PHY 5667 : Quantum Field Theory A, Fall 2018

November 20^{th}, 2018

Assignment # 7

(due Thursday December 6^{th}, 2018)

1. One of the simplest scattering processes in QED, $e^-e^+ \to \mu^-\mu^+$, can be used to derive a broad range of important physical results. This problem shows how it can be used to get the relativistic quantum corrections to Rutherford's formula for the $e^-p \to e^-p$ scattering cross section (where p is a proton).

1.a) Using crossing symmetry, you can easily connect the cross sections for $e^-e^+ \to \mu^-\mu^+$ and $e^-\mu^- \to e^-\mu^-$. This last process can be used to calculate Rutherford scattering ($e^-p \to e^-p$) in QED. Indeed, as far as QED is concerned, the only difference between a proton and a muon is their mass and the sign of their charge, which enters the calculation of the scattering cross section only squared. In the limit in which $m_p \gg m_e$, you can assume the proton at rest before and after the collision, and, without neglecting m_e, show that the differential cross section takes the following form, known as *Mott formula*:

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 v^2 p^2 \sin^4\frac{\theta}{2}}\left(1 - v^2\sin^2\frac{\theta}{2}\right) ,$$

where $p = |\vec{p}_i| = |\vec{p}_f|$ is the magnitude of the momentum of the initial-state and final-state electron (i.e. $p_i = (E, \vec{p}_i)$ for thr initial-state and $p_f = (E, \vec{p}_f)$ for the final-state electron), θ is the scattering angle (such that $\vec{p}_i \cdot \vec{p}_f = p^2\cos\theta = v^2E^2\cos\theta$), and $v = \frac{p}{E} = \sqrt{1 - \frac{m_e 2}{E^2}}$.

1.b) Show that in the limit of $v \ll 1$ (non-relativistic limit) Mott formula reduces to *Rutherford formula*:

$$\frac{d\sigma}{d\Omega} = \frac{e^4 m_e^2}{64\pi^2 p^4 \sin^4\frac{\theta}{2}} ,$$

1.c) In the very high-energy limit, $E \gg m_e$, you can neglect the mass of the electron and set $v = 1$. You can still work in the rest frame of the initial-state proton, but you can no longer assume that the final-state proton is also at rest. Show that in this case:

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 E^2 \sin^4\frac{\theta}{2}} \frac{E}{E'}\left(\cos^2\frac{\theta}{2} + \frac{E - E'}{m_p}\sin^2\frac{\theta}{2}\right) ,$$

where E' is now the energy of the final-state electron. This formula describes the scattering of electrons on pointlike protons. Deviations from this formula should be taken as indication of the fact that the proton has a substructure. Indeed, this evidence was provided by experiments performed at Stanford in the 1950s. Repeating the same experiments at much high-energies showed however still agreement with the form of pointlike scattering, proving the existence of pointlike constituents of the proton (now known as quarks).

2. Problem 5.2 of Peskin and Schroeder's book.