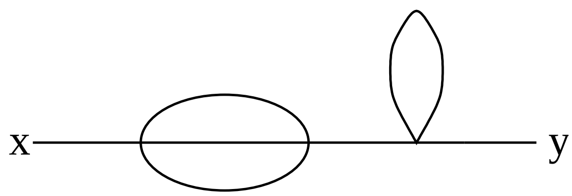
PHY 5667 : Quantum Field Theory A, Fall 2018

October 9^{th}, 2018
Assignment # 4
(due Tuesday October 23^{rd}, 2018)

1. The following Feynman diagram:



represents a contribution to ${}_{\text{out}}\langle 0|T\{\phi(x)\phi(y)\}|0\rangle_{\text{in}}$. Explain which term of the perturbative expansion of ${}_{\text{out}}\langle 0|T\{\phi(x)\phi(y)\}|0\rangle_{\text{in}}$ corresponds to this diagram and write the corresponding analytical contribution both in position space and in momentum space.

2. Problem 7.1 of Schwartz's book.

3. Consider a real scalar field theory with $\mathcal{H}_{\text{int}} = \frac{\lambda}{4!}\phi^4$.

3.a) Derive the tree-level vertex interaction that defines the vertex Feynman rule for this theory in momentum space.

3.b) Given a generic diagram with E external legs, V vertices, and P propagators, what is the relation among E, V, and P for both connected and non-connected diagrams?

3.c) Draw all the fully connected diagrams with $1 \le E \le 4$ and $0 \le V \le 2$, and find their symmetry factors.

4. Repeat the same exercise for the case of a complex scalar field with $\mathcal{H}_{\text{int}} = \frac{\lambda}{4}(\phi^\dagger\phi)^2$. Remember that there are now two kinds of particles (which we can think as positively and negatively charged), and that your rules must have a way to distinguish among them. *Hint*: it seems like you need two kinds of *arrows* to represent the flow of momentum and charge separately. Try to find a more elegant approach that only needs one kind of arrows.