December 7th, 2018
Final Exam
(due by Friday December 14th, 2018)

Consider a quantum field theory containing a fermion field (ψ) and a scalar field (ϕ) with interaction $\mathcal{L}_{int} = -ig\bar{\psi}\gamma^5\psi\phi$, in $d=4$ space-time dimensions.

a) Write the Lagrangian and the corresponding Feynman rules.

b) Calculate the first order corrections to the fermion and scalar propagators, considering only the UV-divergent part. Show how the UV divergences of both propagators can be reabsorbed into a redefinition of the fermion/scalar fields and of their masses.

c) Calculate the first order corrections to the interaction vertex $g\bar{\psi}\gamma^5\psi\phi$, considering only the UV-divergent part. Show how the UV divergences can be reabsorbed into a redefinition of the coupling g.

d) At one loop, there is one more UV-divergent diagram, corresponding to a four-scalar interaction. Draw the diagram and calculate its UV divergence. Explain why this divergence cannot be reabsorbed into a reparametrization of the Lagrangian. How should the theory (i.e. the Lagrangian) be modified for this to be possible?

To calculate the UV-divergent integrals you can use the regularization method you are more familiar with, but make sure to show all your work.