Lesson 5

Building a Standard Model EFT (SMEFT)

After having seen examples of both "bottom-up" and "top-down" applications of the EFT framework and formalism, we want to touch on a truly major application that is particularly relevant these days to interpret and utilize results from the Large Hadron Collider.

↳ As mentioned in the beginning of our excursion through EFT ideas, we would like to systematically interpret LHC results on the basis of a Lagrangian that extends the Lagrangian of the SM ($\mathcal{L}_{SM}$) by introducing "towers" of effective interactions of higher and higher dimension, with "couplings" suppressed by inverse powers of the scale of new physics Λ ($\Lambda \to \lambda_{UV}$).

Comments:

- This is a "bottom-up" approach! need to apply the same spirit and mindset that we saw in the $\gamma\gamma \to \gamma\gamma$ scattering example!

  ↳ we do not know the UV-completion of the SM, and we need a systematic tool to interpret both "indirect" and "direct" footprints of such physics.

  ↳ $\boxed{\text{EFT}}$

  since EFT → QFT, "indirect" effects will percolate through our physical predictions by modifying the parameters and couplings of $\mathcal{L}_{SM}$ itself. At the same time, there will be more "direct" or new effects induced by interactions that are not part of the $\mathcal{L}_{SM}$.

  ↳ it's like exploring QED via the changes induced by the new force of the scattering matrix elements!

Is an EFT approach justified?

↳ as we have seen, an EFT approach is justified when a physical system is characterized by the presence of quite different energy/distance scales, such that a (E/Λ) expansion can be justified.

Since no signal of new physics beyond the SM has been yet detected at the LHC, chances are that $\Lambda_{UV} > \text{TeV}$, hence $\Lambda_{UV} \gtrsim 10\,\Lambda_{EW}$.

similar to flavor-physics in a way

$$\Lambda_{UV} = \text{EW scale} \simeq 100\text{–}200\ \text{GeV}$$
$$\Lambda_{flavor} \simeq m_b - m_c \simeq 5\text{–}10\ \text{GeV}$$

So, we definitely should try to attack the problem using an EFT formalism and be aware, however, that each step should be tested for consistency.

Typical example: care need to be paid to explore new physics in distributions (differential cross-sections) that extend to very-high Energies and interpret them using an EFT formalism.

$\left(\text{ex.: } \dfrac{d\sigma}{dp_T}\right)$

↳ since in these regions the E/Λ expansion may be less justified.

Direct discovery of new physics will greatly boost this game!

↳ as we have seen in our "top-down" example, (flavor physics) knowing the UV completion allows for a full fledged use of the EFT framework to reach more precise determinations of the EFT couplings.

and even improve with respect to Λ_{UV}!

- Is the EFT approach to studying beyond SM (BSM) physics "model independent"?

    ↳ not really, we will have to make assumptions along the way, in order to maintain a decent level of predictivity.

    But: it is certainly more model independent than any other approach.
    One does not pick a model, just makes assumptions that cut across classes of models.

    ↳ a very interesting "complement" would be to study the "projection" of specific models on $\mathcal{L}^{SM}_{EFT}$
    - > which operators do they map onto?
    - > How are they connected by their scale-evolution?
    - > Can we see patterns? Should we look for patterns? Which ones?

- In the following we will assume a knowledge of the SM and how $\mathcal{L}_{SM}$ comes about. (→ QFT B)

    ↳ we will give $\mathcal{L}_{SM}$, but we will not explain how it comes about, just use it for reference to explain what is the impact of the extra EFT structures on it.

The amount of formal discussion given is minimal (only 1hr and 15' to discuss lots of formulas!), just a few examples.

But, you will find extensive typed notes on this website ("Notes on Higgs physics with dimension 6 operators").

Building a SM EFT (from live lesson)

$$\frac{c_W}{\Lambda^2} O_W + \frac{c_G}{\Lambda^2} O_G + \dots \quad \text{(see later)}$$

$$\mathcal{L}_{\substack{SM\\EFT}} = \mathcal{L}_{SM} + \mathcal{L}_5 + \mathcal{L}_6 + \dots$$

this will be our focus today

(only operator $(L^T L)(H^T H) \to \Delta L = 2$ Neutrino Majorana mass)

SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$

(review QFTB)

$$\mathcal{L}_{SM} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} +$$

$$+ (D_\mu H)^\dagger (D^\mu H) + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2$$

$$+ i\left(\bar{L} D_\mu \gamma^\mu L + \bar{e}_R D_\mu \gamma^\mu e_R + \bar{Q} D_\mu \gamma^\mu Q + \bar{u}_R D_\mu \gamma^\mu u_R + \bar{d}_R D_\mu \gamma^\mu d_R\right)$$

$$- \left(\bar{L} Y_e e_R H + \bar{Q} Y_u u_R \tilde{H} + \bar{Q} Y_d d_R H + h.c.\right)$$

useful for the following discussion: $\nearrow SU(3) \quad [T^a, T^b] = i f^{abc} T^c$

