PHY 5669 : Quantum Field Theory B, Spring 2019

April 2^{nd}, 2019
Assignment # 4
(due Thursday April 18^{th}, 2019)

1. **This is problem 17.2 of Peskin and Schroeder's book.**
   In this problem you will learn how to directly probe the spin of the gluon, by comparing the predictions of QCD with those of a model in which the interaction among quarks is mediated by a scalar boson. Let the coupling of the *scalar gluon* (S) to quarks be defined by:

$$\mathcal{L} = gS\bar{q}q\,,$$

   and define $\alpha_g = g^2/(4\pi)$.

   1.a) Compute the tree-level cross section for $e^+e^- \to q\bar{q}S$ as a function of the energies of the final-state particles, which will be represented as energy fractions x_q, $x_{\bar{q}}$, and x_s to the energy of the incoming electron beam. Show that:

$$\frac{d^2\sigma}{dx_q dx_{\bar{q}}}(e^+e^- \to q\bar{q}S) = \frac{4\pi\alpha^2 Q_q^2}{3s}\frac{\alpha_g}{4\pi}\frac{x_s^2}{(1-x_q)(1-x_{\bar{q}})}\,.$$

   1.b) In practice it is very difficult to tell quarks from gluons experimentally, since they both appear as jets of hadrons. It is more convenient to identify the particle (or jet) with the largest energy fraction (call it x_a), the next to the largest energy fraction (call it x_b), and the one with the least energy fraction (call it x_c). Consider the distribution $d^2\sigma/dx_a dx_b$ obtained by summing over the various possibilities. Calculate it for QCD and well as for the *scalar-gluon* model, and show that it can be used to distinguish between the two models.

2. **This is problem 17.3 of Peskin and Schroeder's book.**
   This problem goes through the calculation of quark-antiquark scattering and gluon-gluon scattering at the lowest order in QCD, using some simplifications that allow you to calculate cumbersome expressions in a pretty simple way once you see the structures involved. If you are interested, you could also read Chapter 27 of Schwartz's book and learn a totally different method to reduce this calculation even further. Both methods lend themselves well to a symbolic implementation in any algebraic manipulator. If you use an algebraic manipulator, you can also ignore all simplifications and let the code process the extra terms that will be generated. It is up to you which way you want to take.

   2.a) Compute the differential cross section:

$$\frac{d\sigma}{dt}(q\bar{q} \to gg)\,,$$

at the lowest order in QCD and ignoring quark masses. This is most easily done by computing the amplitudes between states of definite quark and gluon helicity, and using explicit polarization vectors and spinors. For instance, you can use

$$\epsilon^{\mu} = \frac{1}{\sqrt{2}}(0, 1, i, 0)\,,$$

for a right-handed gluon moving in the positive $\hat{z}$ direction. Consider only transversely polarized gluons, and notice that only the $q_L\bar{q}_R$ and $q_R\bar{q}_L$ initial states give a non-zero contribution, and, since QCD respect parity (P), they give identical contributions to the total cross section. In summary, you only have to calculate:

$$\begin{aligned} q_L\bar{q}_R &\to g_L g_L\,, \\ q_L\bar{q}_R &\to g_L g_R\,, \\ q_L\bar{q}_R &\to g_R g_R\,. \end{aligned}$$

After computing these amplitudes, square them and combine them properly with color factors to obtain the various helicity cross sections. The total cross section will be obtained by summing all of them and averaging over initial colors and spins.

2.b) Compute the differential cross section:

$$\frac{d\sigma}{dt}(gg \to gg)\,,$$

at the lowest order in QCD. There are 16 possible helicity amplitudes, but many of them are related to each other by parity and crossing symmetry. All 16 can be built out of the following three amplitudes:

$$\begin{aligned} g_R g_R &\to g_R g_R\,, \\ g_R g_R &\to g_R g_L\,, \\ g_R g_R &\to g_L g_L\,. \end{aligned}$$

Combine these to compute the the spin- and color-averaged differential cross section.