PHY 5669 : Quantum Field Theory B, Spring 2019

January 8^{th}, 2019

Warm-up Project

Notice: this is not a Homework (no due date, not graded). It is just an outline of steps that you may want to follow in reviewing the calculation of the $O(\alpha)$ corrections to the cross section for $e^+e^- \to \mu^+\mu^-$, a process we have been using as a prototype to understand the components of a first-order QED calculation. Chapters 16-20 of M. Schwartz's book should provide you with all the details that you need to support your study(including several explicit calculations). In particular, I suggest that you review points **1)-5)**, and work on **6)** before our lesson on Jan. 15. We will use the results of point **6)** and calculate explicitly point **7)** together starting on Jan. 15.

In QED the cross section for $e^+e^- \to \mu^+\mu^-$ can be calculated perturbatively as an expansion in the QED fine structure constant α (where $\alpha = e^2/(4\pi)$):

$$\sigma(e^+e^- \to \mu^+\mu^-) = \sigma_0 + \sigma_1 + \cdots$$

where σ_0 is of $O(\alpha^2)$, σ_1 is of $O(\alpha^3)$, etc.

1. Compute the lowest-order or tree-level cross section σ_0.

2. σ_1 receives contributions both from $O(\alpha)$ one-loop corrections to the tree-level $e^+e^- \to \mu^+\mu^-$ scattering amplitude (also known as *virtual corrections*), and from the tree-level one-photon emission process, $e^+e^- \to \mu^+\mu^- + \gamma$ (also known as *real corrections*), when the extra photon is not tagged or detected, i.e. when one measures the *inclusive* $e^-\mu^- \to e^-\mu^- + X$ cross section (where X denotes anything that is not specifically identified). Show that both contributions appear at $O(\alpha^3)$ in the cross section and draw the corresponding Feynman diagrams.

3. Write down the analytic expression of each *virtual* correction and show, by only performing the corresponding loop integrations to the extent needed, which corrections contains ultraviolet (UV) divergences and which ones are finite. I suggest you use Dimensional Regularization as regularization scheme.

4. Take some time to review the systematic renormalization of QED and use this opportunity to understand the two main approaches that you could take: using *physical observables* or *counterterms* to systematically implement the explicit cancellation of UV divergences. Also take the time to appreciate the role played by the choice of a subtraction scheme (e.g. on-shell versus Minimal-Subtraction scheme). In particular, using a counterterm approach, make sure you understand how the renormalization of QED (in terms of fields, mass, and charge renormalization) provides a systematic way to subtract all UV divergences. Calculate how this is implemented by picking a specific subtraction scheme, and notice how this choice affects the form of the counterterms (finite parts only).

5. Show how to implement the calculation of the *renormalized* cross section σ_1. Using a counterterm approach, specify what diagrams and CT-diagrams do you need to include. Does this depend on the subtraction scheme you choose?

6. Show that upon subtraction of all UV divergences, the *virtual correction* part of σ_1 (σ_1^{virt}) still contains infrared (IR) divergences. Calculate them using either a mass regulator (m_γ), or dimensional regularization. I suggest you try both. Also, try setting the fermion masses to zero and notice what changes. Chapter 20 of M. Schwartz's book will be very helpful.

7. Calculate the *real* correction part of σ_1 (σ_1^{real}) to the extent needed to show that it also contains IR divergences and that they exactly cancel against the (*virtual*) IR divergences that you have calculated in point **5)**.

8. Given all previous results, explain what parts of the calculation outlined so far you will have to code (if you had to!) in order to compute $\sigma(e^+e^- \to \mu^+\mu^-)$ (UV and IR finite) including $O(\alpha)$ corrections.