> $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$

> $W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g \epsilon^{ijk} W^j_\mu W^k_\nu$

$\hookrightarrow SU(2) \quad \left[\frac{\sigma^i}{2}, \frac{\sigma^j}{2}\right] = i\epsilon^{ijk} \frac{\sigma^k}{2}$

> $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$\to$ > $D_\mu q_L = \left(\partial_\mu - i g_s T^a G^a_\mu - i g \frac{\sigma^i}{2} W^i_\mu - i g' Y_q B_\mu\right) q_L$

$\to$ > $L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$

"Higgs field"

$\boxed{EWSB} \longrightarrow \quad H = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 + \frac{h}{v} \end{pmatrix}, \quad \tilde{H} = i\sigma^2 H^*$

(unitary gauge)

$\boxed{\mathcal{L}_6}$ using GIMR basis. → 59 operators

→ [arXiv: 1008.4884, JHEP 1010 (2010) 085]

simplifying assumptions

- → gauge invariant
- → $\Delta B = \Delta L = 0$
- → CP even → 27 operators
- → flavor diagonal and family universal.

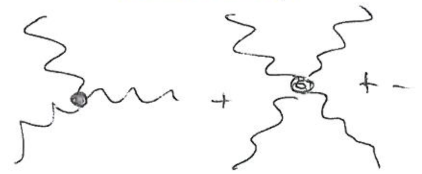
] assumptions like these depend on the kind of physics you are interested in.

Examples: ⟹ mainly from EWPO + Higgs Physics

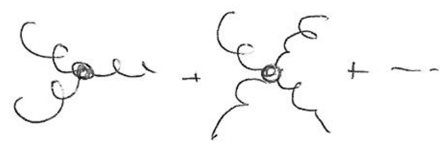
↳ (i.e. operator that affect EW + Higgs physics)

(1) Gauge operators:

$$\epsilon_{ijk} W_\mu^{i,\nu} W_\nu^{j,\rho} W_\rho^{k,\mu} \longrightarrow O_W \longrightarrow$$



$$f_{abc} G_\mu^{a,\nu} G_\nu^{b,\rho} G_\rho^{c,\mu} \longrightarrow O_G \longrightarrow$$



(2) other bosonic operators

$$O_{HG} = (H^\dagger H) G^a_{\mu\nu} G^{a,\mu\nu}$$

$$O_{HW} = (H^\dagger H) W^i_{\mu\nu} W^{i,\mu\nu}$$

$$O_{HB} = (H^\dagger H) B_{\mu\nu} B^{\mu\nu}$$

} affect triple gauge-boson and H-gaugeboson couplings

↳ see following example

$$O_{HWB} = (H^\dagger \sigma^i H) W^i_{\mu\nu} B^{\mu\nu}$$

$$O_{HD} = (H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$$

} affect W/Z propagators and oblique corrections (S, T)

$$O_{H\Box} = (H^\dagger H) \underset{\partial^\mu \partial_\mu}{\Box} (H^\dagger H) \longrightarrow \text{modifies } H \text{ wave function}$$

$$O_H = (H^\dagger H)^3 \longrightarrow \text{modifies Higgs potential } (\lambda, v)$$

(3) single-fermion-current operators

$$O^{(1)}_{HL} = (H^{\dagger} i \overleftrightarrow{D}_{\mu} H)(\bar{L} \gamma^{\mu} L)$$

$$O^{(3)}_{HL} = (H^{\dagger} i \overleftrightarrow{D}^{i}_{\mu} H)(\bar{L} \sigma^{i} \gamma^{\mu} L)$$

$$O_{He} = (H^{\dagger} i \overleftrightarrow{D}_{\mu} H)(\bar{e}_R \gamma^{\mu} e_R)$$

Modifies H-fermion interactions and Z-boson couplings to f. W-boson couplings to f.

⋮

same for quark fields

⋮

$$O_{eH} = (H^{\dagger} H)(\bar{L} e_R H)$$

→ change in Yukawa couplings and Higgs interactions

⋮ same for quarks.

(4) Four-fermion operators.

$$O_{LL} = (\bar{L} \gamma_{\mu} L)(\bar{L} \gamma^{\mu} L) \quad \rightarrow \quad G_F$$

Upon EWSB these operators modify the SM Lagrangian parameters and couplings.

↳ need to specialize them to $H = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 + \frac{h}{v} \end{pmatrix}$ to see physical content.

