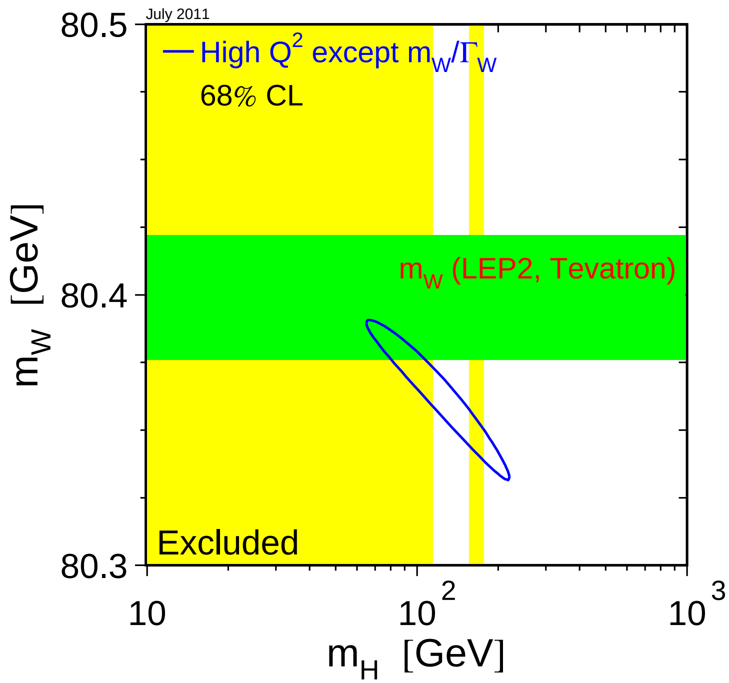
$$M_H > 114.4 \text{ GeV (LEP)}$$

$$M_H \neq 156 - 177 \text{ GeV (Tevatron)}$$

focus is now on exclusion limits and discovery!

Disentangling $m_t - M_H$ and $M_W - M_H$ correlations





$$
\begin{aligned}
M_W/(\mathrm{GeV}) &= 80.409 - 0.507\left(\frac{\Delta\alpha_h^{(5)}}{0.02767} - 1\right) + 0.542\left[\left(\frac{m_t}{178\,\mathrm{GeV}}\right)^2 - 1\right] \\
&- 0.05719\ln\left(\frac{M_H}{100\,\mathrm{GeV}}\right) - 0.00898\ln^2\left(\frac{M_H}{100\,\mathrm{GeV}}\right)
\end{aligned}
$$

A. Ferroglia, G. Ossola, M. Passera, A. Sirlin, PRD 65 (2002) 113002

W. Marciano, hep-ph/0411179