

The Standard Model of Particle Physics

Lecture III

Probing the Higgs mechanism of electroweak symmetry breaking

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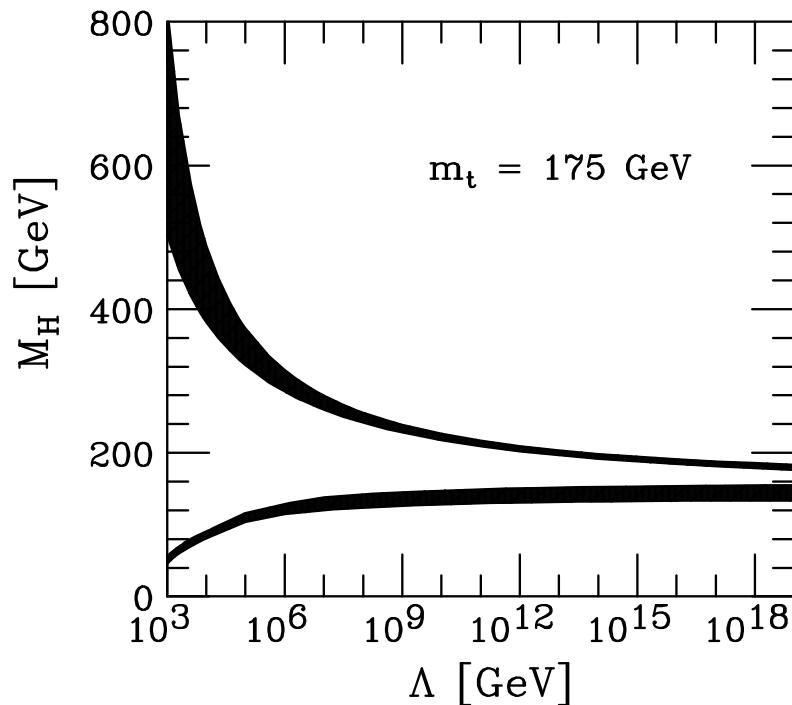
Maria Laach School, Bautzen, September 2011

Outline of Lecture III

- SM Higgs so constrained that it can point to scale of new physics.
 - constrain from precision fits (see [Lecture II](#));
 - validity of the SM up to a scale Λ translate into bounds on M_H (and viceversa $M_H \rightarrow \Lambda$).
- Looking for a SM Higgs boson at hadron colliders:
 - parton level production processes;
 - branching ratios;
 - Tevatron Higgs physics program;
 - LHC Higgs physics program; ([→ see S Caron's lecture this morning](#))
 - preparing the ground for tomorrow's lecture: overview of inclusive theoretical predictions;
 - what we haven't discussed ...
- SM Higgs physics at linear colliders: brief glance

Theoretical constraints on M_H in the Standard Model

SM as an effective theory valid up to a scale Λ . The Higgs sector of the SM actually contains two unknowns: M_H and Λ .



Bounds given by:

- unitarity
- triviality
- vacuum stability
- fine tuning

$M_H^2 = 2\lambda v^2$ → M_H determines the weak/strong coupling behavior of the theory, i.e. the limit of validity of the perturbative approach.

Unitarity: longitudinal gauge boson scattering cross section at high energy grows with M_H .

Electroweak Equivalence Theorem:
in the high energy limit ($s \gg M_V^2$)

$$\mathcal{A}(V_L^1 \dots V_L^n \rightarrow V_L^1 \dots V_L^m) = (i)^n (-i)^m \mathcal{A}(\omega^1 \dots \omega^n \rightarrow \omega^1 \dots \omega^m) + O\left(\frac{M_V^2}{s}\right)$$

(V_L^i =longitudinal weak gauge boson; ω^i =associated Goldstone boson).

Example: $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$

$$\mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) \sim -\frac{1}{v^2} \left(-s - t + \frac{s^2}{s - M_H^2} + \frac{t^2}{t - M_H^2} \right)$$

$$\mathcal{A}(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) = -\frac{M_H^2}{v^2} \left(\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right)$$

⇓

$$\mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \mathcal{A}(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) + O\left(\frac{M_W^2}{s}\right)$$

Using partial wave decomposition:

$$\mathcal{A} = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) a_l$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{A}^2| \rightarrow \sigma = \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1) |a_l|^2 = \frac{1}{s} \text{Im} [\mathcal{A}(\theta = 0)]$$

↓

$$|a_l|^2 = \text{Im}(a_l) \rightarrow |\text{Re}(a_l)| \leq \frac{1}{2}$$

Most constraining condition for $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ from

$$a_0(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) = -\frac{M_H^2}{16\pi v^2} \left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log \left(1 + \frac{s}{M_H^2} \right) \right] \xrightarrow{s \gg M_H^2} -\frac{M_H^2}{8\pi v^2}$$

$$|\text{Re}(a_0)| < \frac{1}{2} \rightarrow M_H < 870 \text{ GeV}$$

Best constraint from coupled channels ($2W_L^+ W_L^- + Z_L Z_L$):

$$a_0 \xrightarrow{s \gg M_H^2} -\frac{5M_H^2}{32\pi v^2} \rightarrow M_H < 780 \text{ GeV}$$

Observe that: if there is no Higgs boson, i.e. $M_H \gg s$:

$$a_0(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) \xrightarrow{M_H^2 \gg s} -\frac{s}{32\pi v^2}$$

Imposing the unitarity constraint $\rightarrow \boxed{\sqrt{s_c} < 1.8 \text{ TeV}}$

Most restrictive constraint $\rightarrow \boxed{\sqrt{s_c} < 1.2 \text{ TeV}}$

\Downarrow

New physics expected at the TeV scale

Triviality: a $\lambda\phi^4$ theory cannot be perturbative at all scales unless $\lambda=0$.

In the SM the scale evolution of λ is more complicated:

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2g^2 + \frac{9}{8}g^4 - 24y_t^4 + \dots$$

($t=\ln(Q^2/Q_0^2)$, $y_t = m_t/v \rightarrow$ top quark Yukawa coupling).

Still, for large λ (\leftrightarrow large M_H) the first term dominates and (at 1-loop):

$$\lambda(Q) = \frac{\lambda(Q_0)}{1 - \frac{3}{4\pi^2}\lambda(Q_0)\ln\left(\frac{Q^2}{Q_0^2}\right)}$$

when Q grows	\longrightarrow	$\lambda(Q)$ hits a pole \rightarrow triviality
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Imposing that $\lambda(Q)$ is finite, gives a scale dependent bound on M_H :

$\frac{1}{\lambda(\Lambda)} > 0 \quad \longrightarrow \quad M_H^2 < \frac{8\pi^2 v^2}{3 \log\left(\frac{\Lambda^2}{v^2}\right)}$
--

where we have set $Q \rightarrow \Lambda$ and $Q_0 \rightarrow v$.

Vacuum stability: $\boxed{\lambda(Q) > 0}$

For small λ (\leftrightarrow small M_H) the last term in $d\lambda/dt = \dots$ dominates and:

$$\lambda(\Lambda) = \lambda(v) - \frac{3}{4\pi^2} y_t^4 \log \left(\frac{\Lambda^2}{v^2} \right)$$

from where a first rough lower bound is derived:

$$\boxed{\lambda(\Lambda) > 0 \quad \longrightarrow \quad M_H^2 > \frac{3v^2}{2\pi^2} y_t^4 \log \left(\frac{\Lambda^2}{v^2} \right)}$$

More accurate analyses use 2-loop renormalization group improved V_{eff} .

Fine-tuning: M_H is unstable to ultraviolet corrections

$$M_H^2 = (M_H^0)^2 + \frac{g^2}{16\pi^2} \Lambda^2 \cdot \text{constant} + \text{higher orders}$$

$M_H^0 \rightarrow$ fundamental parameter of the SM

$\Lambda \rightarrow$ UV-cutoff scale

Unless $\Lambda \simeq$ EW-scale, fine-tuning is required to get $M_H \simeq$ EW-scale.

