PHY 5667 : Quantum Field Theory A, Fall 2019

December 1^{st}, 2019

Final Exam

(due by Friday December 13^{th}, 2019)

Consider the case of scalar electrodynamics of two complex scalar fields ϕ_1 and ϕ_2 with interaction:

$$\mathcal{L}_{int} = -\frac{\lambda_1}{4}(\phi_1^\dagger\phi_1)^2 - \frac{\lambda_2}{4}(\phi_2^\dagger\phi_2)^2 - \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)\,.$$

a) Write the complete Lagrangian, and justify the origin and role of its various components.

b) Write the Feynman rules for propagators and vertices, including counterterm vertices.

c) Calculate the field, mass, and coupling renormalization constants after having imposed well-defined renormalization conditions.

d) Calculate the scale evolution of each coupling by calculating the corresponding beta function.

e) **Extra credit**: consider the $\tilde{e}_1^+\tilde{e}_1^- \to \tilde{e}_2^+\tilde{e}_2^-$ scattering process, where $\tilde{e}_1$ and $\tilde{e}_2$ are the scalar particles associated to fields ϕ_1 and ϕ_2 respectively (assume $m_1 \neq m_2$).

e.1) Compute the lowest order (tree-level) cross section.

e.2) Explain schematically what enters the calculation of the cross section at the first non-trivial order in α ($\alpha = e^2/(4\pi)$), λ_1, λ_2 and λ_3.

e.3) Calculate the cross section at $O(\alpha\lambda_3^2)$, explaining in detail the treatment of both the UV and IR regions of the integration over loop momenta or final-state momenta (phase-space integration). Note: it is useful to work out some of the integrals till they allow you to match contributions from different parts of the calculation. Beyond that, finite integrals can be left as symbolic.