↳ see examples to follow and posted typed notes

Example: $\mathcal{L}_6 = \dots + \frac{c_{HG}}{\Lambda^2} O_{HG} + \dots$

$$\boxed{O_{HG}} = (H^\dagger H) G^a_{\mu\nu} G^{a,\mu\nu} = \frac{v^2}{2}\left(1 + \frac{2h}{v} + \frac{h^2}{v^2}\right) G^a_{\mu\nu} G^{a,\mu\nu}$$

$$H = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 + \frac{h}{v} \end{pmatrix} \qquad = \underbrace{\frac{v^2}{2} G^a_{\mu\nu} G^{a,\mu\nu}} + \underbrace{v \cdot h \cdot G^a_{\mu\nu} G^{a,\mu\nu}} + \dots$$

$\hookrightarrow$ affect hgg, $hggg$, ... coupling(s)

$$\left\{ \hat{C}_i \equiv v^2 \frac{C_i}{\Lambda^2} \right\}$$

$\mathcal{L}_{kin} = -\frac{1}{4}(1 - 2\hat{C}_{HG}) G^a_{\mu\nu} G^{a,\mu\nu} \leftarrow$ affects kinetic term

$\frac{C_{HG}}{\Lambda^2} O_{HG}$

$$\tilde{G}^a_{\mu\nu} = G^a_{\mu\nu}(1 - \hat{C}_{HG})$$

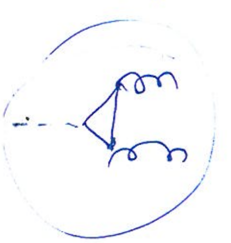
| through D_μ

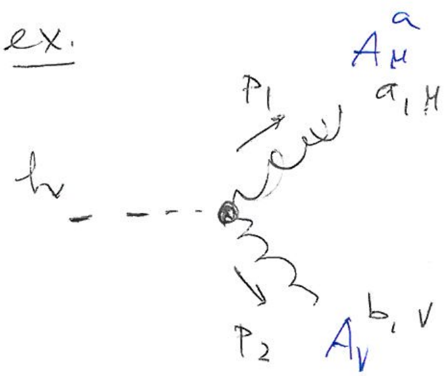
same for O_{HB}, O_{HW}

$$\tilde{g}_s = g_s(1 + \hat{C}_{HG})$$

ex.

SM



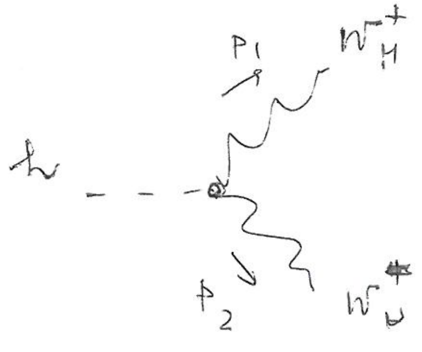


$$= \frac{4i}{v} \hat{C}_{HG} \delta^{ab} (p_{2,\mu} p_{1,\nu} - p_1 \cdot p_2 \, g_{\mu\nu})$$

similarly:

$\boxed{O_{HW}}$ < kinetic term $\to$ coupling; hW interaction

direct



$$\frac{4i}{v} \hat{C}_{HW} (p_{2\mu} p_{1\nu} - g_{\mu\nu} p_1 \cdot p_2) +$$

corrections to M_Z^2

$$+ 2i(\sqrt{2} G_F)^{1/2} c_W^2 M_Z^2 \times$$

$$\left[1 + \delta_h - \frac{1}{2(c_W^2 - s_W^2)} \left(4 s_W c_W \hat{C}_{HWB} + c_W^2 \hat{C}_{HD} + \delta_{G_F} \right) \right]$$

$$\delta_h = -\frac{1}{4}\hat{C}_{HD} + \hat{C}_{H\Box}$$

- $O_{HD} = (H^\dagger D_\mu H)^*(H^\dagger D^\mu H) =$

$$= \frac{v^2}{4}\left(1+\frac{2h}{v}+\frac{h^2}{v^2}\right)(\partial^\mu h)(\partial_\mu h) + \frac{g^2 v^4}{16 c_W^2} Z_\mu Z^\mu \left(1+4\frac{h}{v}+6\frac{h^2}{v^2}+4\frac{h^3}{v^3}+\frac{h^4}{v^4}\right)$$

changes the Z propagator and Z-h interactions.

$h = \tilde{h}(1+\delta h)$

- $O_{H\Box} = (H^\dagger H)\Box(H^\dagger H) =$

$$= -(v^2+4vh+4h^2)(\partial_\mu h)(\partial^\mu h)$$

same

Go systematically through it using the typed notes.

$$\delta_{G_F} = \hat{C}_{HL}^{(3)11} + \hat{C}_{HL}^{(3)22} - \frac{1}{2}\left(\hat{C}_{LL}^{1221} + \hat{C}_{LL}^{2112}\right)$$

$\hookrightarrow \tilde{G}_F = G_F(1+\delta G_F)$

etc. ; need to account for both direct and indirect corrections (M_W, M_Z, s_W, G_F, α, M_H, ...)

Constraints from:

- precision EW observables (indirect)
- Higgs measurements $\left\langle \boxed{\sigma,\ \Gamma} \quad \boxed{d\sigma/dX} \right.$
- all available processes with sufficient exp. precision.

$\rightarrow$ see $\boxed{\text{HEP talk on Oct. 5, 2018.}}$ Slides posted on this website. Ask more if interested!