PHY 5669: Quantum Field Theory B, Spring 2019

April 22^{nd}, 2019
Final Exam
(due May 3^{rd}, 2019)

To complete your Final Exam you should choose and solve only one of the problems proposed in the following. Parts of each one will not be added up for the final grade.

1. Decay modes of W and Z bosons.

1.a Compute the partial decay widths of the W boson into pairs of quarks and leptons. Assume all fermions are massless except the top quark. The decay widths to quarks are enhanced by QCD corrections. Show that QCD corrections to order α_s are similar to what encountered for $e^+e^- \to$ hadrons, where we saw that to order α_s:

$$\sigma(e^+e^- \to \text{hadrons}) = \sigma_0 \left(\sum_i 3Q_f^2\right)\left[1 + \frac{\alpha_s}{\pi} + O(\alpha_s^2)\right] .$$

Using $\sin^2\theta_W = 0.23$ find a numerical value for the total width of the W^+.

1.b Compute the partial decay widths of the Z boson into pairs of quarks and leptons, under the same assumptions as in part **1.a)**. Determine the total width of the Z and the fractions of the decays that give hadrons, charged leptons, and invisible modes $\nu\bar{\nu}$.

2. Decays modes of the Higgs boson.

2.a Calculate the rate for $H \to Q\bar{Q}$ where Q is a generic quark. Notice that this result can be easily extended to massive leptons as well.

2.b In the case of a SM Higgs ($M_H \approx 125$ GeV), $M_H < 2M_W$ and $M_H < 2M_Z$, but in Fig. 1 you can see that $\text{Br}(H \to WW)$ and $\text{Br}(H \to ZZ)$ are given also for $M_H \approx 125$ GeV and lower. What do they correspond to? How could you calculate the corresponding rates?

2.c The Higgs boson can also decay into gluons ($H \to gg$). This cannot happen at tree level. How can it be induced at the one-loop level? Calculate the corresponding rate. You can leave your result in a form that depends on an a Feynman-parameter integral, or you can solve it (it is not too complicated!). Results are in the literature and you should feel free to look them up in order to know what the result should be,

2.d At the one-loop level, the Higgs boson can also decay into two photons ($H \to \gamma\gamma$) and a photon and a Z boson ($H \to \gamma Z$). Calculate the corresponding rates. You can leave your result in a form that depends on one or more Feynman-parameter integrals (see comments in **2.c)**.

2.e In Fig. 1 you can see plotted the theoretical predictions for the SM Higgs-boson branching ratios (l.h.s.) and total width (r.h.s.). Remember that $\text{Br}(H \to XX) = \Gamma(H \to XX)/\Gamma_{\text{tot}}$ where Γ_{tot} is the sum of all the rates. How do your results compare to the

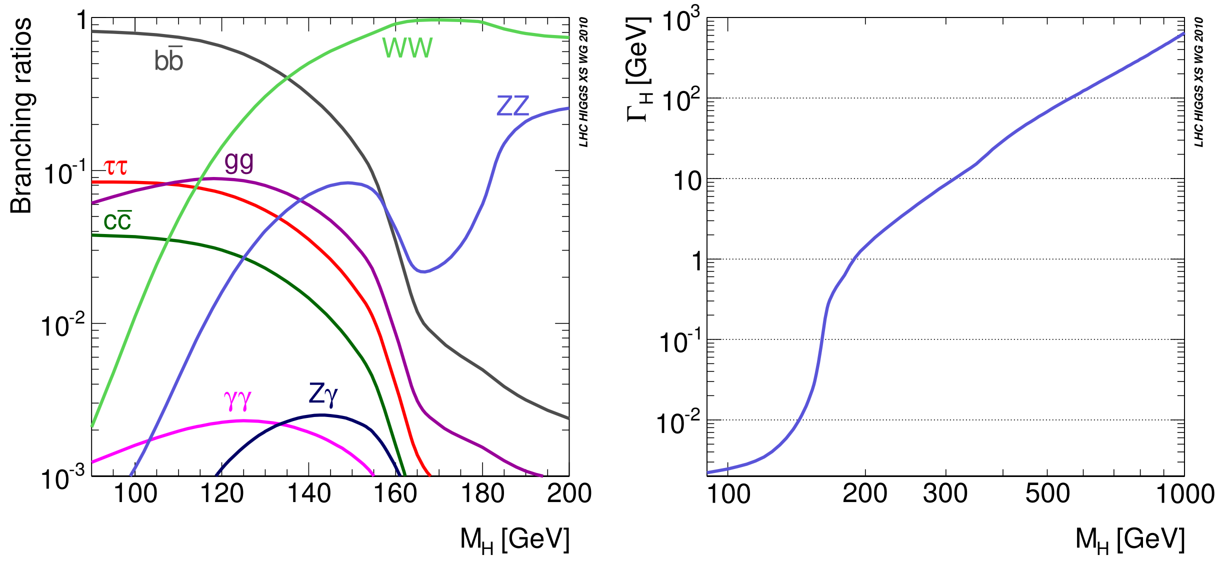

Figure 1: **L.H.S.**: Branching ratios of the SM Higgs boson as a function of its mass. **R.H.S.**: width of the SM Higgs boson as a function of its mass.

numbers that you can extract from the plots? Assuming that your results are correct (you can cross check with the literature), they will probably not perfectly agree with the plot on the l.h.s. Can you tell what could cause such difference?

3. The Two-Higgs-Doublet Model.

3.a Consider a model with two scalar fields ϕ_1 and ϕ_2, which transform as $SU(2)$ doublets with hypercharge $Y = 1/2$. Assume that the two doublets acquire *parallel* vacuum expectation values of the form $(0, v/\sqrt{2})^T$, with $v = v_1$ or $v = v_2$ respectively. Show that these vacuum expectation values produce the same gauge-boson mass matrix that in the Standard Model with one scalar field of the same kind, provided one substitutes:

$$v^2 \to v_1^2 + v_2^2\,.$$

3.b The most general potential function for such model that satisfies the additional discrete symmetry $\phi_1 \to -\phi_1$ and $\phi_2 \to -\phi_2$ is:

$$\begin{aligned} V(\phi_1, \phi_2) &= -\mu_1^2 \phi_1^\dagger \phi_1 - \mu_2^2 \phi_2^\dagger \phi_2 + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) \\ &\quad + \lambda_4 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + \lambda_5 (\phi_1^\dagger \phi_2)^2 + \text{h.c.}\,. \end{aligned}$$

Find conditions on the parameters μ_i and λ_i under which the configuration of vacuum expectation values given in part **1.a)** is a locally stable minimum configuration of such potential.

3.c In the unitary gauge, one linear combination of the upper components of ϕ_1 and ϕ_2 is eliminated (or reabsorbed into the longitudinal degree of freedom of the corresponding gauge boson), while the other remains as a physical charged scalar field. Show that the charged physical scalar field has the form:

$$\phi^+ = \sin\beta\,\phi_1^+ - \cos\beta\,\phi_2^+\,,$$

where μ_1^2, μ_2^2, and all the λ_i are positive, while β is defined by the relation:

$$\tan\beta = \frac{v_2}{v_1}\,.$$