More generally, the all order calculation of V_{eff} would give:

$$\bar{\mu}^2 = \mu^2 + \Lambda^2 \sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/Q)$$

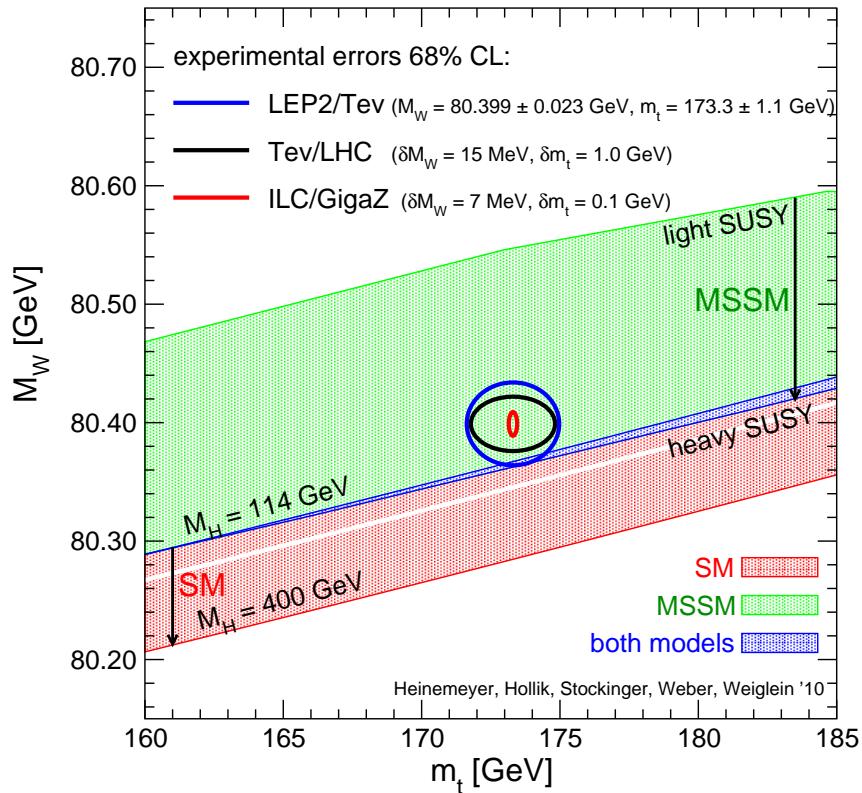
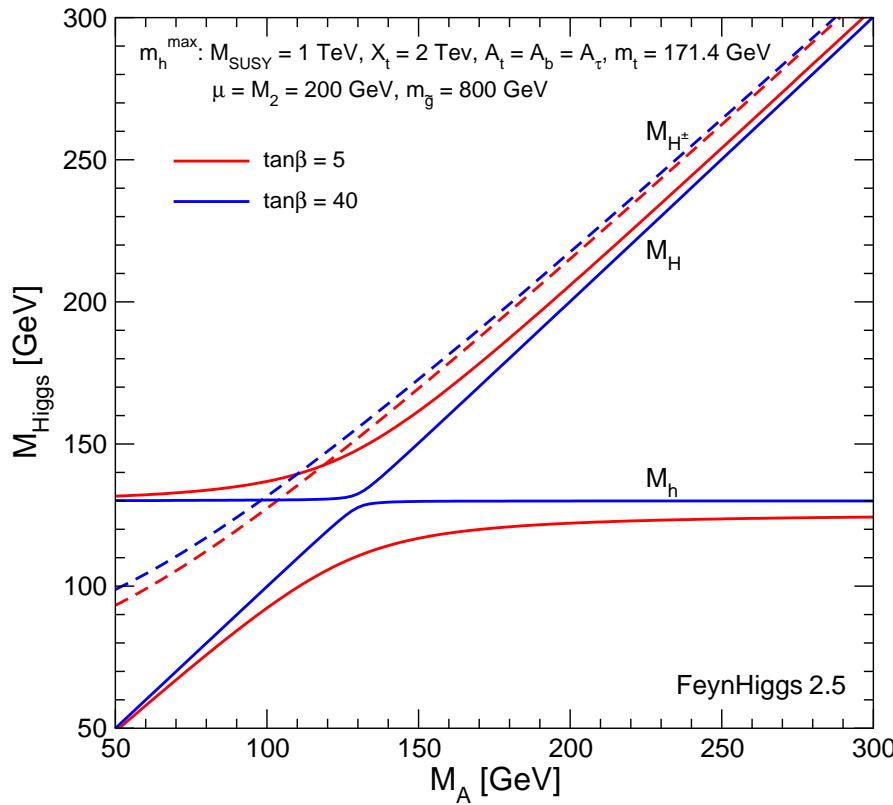
Veltman condition: the absence of large quadratic corrections is guaranteed by:

$$\sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/M_H) = 0 \quad \text{or better} \quad \sum_{n=0}^{n_{max}} c_n(\lambda_i) \log^n(\Lambda/M_H) < \frac{v^2}{\Lambda^2}$$

where: $n_{max} = 0, 1, 2 \rightarrow \Lambda \simeq 2, 15, 50 \text{ TeV.}$

Beyond SM: new physics at the TeV scale can be a better fit

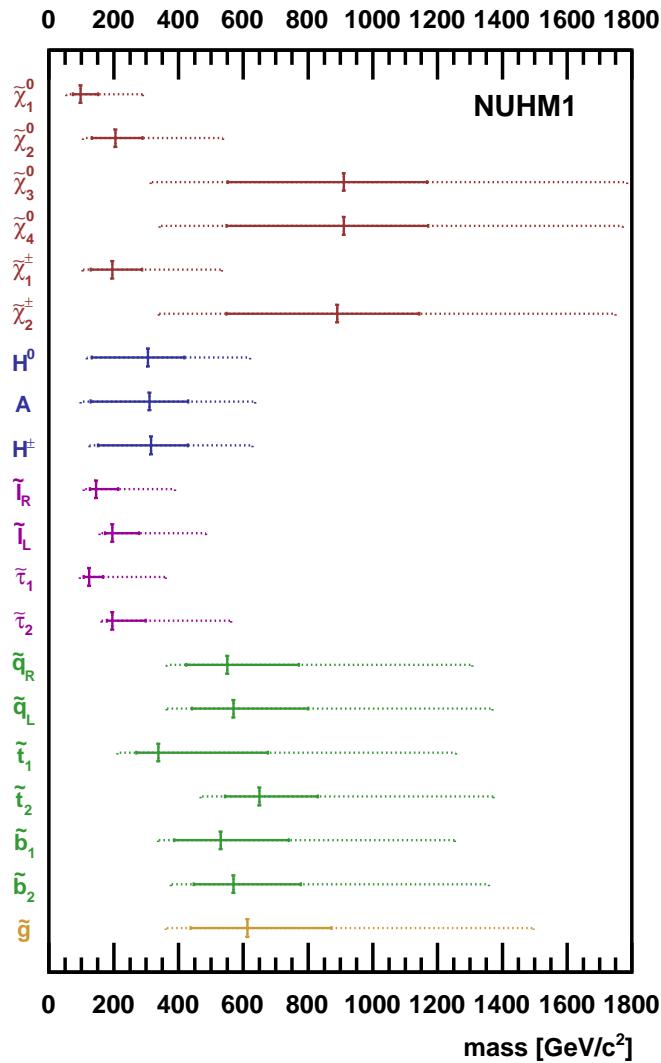
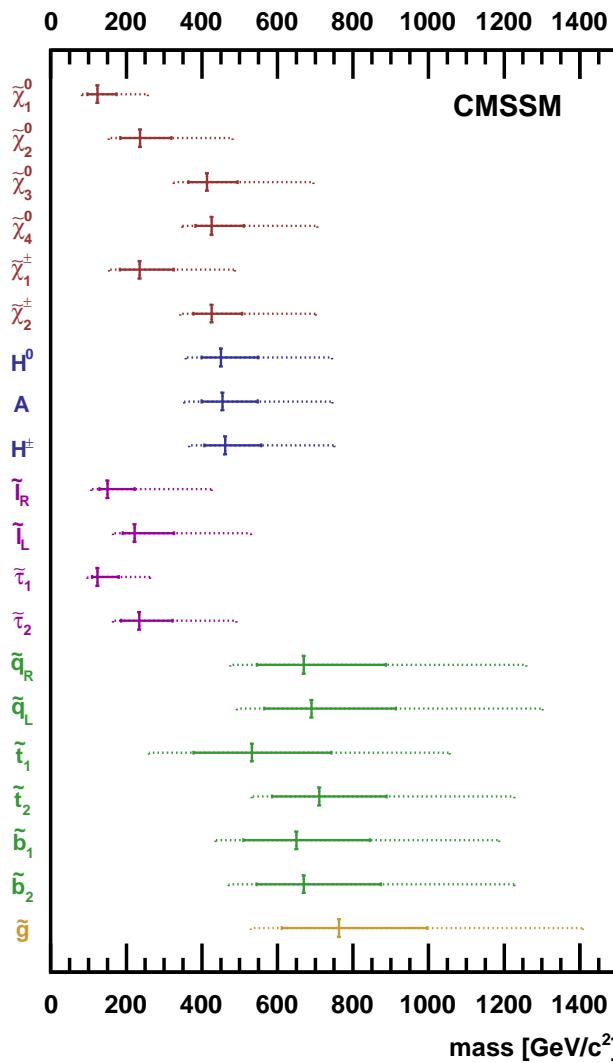
Example: MSSM



- ▷ a light scalar Higgs boson, along with a heavier scalar, a pseudoscalar and a charged scalar;
- ▷ similar although less constrained pattern in any 2HDM;
- ▷ MSSM main uncertainty: unknown masses of SUSY particles.
- ▷ precise measurement of mass spectrum and couplings will be crucial.

