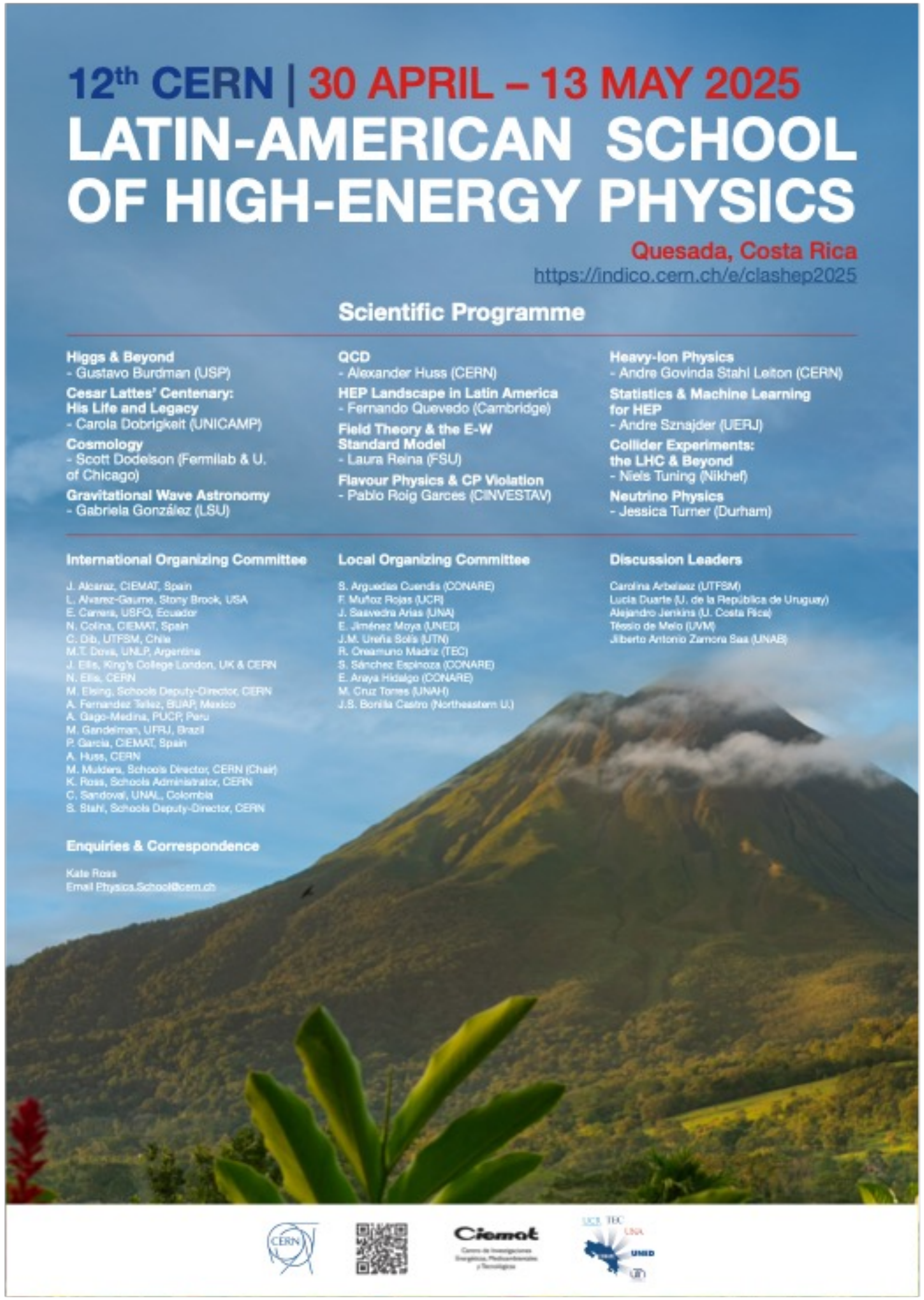


Field Theory
and
the EW Standard Model

Laura Reina

Outline

Essential elements of field theory

- General framework.
- Global and local symmetries.
- From classical to quantum field theory.

The Standard Model (SM)

- Building the SM Lagrangian: first principles and phenomenological evidence.
- Testing the SM consistency.

The Standard Model in the LHC era

- Strengths and weaknesses.
- Probing SM predictions at the LHC.

