PHY 5667 : Quantum Field Theory A, Fall 2019

September 5^{th}, 2019
Assignment # 2
(due Thursday September 19^{th}, 2019)

1. It is often possible to derive a field theory as the limit of a discrete system. Perhaps the simplest example is an infinite system of aligned point masses, m, separated by massless springs of spring constant k and equilibrium length a. This model can be used to (approximately) describe both the longitudinal vibrations of an elastic rod and the transverse oscillations of a stretched string. Let η_i be the displacement from equilibrium of the ith point mass. Derive the exact Lagrangian and the Euler-Lagrange equations for this system. Then consider the limit

$$m, a \to 0 \ , \ k \to \infty \ , \ \mu = m/a \text{ and } Y = ka \text{ fixed} \ .$$

Replacing η_i by a smooth function $\eta(x,t)$, show that in this limit the Lagrangian may be written in the density form

$$L = \int dx \frac{1}{2} \left[\mu \left(\frac{\partial \eta}{\partial t} \right)^2 - Y \left(\frac{\partial \eta}{\partial x} \right)^2 \right]$$

and write down the corresponding Euler-Lagrange equation. This is a notable equation (which one?) that you have obtained as the continuous limit of a discrete system of coupled harmonic oscillators!

2. The electromagnetic field may be specified by a vector $A^\mu(\mathbf{x}, t)$, in terms of which the Lagrange density of the field is

$$\mathcal{L}(x) = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \ ,$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ .$$

Derive the Lagrange equations for this system, and express them in terms of the free-space field strengths $\mathbf{E} = -\partial_0 \mathbf{A} - \nabla A_0$ and $\mathbf{B} = \nabla \times \mathbf{A}$. How many of Maxwell's equations does this give, and why are the others also satisfied?

3. Problem 3.5 of Srednicki's book.

4. Problem 3.4 of Srednicki's book. **Notice**: there was a typo in the first version of this homework, for which Problem 3.4 was assigned instead of Problem 3.5. If you have already done Problem 3.4, feel free to turn it in for extra credit. If you have not done it you can choose to do it (for extra credit) or not.