... mass spectrum at a glance: $\Lambda \sim \text{TeV}$

(MasterCode by Buchmüller et al., '09)



- ▷ CMSSM/NUHM1 (different choice of soft SUSY breaking mass terms);
- ▷ all available data (exp.) and all known corrections (th.) included in fit;
- ▷ most masses accessible to early LHC.

The Higgs bosons of the MSSM: example of 2HDM (some basics)

Two complex $SU(2)_L$ doublets, with hypercharge $Y = \pm 1$:

$$\Phi_u = \begin{pmatrix} \phi_u^+ \\ \phi_u^0 \end{pmatrix}, \quad \Phi_d = \begin{pmatrix} \phi_d^0 \\ \phi_d^- \end{pmatrix}$$

and (super)potential (Higgs part only):

$$\begin{aligned} V_H &= (|\mu|^2 + m_u^2)|\Phi_u|^2 + (|\mu|^2 + m_d^2)|\Phi_d|^2 - \mu B \epsilon_{ij} (\Phi_u^i \Phi_d^j + h.c.) \\ &+ \frac{g^2 + g'^2}{8} (|\Phi_u|^2 - |\Phi_d|^2)^2 + \frac{g^2}{2} |\Phi_u^\dagger \Phi_d|^2 \end{aligned}$$

The EW symmetry is spontaneously broken by choosing:

$$\langle \Phi_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}$$

normalized to preserve the SM relation: $M_W^2 = g^2(\textcolor{red}{v}_u^2 + v_d^2)/4 = g^2 \textcolor{red}{v}^2/4$.

Five physical scalar/pseudoscalar degrees of freedom:

$$\begin{aligned} h^0 &= -(\sqrt{2}\text{Re}\Phi_d^0 - v_d) \sin \alpha + (\sqrt{2}\text{Re}\Phi_u^0 - v_u) \cos \alpha \\ H^0 &= (\sqrt{2}\text{Re}\Phi_d^0 - v_d) \cos \alpha + (\sqrt{2}\text{Re}\Phi_u^0 - v_u) \sin \alpha \end{aligned}$$

$$A^0 = \sqrt{2} (\text{Im}\Phi_d^0 \sin \beta + \text{Im}\Phi_u^0 \cos \beta)$$

$$H^\pm = \Phi_d^\pm \sin \beta + \Phi_u^\pm \cos \beta$$

where $\boxed{\tan \beta = v_u/v_d}$.

All masses can be expressed (at tree level) in terms of $\boxed{\tan \beta \text{ and } M_A}$:

$$M_{H^\pm}^2 = M_A^2 + M_W^2$$

$$M_{H,h}^2 = \frac{1}{2} \left(M_A^2 + M_Z^2 \pm ((M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta)^{1/2} \right)$$

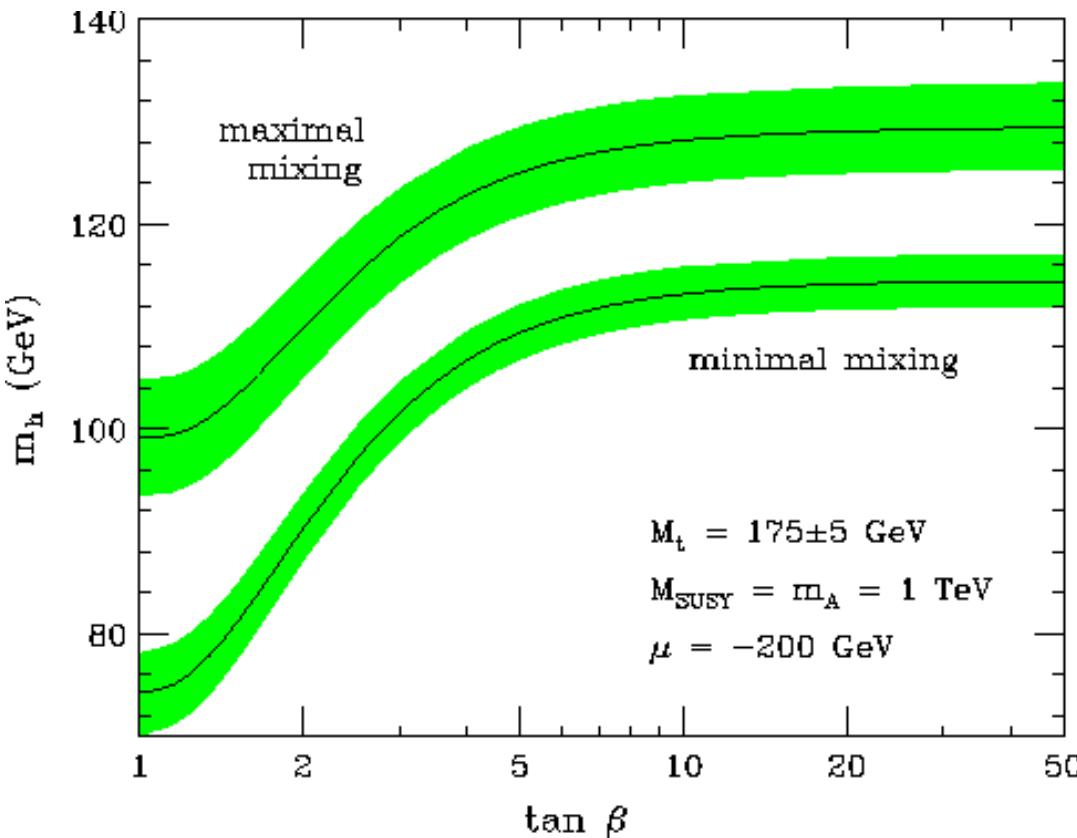
Notice: tree level upper bound on M_h : $\boxed{M_h^2 \leq M_Z^2 \cos 2\beta \leq M_Z^2}$!

Higgs masses greatly modified by radiative corrections.

In particular, the upper bound on M_h becomes:

$$M_h^2 \leq M_Z^2 + \frac{3g^2 m_t^2}{8\pi^2 M_W^2} \left[\log \left(\frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left(1 - \frac{X_t^2}{12M_S^2} \right) \right]$$

where $M_S \equiv (M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2)/2$ while X_t is the top squark mixing parameter:



$$\begin{pmatrix} M_{Q_t}^2 + m_t^2 + D_L^t & m_t X_t \\ m_t X_t & M_{R_t}^2 + m_t^2 + D_R^t \end{pmatrix}$$

with $X_t \equiv A_t - \mu \cot \beta$.

$$D_L^t = (1/2 - 2/3 \sin \theta_W) M_Z^2 \cos 2\beta$$

$$D_R^t = 2/3 \sin^2 \theta_W M_Z^2 \cos 2\beta$$

Higgs boson couplings to SM gauge bosons:

Some phenomenologically important ones:

$$g_{hVV} = g_V M_V \sin(\beta - \alpha) g^{\mu\nu} , \quad g_{HVV} = g_V M_V \cos(\beta - \alpha) g^{\mu\nu}$$

where $g_V = 2M_V/v$ for $V = W, Z$, and

$$g_{hAZ} = \frac{g \cos(\beta - \alpha)}{2 \cos \theta_W} (p_h - p_A)^\mu , \quad g_{HAZ} = -\frac{g \sin(\beta - \alpha)}{2 \cos \theta_W} (p_H - p_A)^\mu$$

Notice: $\boxed{g_{AZZ} = g_{AWW} = 0}$, $\boxed{g_{H^\pm ZZ} = g_{H^\pm WW} = 0}$

Decoupling limit: $\boxed{M_A \gg M_Z}$ $\rightarrow \begin{cases} M_h \simeq M_h^{max} \\ M_H \simeq M_{H^\pm} \simeq M_A \end{cases}$

$$\cos^2(\beta - \alpha) \simeq \frac{M_Z^4 \sin^2 4\beta}{M_A^4} \rightarrow \begin{cases} \cos(\beta - \alpha) \rightarrow 0 \\ \sin(\beta - \alpha) \rightarrow 1 \end{cases}$$

The only low energy Higgs is $h \simeq H_{SM}$.