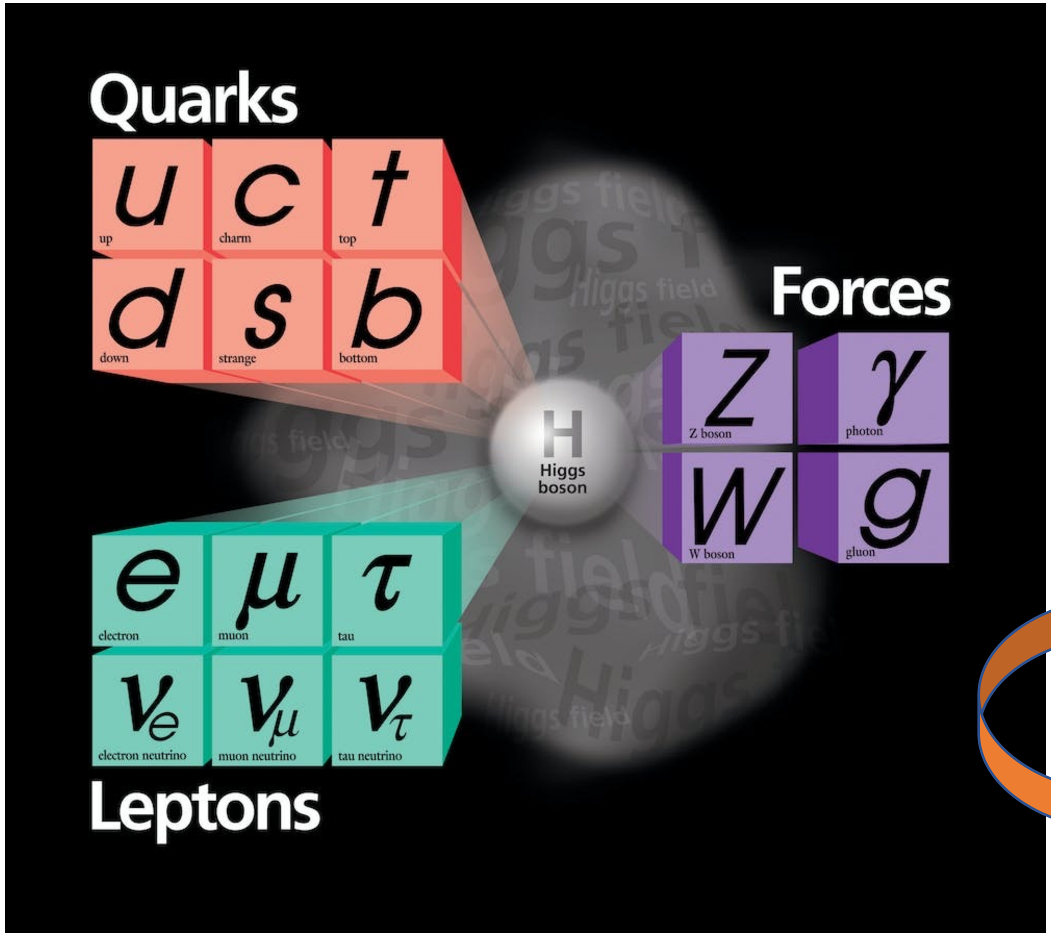
The Standard Model

- Building the SM Lagrangian: first principles and phenomenological evidence
  - Steps towards the SM Lagrangian
  - Main building blocks
  - Main phenomenological consequences

- Testing the SM consistency
  - Global fit of precision observables
  - Constraining new physics

- SM limits and problematics aspects
  -

The Standard Model of particle physics: the artist rendering



A very minimal quantum field theory describing strong, weak, and electromagnetic interactions, based on a local (gauge) symmetry

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$$

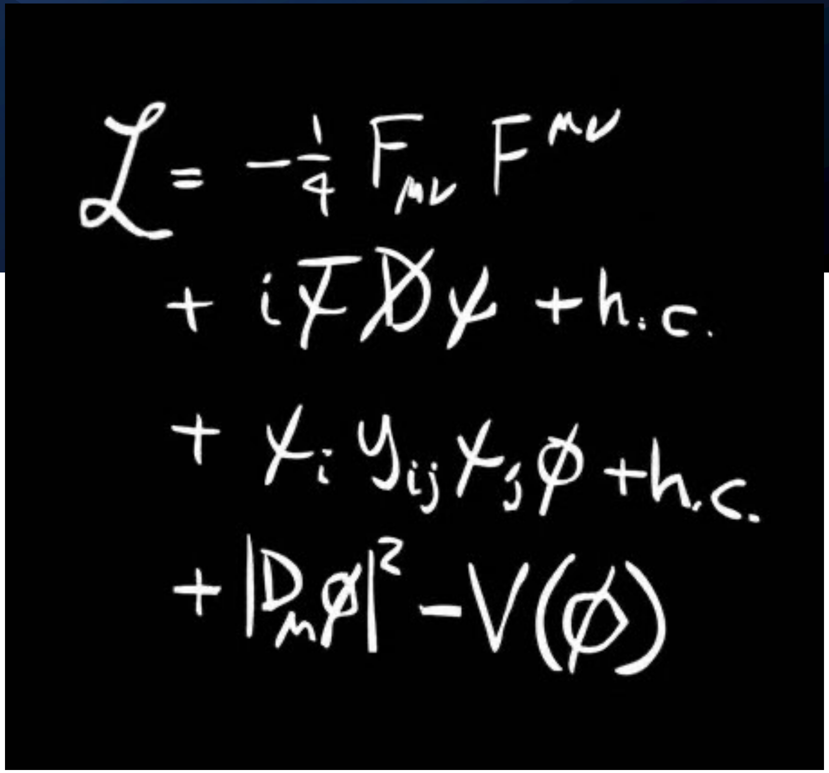
Strong interactions: gluons → $m_g = 0$
Electromagnetic interactions: photon → $m_\gamma = 0$
Weak interactions: $W^\pm$ and Z → $M_W, M_Z \neq 0$

