PHY 5667 : Quantum Field Theory A, Fall 2017

November 21^{st}, 2017
Assignment # 6
(due Thursday November 30^{th}, 2017)

1. Show that, for spinors $u^s(p)$ and $v^s(p)$,

$$\sum_{s=1,2} u^s(p)\bar{u}^s(p) = \gamma\cdot p + m = \not{p} + m \ ,$$

and

$$\sum_{s=1,2} v^s(p)\bar{v}^s(p) = \gamma\cdot p - m = \not{p} - m \ .$$

2. Following the example of the scalar-field (Feynman) propagator that we discussed in detail in class, calculate the Feynman propagator for Dirac spinor fields

$$S_F(x-y) \equiv <0|T\psi(x)\bar{\psi}(y)|0> \ ,$$

and show that

$$S_F(x-y) = \int \frac{d^4p}{(2\pi)^4}\tilde{S}_F(p)e^{-ip(x-y)} \ ,$$

where

$$\tilde{S}_F(p) = \frac{i(\not{p}+m)}{p^2-m^2+i\epsilon} \ ,$$

while the meaning of the $i\epsilon$ term in the denominator should be clear from the discussion of the scalar-field propagator.

3. Consider the two-fermion scattering process:

$$\text{fermion}(p) + \text{fermion}(k) \rightarrow \text{fermion}(p') + \text{fermion}(k')$$

in the context of the Yukawa theory ($\mathcal{L}_{\text{int}} = -g\bar{\psi}\psi\phi$). Calculate the differential cross section $\left(\frac{d\sigma}{d\Omega}\right)_{CM}$ and the total cross section σ at the lowest order in g (a.k.a. *tree level*).

4. Consider the decay $\phi \rightarrow e^+ + e^-$, where ϕ is a generic particle.

4.a) Show that the total rate for such decay is,

$$\Gamma(\phi\rightarrow e^+ + e^-) = \frac{1}{16\pi M_\phi}\sqrt{1-\frac{4m_e^2}{M_\phi^2}}\,|\mathcal{M}|^2 \,,$$

where $\mathcal{M}$ is the corresponding invariant matrix element.

4.b) Evaluate $\Gamma(\phi\rightarrow e^+ + e^-)$ when:

4.b.1) ϕ is a *scalar*, with interaction $g_S\phi\bar{\psi}\psi$;

4.b.2) ϕ is a *pseudoscalar*, with interaction $ig_P\phi\bar{\psi}\gamma_5\psi$;

4.b.3) ϕ is a *vector*, with interaction $g_V\phi^\mu\bar{\psi}\gamma_\mu\psi$;

4.b.4) ϕ is a *axial vector*, with interaction $ig_A\phi^\mu\bar{\psi}\gamma_\mu\gamma_5\psi$.

4.c) Imagine a collider reports evidence of a particle that decays only to leptons (e, μ, τ) whose mass is around 4 GeV. If about 25% of the time it decays into $\tau^+\tau^-$, what spin and parity might the particle have?