Higgs boson couplings to quarks and leptons:

Yukawa type couplings, Φ_u to up-component and Φ_d to down-component of $SU(2)_L$ fermion doublets. Ex. (3rd generation quarks):

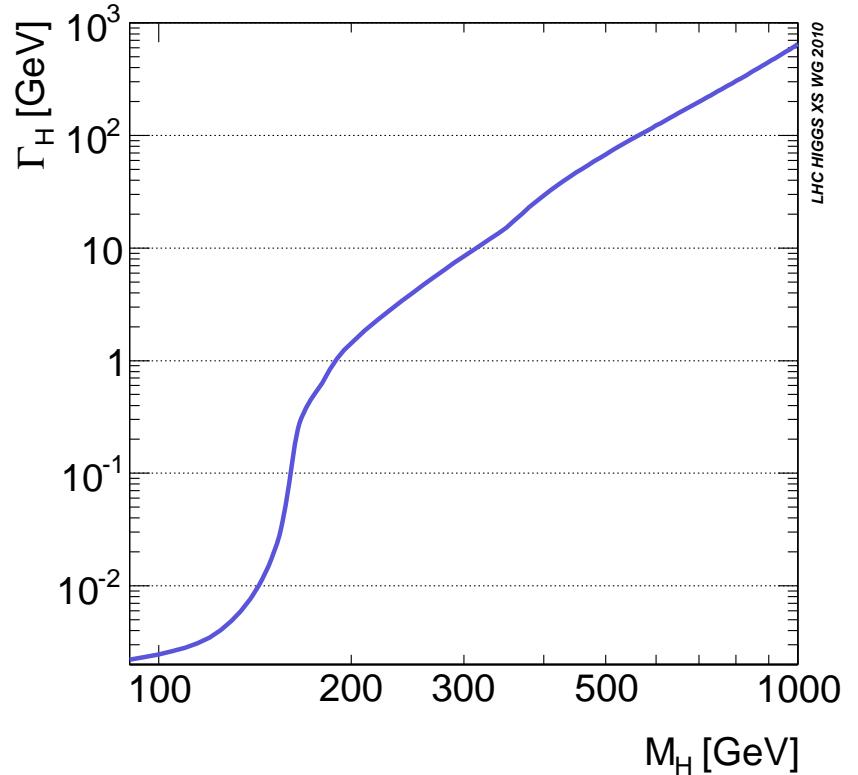
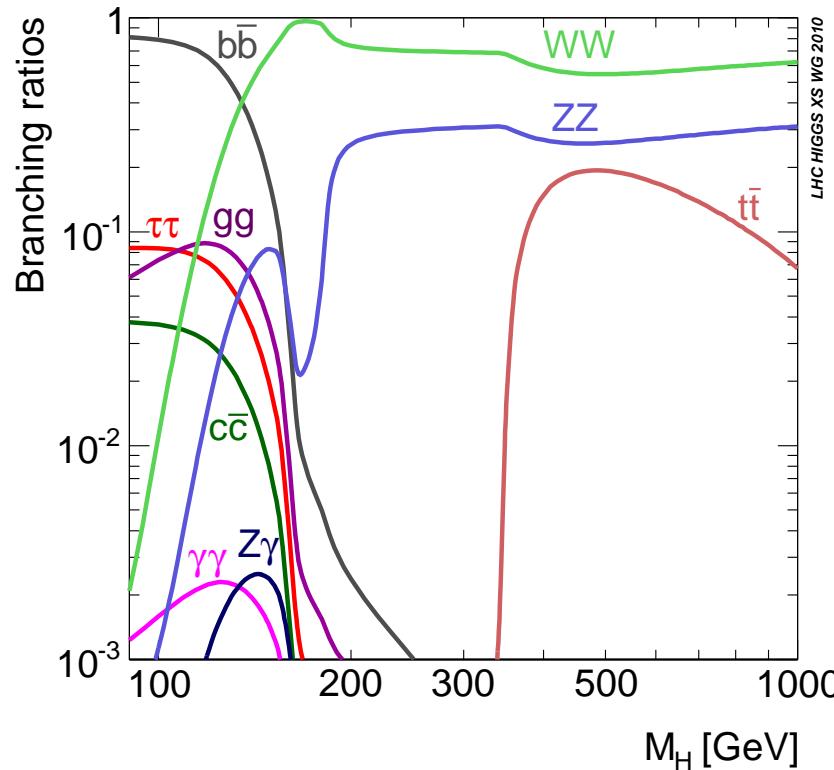
$$\mathcal{L}_{Yukawa} = h_t [\bar{t}P_L t\Phi_u^0 - \bar{t}P_L b\Phi_u^+] + h_b [\bar{b}P_L b\Phi_d^0 - \bar{b}P_L t\Phi_d^-] + \text{h.c.}$$

and similarly for leptons. The corresponding couplings can be expressed as ($y_t, y_b \rightarrow \text{SM}$):

$$\begin{aligned} g_{ht\bar{t}} &= \frac{\cos \alpha}{\sin \beta} y_t = [\sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha)] y_t \\ g_{hb\bar{b}} &= -\frac{\sin \alpha}{\cos \beta} y_b = [\sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha)] y_b \\ g_{Ht\bar{t}} &= \frac{\sin \alpha}{\sin \beta} y_t = [\cos(\beta - \alpha) - \cot \beta \sin(\beta - \alpha)] y_t \\ g_{Hb\bar{b}} &= \frac{\cos \alpha}{\cos \beta} y_b = [\cos(\beta - \alpha) + \tan \beta \sin(\beta - \alpha)] y_b \\ g_{At\bar{t}} &= \cot \beta y_t , \quad g_{Ab\bar{b}} = \tan \beta y_b \\ g_{H^\pm t\bar{b}} &= \frac{g}{2\sqrt{2}M_W} [m_t \cot \beta (1 + \gamma_5) + m_b \tan \beta (1 - \gamma_5)] \end{aligned}$$

Notice: consistent decoupling limit behavior.