Due to the presence of a scalar field whose potential spontaneously breaks the gauge symmetry of weak interactions and gives origin to massive gauge bosons (W,Z)

The Higgs boson (H) is the physical particle associated with such field

Let's build it step by step!

Building the Standard Model Lagrangian



- Steps towards the SM Lagrangian.
- Main building blocks.
- Main phenomenological consequences.

Towards the SM of particle physics

Translating experimental evidence of particle interactions into the right gauge symmetry group: one of the most fascinating story in particle physics

- Electromagnetic interactions → Quantum Electrodynamics (QED) – $U(1)_{QED}$
  - Plenty of **phenomenological evidence** to go beyond E&M and classical FT **very early on**: **Lamb shift** in atomic levels, anomalous magnetic moment of the electron ($\boldsymbol{g_e}$), …
  - The **true testing ground of QFT ideas**, paved by phenomenological success.
  - Remarkably tested to this days at lepton and hadron colliders.
  - Still, not everything can be explained by an **exact abelian gauge theory**!

- Strong interactions → Quantum Chromodynamics (QCD) - $SU(3)_C$
  - Evidence for *strong* force in *hadronic interactions.*
  - Gell-Mann-Nishijima **quark model** interprets hadron spectroscopy.
  - Need for **3-fold quantum number** (color) ↔ hadron spectroscopy, $e^+e^- \to$ hadrons.
  - DIS experiments → confirm **parton model** based on $SU(3)_C$.
  - **Exact non abelian gauge theory** explains **confinement vs asymptotic freedom.**
  - Much more (the whole physics program of **hadron colliders**!).

❑ Weak interactions – quite puzzling …

- *Discovered* in **neutron β-decay**: $n \rightarrow p + e^- + \bar{\nu}_e$.
- **New force**: small rates/long lifetimes.
- **Universal**: same strength in both hadronic and leptonic decays:
  - $n \rightarrow p\, e^- \bar{\nu}_e,\ \mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu, \pi^- \rightarrow \mu^- \bar{\nu}_\mu, \ldots$
- **Violate parity** (P).
- **Charged currents** only **left-handed.**
- **Neutral currents not** of **electromagnetic** origin.
- First description by **Fermi Theory** as a four-fermion interaction

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} (\bar{p}\gamma_\mu(1-\gamma_5)n)(\bar{e}\gamma^\mu(1-\gamma_5)\nu_e)$$

$G_F \rightarrow$ Fermi constant

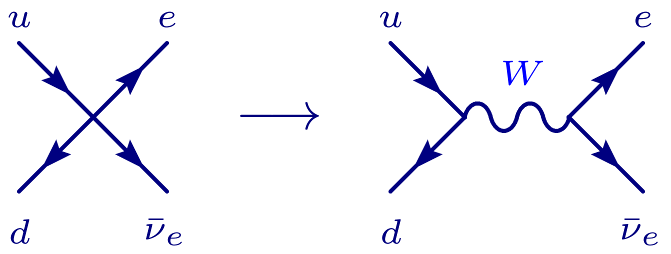
$[G_F] = [m]^{-2}$

(in units of $c = \hbar = 1$)

