PHY 5669 : Quantum Field Theory B, Spring 2019

March 12^{th}, 2019
Assignment # 3
(due Thursday March 28^{th}, 2019)

1. Consider a non-abelian gauge theory with gauge group G coupled to a complex scalar field in the representation r, i.e. consider the case of a scalar non-abelian gauge theory.

1.a) Derive the Feynman rules for the scalar field and show that they are a simple modification of the scalar QED Feynman rules.

1.b) Compute the contribution of this scalar field to the β function, and show that the full β function for this theory is

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3}C_2(\text{adj}) - \frac{1}{3}C(r)\right]$$

where g is the gauge coupling (defined in the covariant derivative: $D_\mu = \partial_\mu - igA^a_\mu T^a$), while $C(r)$ is the *index* of the representation r (s.t. $Tr(T^a_r T^b_r) = C(r)\delta^{ab}$) and $C_2(r)$ is the *quadratic Casimir* of the representation r (s.t. $T^a_r T^a_r = C_2(r)\cdot\mathbf{1}$).

2. Extra Credit: Calculate the counterterms of QCD (see Eqs. (26.80)-(26.87) of Schwartz's book) that are not already calculated in the book, using Feynman gauge and $\overline{\text{MS}}$ subtraction. For this problem, I would like to see a very streamlined calculation, where you have identified the relevant steps and you do not spend time calculating finite parts that are not contributing to the counterterms. Even better, I suggest you use some software for symbolic computations. The best one for this kind of calculations is called FORM and can be found on `https://www.nikhef.nl/~form/`. It is free and easy to use. All the documentation is on-line. You can skim through the manual, look at some examples, and focus in particular on the sections that you need. Of course, other more complex tools (such as Maple or Mathematica) are also fine, but probably more difficult to specialize to the scope. I will be glad to discuss it more with you if you are interested.