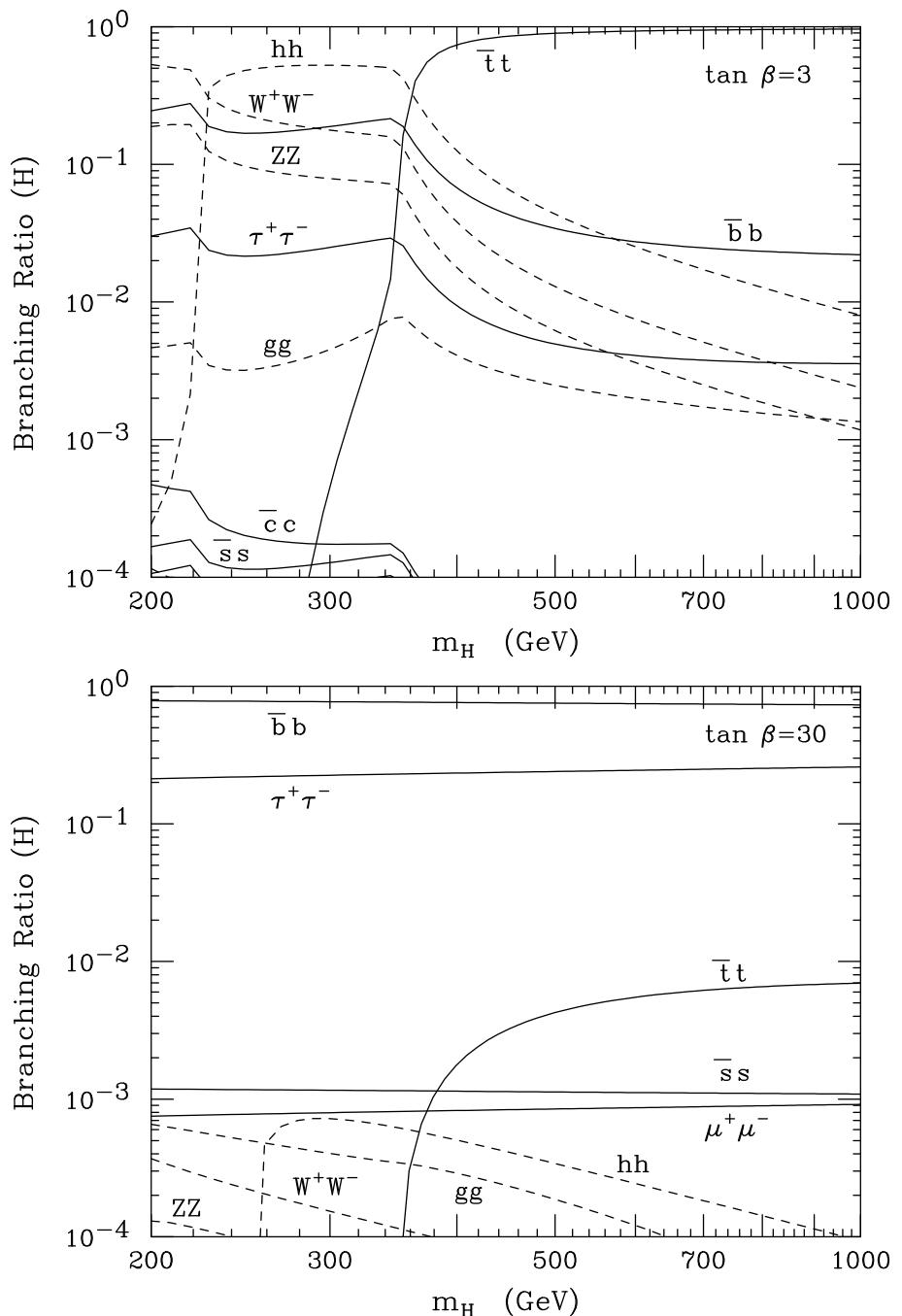
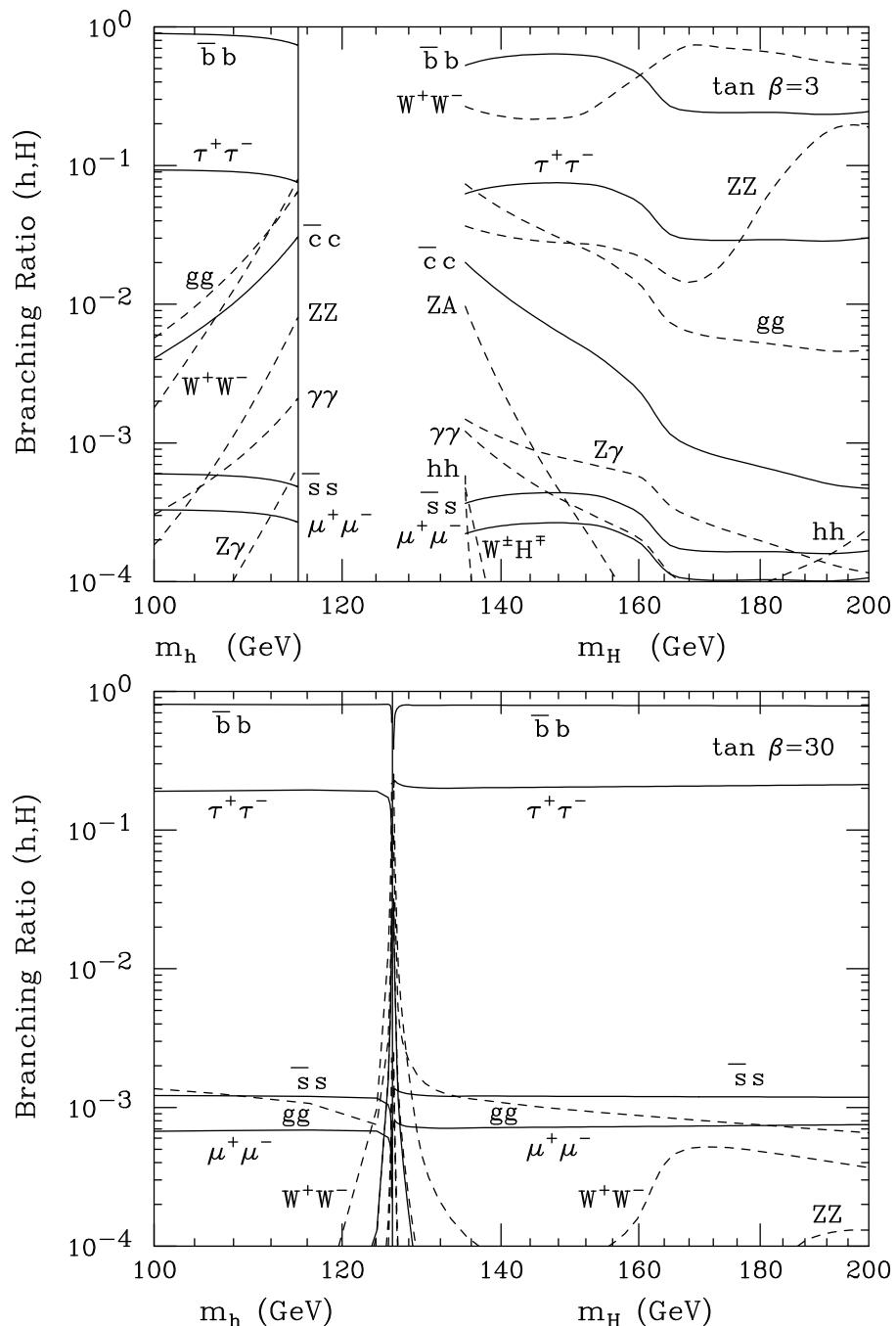
SM Higgs boson decay branching ratios and width



Observe difference between light and heavy Higgs

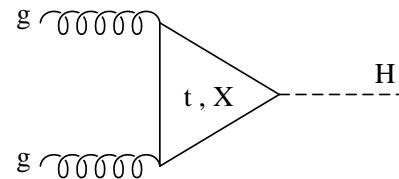
These curves include: tree level + QCD and EW loop corrections.

MSSM Higgs boson branching ratios, possible scenarios:

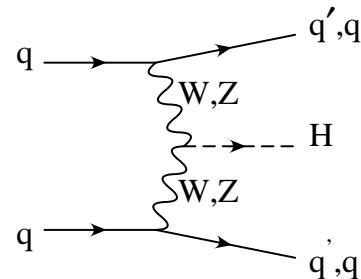


$p\bar{p}, pp$ colliders: SM Higgs production modes

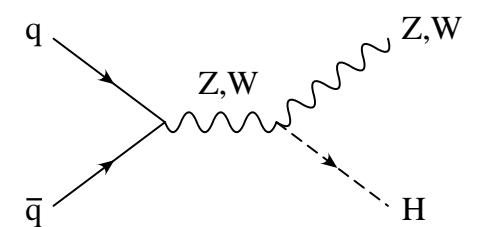
$gg \rightarrow H$



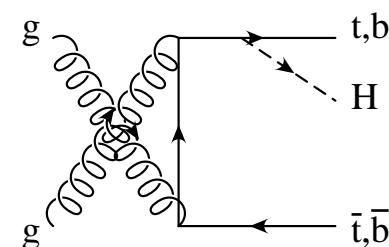
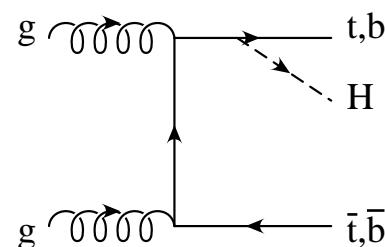
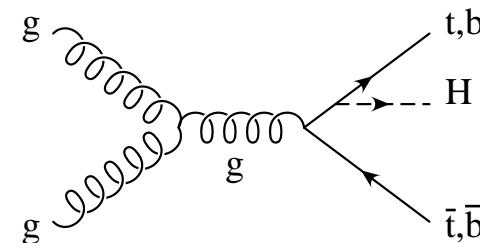
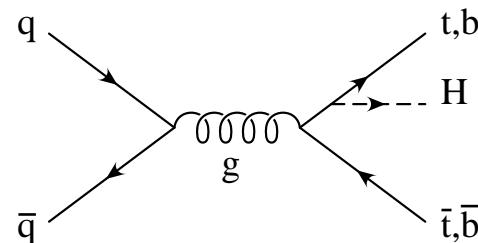
$qq \rightarrow qqH$



$qq \rightarrow WH, ZH$



$q\bar{q}, gg \rightarrow t\bar{t}H, b\bar{b}H$



Schematically . . .

