PHY 5667 : Quantum Field Theory A, Fall 2017

December 7^{th}, 2017

Final Exam

(due by Friday December 14^{th}, 2017)

1. Show explicitly that at one loop in QED:

1.a) the three-photon vertex is zero;

1.b) the four-photon vertex is UV finite, i.e. it does not give origin to ultraviolet divergences.

Explain why this is important for the renormalizability of the theory.

2. Consider a quantum field theory containing a fermion field (ψ) and a scalar field (ϕ) with interaction $\mathcal{L}_{int} = -ig\bar{\psi}\gamma^5\psi\phi$, in $d=4$ space-time dimensions.

2.a) Write the Lagrangian and the corresponding Feynman rules.

2.b) Calculate the first order corrections to the fermion and scalar propagators, considering only the UV-divergent part. Show how the UV divergences of both propagators can be reabsorbed into a redefinition of the fermion/scalar fields and of their masses.

2.c) Calculate the first order corrections to the interaction vertex $g\bar{\psi}\gamma^5\psi\phi$, considering only the UV-divergent part. Show how the UV divergences can be reabsorbed into a redefinition of the coupling g.

2.d) At one loop, there is one more UV-divergent diagram, corresponding to a four-scalar interaction. Draw the diagram and calculate its UV divergence. Explain why this divergence cannot be reabsorbed into a reparametrization of the Lagrangian. How should the theory (i.e. the Lagrangian) be modified for this to be possible?

To calculate the UV-divergent integrals you can use the regularization method you are more familiar with, but make sure to show all your work.