PHY 5667 : Quantum Field Theory A, Fall 2017

September 21^{st}, 2017

Assignment # 2

(due Thursday October 5^{th}, 2017)

1. Classical electromagnetism (with no sources) follows from the action,

$$S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) \ , \quad \text{where } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ ,$$

as you have derived in your first homework. Construct the energy-momentum tensor for this theory. Notice that the usual procedure does not result in a symmetric tensor. Show that you can remedy that using a tensor $K^{\lambda\mu\nu}$ antisymmetric in its first two indices (you will have to build a new energy-momentum tensor $\hat{T}^{\mu\nu} = T^{\mu\nu} + \ldots$ where the dots stays for a function of $K^{\lambda\mu\nu}$). In particular show that

$$K^{\lambda\mu\nu} = F^{\mu\lambda} A^\nu \ ,$$

leads to the standard formulae for the electromagnetic energy and momentum densities:

$$\mathcal{E} = \frac{1}{2}(E^2 + B^2) \ , \qquad \mathbf{S} = \mathbf{E} \times \mathbf{B} \ .$$

2. Consider a real scalar field $\phi(x)$. Recall that $T(a)^{-1}\phi(x)T(a) = \phi(x - a)$, where $T(a) = \exp(iP^\mu a_\mu)$ is the spacetime translation operator, and P^0 is identified as the hamiltonian H.

2.a) Let a^μ be infinitesimal, and derive an expression for $[\phi(x), P^\mu]$.

2.b) Show that the time component of your result is equivalent to the Heisenberg equation of $i\dot{\phi} = [\phi, H]$.

2.c) For a free field, use the Heisenberg equation to derive the Klein- Gordon equation.

2.d) Define a spatial momentum operator

$$\mathbf{P} = -\int d^3x \, \pi(x) \nabla \phi(x) \ ,$$

Use the canonical commutation relations to show that P obeys the relation you derived in part **3.a)**.

2.e) Express $\mathbf{P}$ in terms of $a(\mathbf{k})$ and $a^\dagger(\mathbf{k})$.

3. Problem 2.2 of Peskin and Schroeder's book.