The hard cross section is calculated perturbatively

$$\hat{\sigma}(ij \rightarrow X) = \alpha_s^k \sum_{m=0}^n \hat{\sigma}_{ij}^{(m)} \alpha_s^m$$

$n=0$: Leading Order (LO), or tree level or Born level

$n=1$: Next to Leading Order (NLO), include $O(\alpha_s)$ corrections

.....

and convoluted with initial state parton densities at the same order.

Renormalization and factorization scale dependence left at any fixed order.

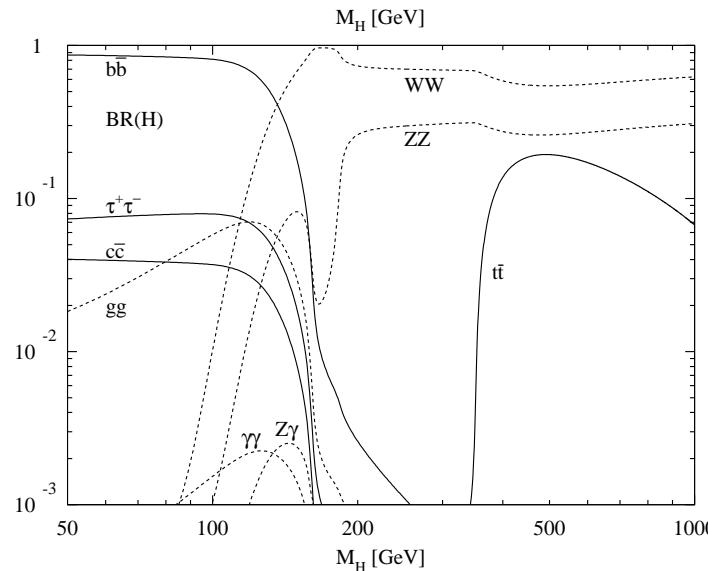
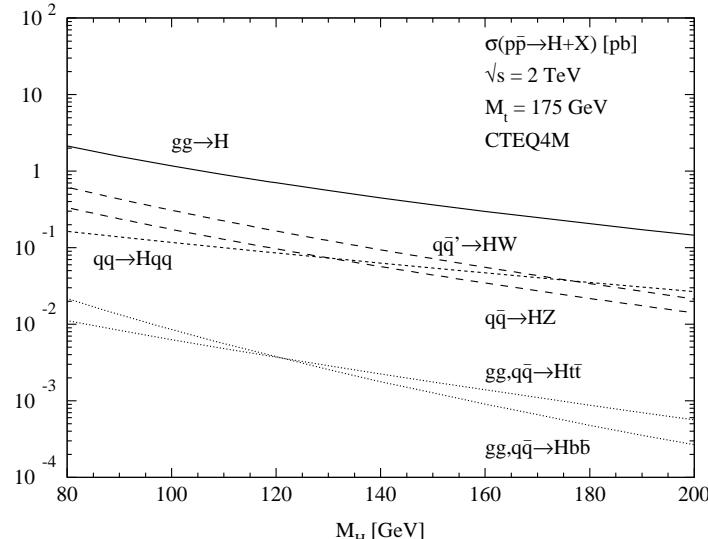
Setting $\boxed{\mu_R = \mu_F = \mu}$:

$$\sigma(pp, p\bar{p} \rightarrow X) = \sum_{ij} \int dx_1 dx_2 f_i^p(x_1, \mu) f_j^{p, \bar{p}}(x_2, \mu) \sum_{m=0}^n \hat{\sigma}_{ij}^{(m)}(\mu, Q^2) \alpha_s^{m+k}(\mu)$$

Systematic theoretical error from:

- ▷ PDF and $\alpha_s(\mu)$;
- ▷ left over scale dependence;
- ▷ input parameters.

Tevatron: great potential for a light SM-like Higgs boson



(M. Spira, Fortsch.Phys. 46 (1998) 203)

Lower mass region:

$$q\bar{q}' \rightarrow WH, H \rightarrow b\bar{b}$$

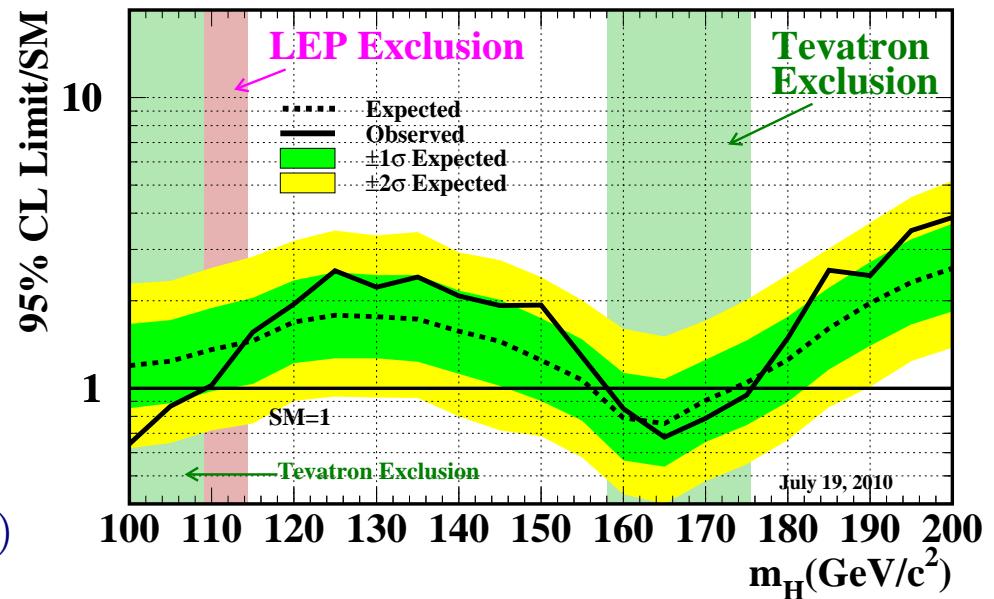
Higher mass region:

$$gg \rightarrow H, H \rightarrow W^+W^-$$

(smaller impact:

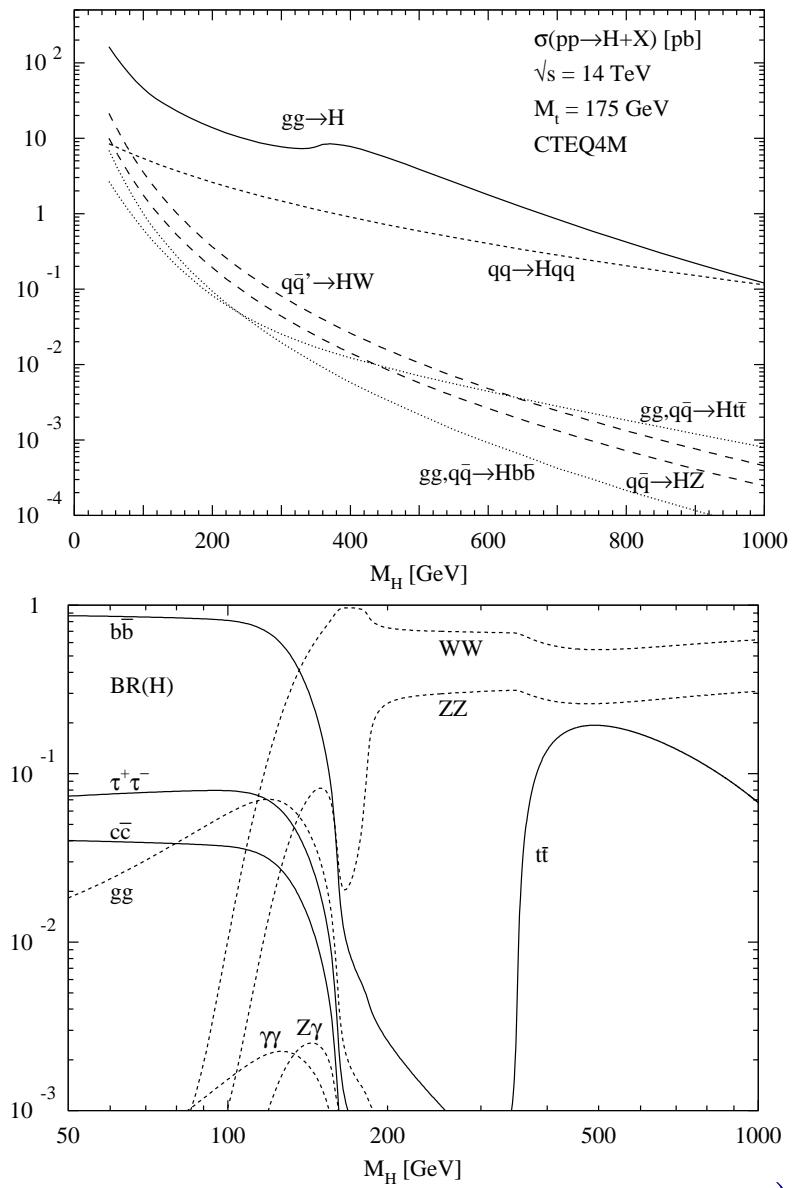
$$q\bar{q} \rightarrow q'\bar{q}'H, q\bar{q}, gg \rightarrow t\bar{t}H)$$

Tevatron Run II Preliminary, $\langle L \rangle = 5.9 \text{ fb}^{-1}$



→ Exclusion region very important for LHC search strategies.

LHC: entire SM Higgs-boson mass range accessible



(M. Spira, Fortsch.Phys. 46 (1998) 203)

Many channels have been studied:

Below 130-140 GeV:

- $gg \rightarrow H, H \rightarrow \gamma\gamma, WW, ZZ$
- $qq \rightarrow qqH, H \rightarrow \gamma\gamma, WW, ZZ, \tau\tau$
- $q\bar{q}, gg \rightarrow t\bar{t}H, H \rightarrow \gamma\gamma, b\bar{b}, \tau\tau$
- $q\bar{q}' \rightarrow WH, H \rightarrow \gamma\gamma, b\bar{b}$

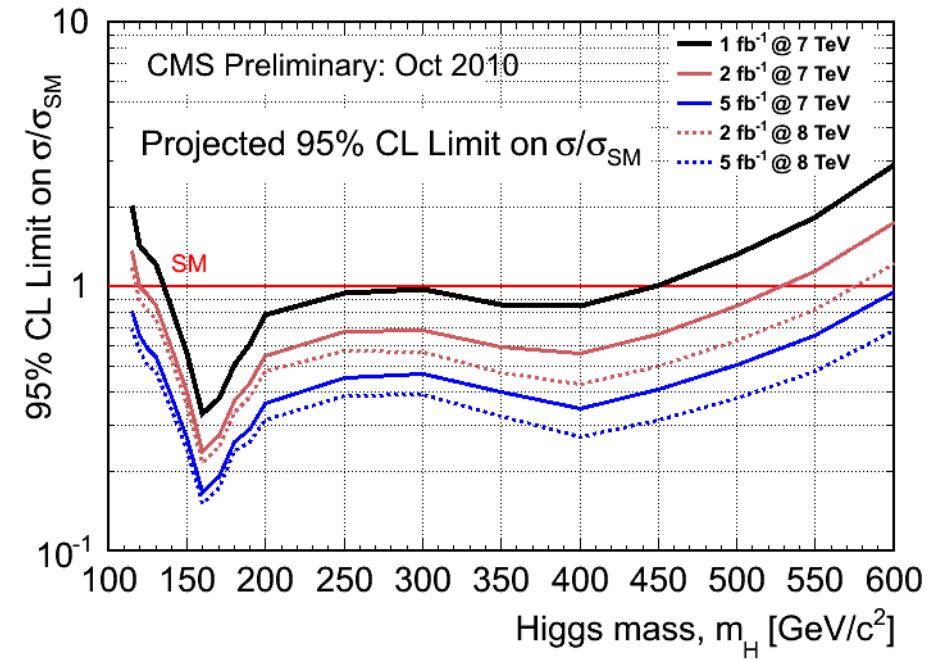
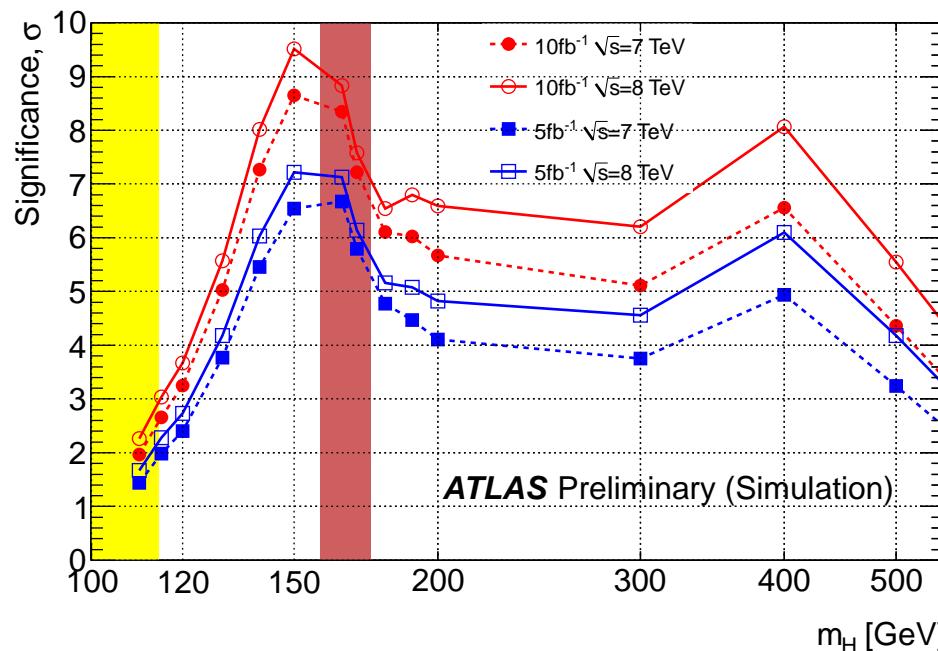
Above 130-140 GeV:

- $gg \rightarrow H, H \rightarrow WW, ZZ$
- $qq \rightarrow qqH, H \rightarrow \gamma\gamma, WW, ZZ$
- $q\bar{q}, gg \rightarrow t\bar{t}H, H \rightarrow \gamma\gamma, WW$
- $q\bar{q}' \rightarrow WH, H \rightarrow WW$

With $\sqrt{s} = 7$ TeV and a few fb^{-1} ...

Combining only $H \rightarrow W^+W^-$, $H \rightarrow ZZ$, $H \rightarrow \gamma\gamma$, ATLAS and CMS indicate that,

- if no signal, the SM Higgs can be excluded up to 500 GeV;
- a 5σ significance for a SM Higgs in the $140 - 170$ GeV mass range;
- in the low mass region (\leftarrow new strategies, new ideas).



where also WH , $H \rightarrow b\bar{b}$ (highly boosted) and VBF with $H \rightarrow \tau\tau$ were used.

We have not discussed: study of Higgs properties

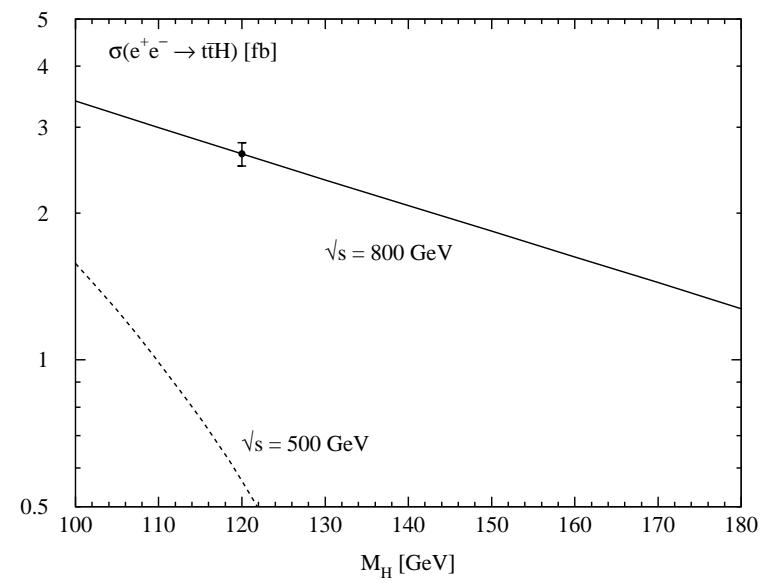
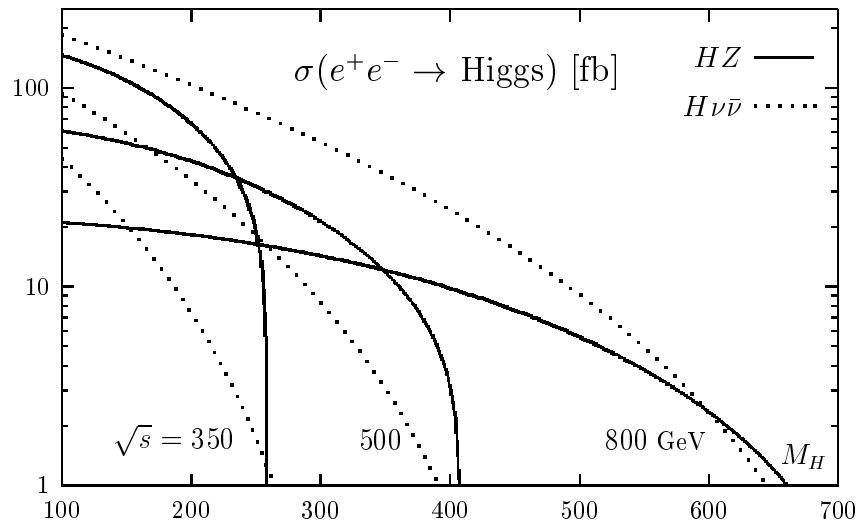
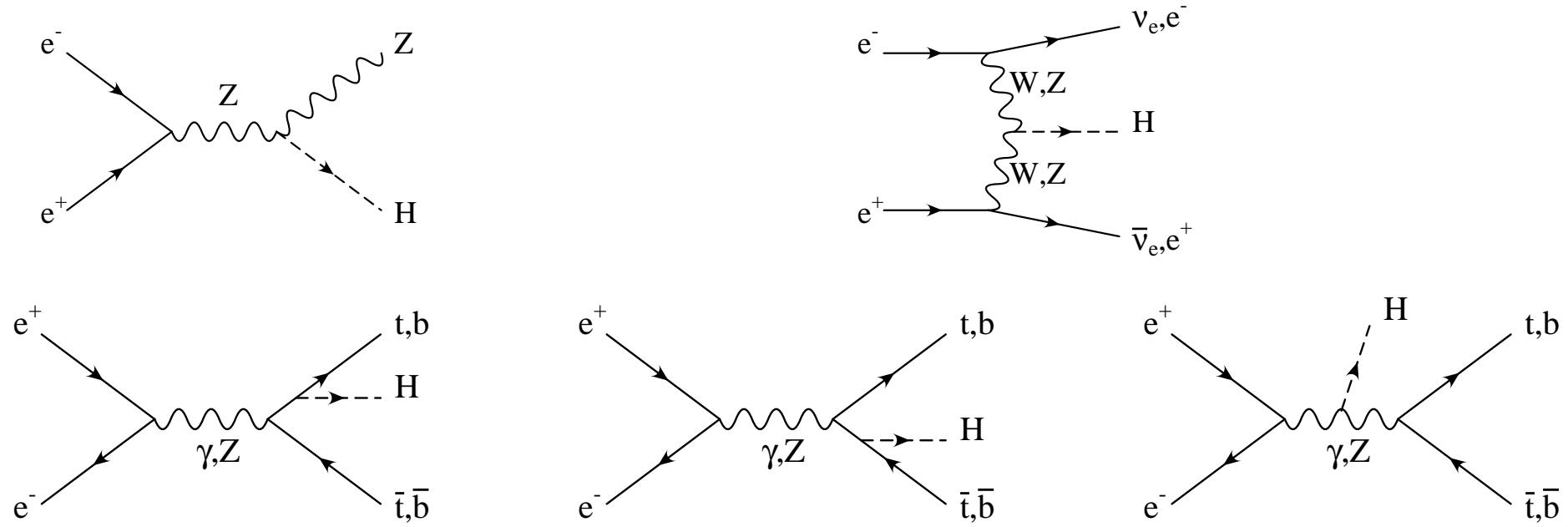
At the LHC:

- Color and charge will be given by the measurement of a given (production+decay) channel.
- The Higgs boson mass will be measured with 0.1% accuracy in $H \rightarrow ZZ^* \rightarrow 4l^\pm$, complemented by $H \rightarrow \gamma\gamma$ in the low mass region. Above $M_H \simeq 400$ GeV precision deteriorates to $\simeq 1\%$ (lower rates).
- The Higgs boson width can be measured in $H \rightarrow ZZ^* \rightarrow 4l^\pm$ above $M_H \simeq 200$ GeV. The best accuracy of $\simeq 5\%$ is reached for $M_H \simeq 400$ GeV.
- The Higgs boson spin could be measured through angular correlations between fermions in $H \rightarrow VV \rightarrow 4f$: need for really high statistics.
- The Higgs boson couplings will be measured combining multiple channels:

$$(\sigma_p(H)\text{Br}(H \rightarrow dd))^{exp} = \frac{\sigma_p^{th}(H)}{\Gamma_p^{th}} \frac{\Gamma_d \Gamma_p}{\Gamma_H}$$

Higgs self-couplings will be very hard!

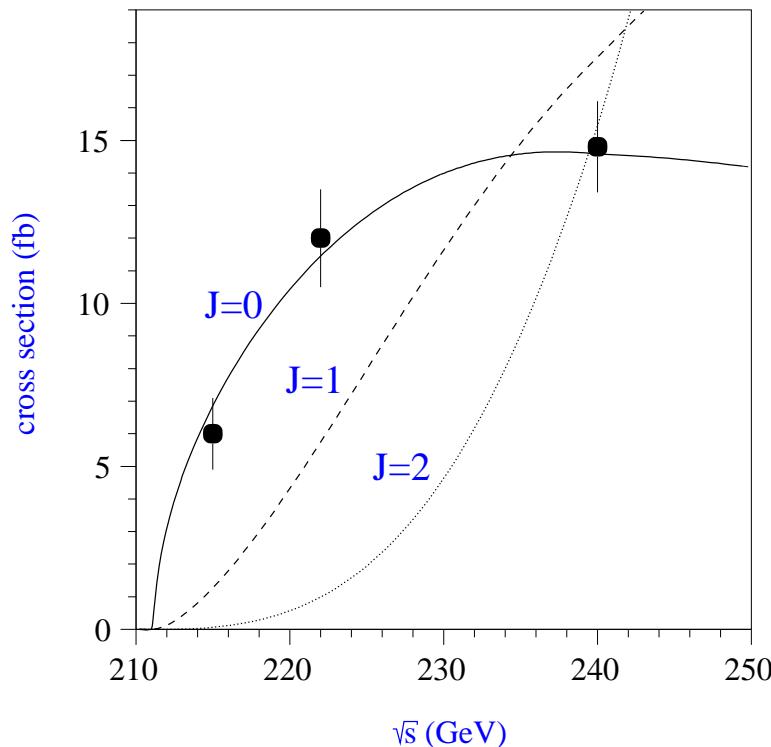
e^+e^- colliders: SM Higgs production modes



Program:

One or more Higgs bosons will be observed over the entire mass spectrum. A high energy e^+e^- collider will then have the unique possibility of:

- Measure $\sigma(e^+e^- \rightarrow ZH)$ at the 2% level: extract $\text{Br}(H \rightarrow xx)$ in model independent way!
- Measure M_H from the recoiling $f\bar{f}$ mass in $ZH \rightarrow Hff\bar{f}$. Accuracies of the order of 50-80 MeV can be obtained.
-



spin measured by:

- ▷ onset slope of the $e^+e^- \rightarrow ZH$
- ▷ correlations in $e^+e^- \rightarrow ZH \rightarrow 4f, \dots$
- ▷ phase space distributions in $e^+e^- \rightarrow t\bar{t}H$

- Measure Γ_H below $M_H \simeq 200$ Gev combining $\text{Br}(H \rightarrow W^+W^-)$ (from $e^+e^- \rightarrow ZH$) and g_{HWW} (from $e^+e^- \rightarrow H\nu\bar{\nu}$), with a $\simeq 6\%$ accuracy.

Ex.: SM Higgs boson, $\sqrt{s}=500$ GeV, 500 fb^{-1} (Except HHH , 1 ab^{-1})

Coupling:	$Hb\bar{b}$	$H\tau^+\tau^-$	$Hc\bar{c}$	HWW	HZZ	$Ht\bar{t}$	HHH
$(M_H=120 \text{ GeV})$	2.2%	3.3%	3.7%	1.2%	1.2%	3%	22%
$(M_H=140 \text{ GeV})$	2.2%	4.8%	10%	2.0%	1.3%	6%	30%
Theory	1.4%	2.3%	23%	2.3%	2.3%	5%	

(Djouadi, '05, using HFITTER/HDECAY)