- Easily accommodate a **massive intermediate vector boson**

$$\mathcal{L}_{IVB} = \frac{g}{\sqrt{2}} W_\mu^+ J_\mu^- + \text{h.c.}$$

$$J_\mu^- = \bar{u}\gamma_\mu \frac{1-\gamma_5}{2} d + \bar{\nu_e}\gamma^\mu \frac{1-\gamma_5}{2} e$$



$$\frac{G_F}{\sqrt{2}} = \frac{{g_W}^2}{8M_W^2} \longleftarrow q^2 \ll M_W^2$$

- New force with massive mediators → SSB gauge symmetry

  - 3 gauge bosons (2 charged+1 neutral), chiral interactions → $SU(2)_L$
  - Cannot be the whole story otherwise they would have the same mass upon SSB (but $M_W \neq M_Z$)
  - Only possibility, without extending to a gauge group that would have many more gauge bosons is

$$SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_Q$$

$U(1)_{QED}$

Y → *hypercharge*, $Y = Q - T_3$
Q → charge (QED)
T_3 → $SU(2)$ generator

- Strong, electromagnetic, and weak interactions → $SU(3)_c \times SU(2)_L \times U(1)_Y$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{SSB} SU(3)_C \times U(1)_Q$$

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm EW}$$

$$\mathcal{L}_{\rm EW} = \mathcal{L}_{\rm EW}^{\rm ferm} + \mathcal{L}_{\rm EW}^{\rm gauge} + \mathcal{L}_{\rm EW}^{\rm SSB} + \mathcal{L}_{\rm EW}^{\rm Yukawa}$$

Strong interactions: Quantum Chromodynamics (QCD)

See lectures by A. Huss

Exact $SU(3)$ Yang-Mills theory - $SU(3)_C$

$$\mathcal{L}_{\rm QCD} = \sum_i \bar{Q}_i (i\not{D} - m_i)\, Q_i - \frac{1}{4} F^{a,\mu\nu} F^a_{\mu\nu}$$

$$D_\mu = \partial_\mu + i g_s A^a_\mu T^a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

- $Q_i \to (i = 1, \ldots, 6 \to u, d, s, c, b, t)$ fundamental representation of $SU(3)_C$ (dim=3) → *quark* triplets

$$Q_i = \begin{pmatrix} Q_i \\ Q_i \\ Q_i \end{pmatrix}$$

- A^a_μ → adjoint representation of $SU(3)_C$ (dim = $N^2 - 1, N = 3$) → 8 massless *gluons* (gauge fields)
- $T^a \to SU(3)_C$ generators (Gell-Mann matrices)
- g_s → strong coupling constant – gauge coupling of QCD ($\alpha_s = g_s^2/(4\pi)^2$)

- All other fermion fields are $SU(3)_C$ singlets

Electromagnetic and weak interactions: unified into the Glashow-Weinberg-Salam theory

Spontaneously broken Yang-Mills theory based on $SU(2)_L \times U(1)_Y$

- $SU(2)_L$ → **weak isospin** group, gauge **coupling g**
  - 3 generators $T^a = \sigma^a/2$ (σ^a Pauli matrices, a= 1,2,3)
  - 3 gauge bosons: $W_1^\mu, W_2^\mu, W_3^\mu$
  - $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$ fields are doublets of $SU(2)$
  - $\psi_R = \frac{1}{2}(1+\gamma_5)\psi$ fields are singlets of $SU(2)$

$\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L$ mass terms are forbidden since not gauge invariant

- $U(1)_Y$→ **weak hypercharge** group ($Y = Q - T_3$), gauge **coupling g'**
  - 1 generator → each field has a Y quantum number (charge)
  - 1 gauge boson: B^μ

Three generations (*families*) of fermion fields – Summary of quantum numbers

				$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_Q$
$Q_L^i =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$	$\frac{2}{3}$ $-\frac{1}{3}$
$u_R^i =$	u_R	c_R	t_R	3	1	$\frac{2}{3}$	$\frac{2}{3}$
$d_R^i =$	d_R	s_R	b_R	3	1	$-\frac{1}{3}$	$-\frac{1}{3}$
$L_L^i =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$	0 -1
$e_R^i =$	e_R	μ_R	τ_R	1	1	-1	-1
$\nu_R^i =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0	0

Last line (right-handed neutrinos) **is not part of the SM**. **Why?** **More to come …**
Interesting to notice that ν_R^i has zero charge under the entire SM group!

1- Lagrangian of the fermion fields

For each generation (here specialized to the first generation)

$$\mathcal{L}_{EW}^{\text{ferm}} = \bar{L}_L(i\not{D})L_L + \bar{e}_R(i\not{D})e_R + \bar{Q}_L(i\not{D})Q_L + \bar{u}_R(i\not{D})u_R + \bar{d}_R(i\not{D})d_R$$

With covariant derivative

$$D_\mu = \partial_\mu - igW_\mu^a T^a - ig'\frac{1}{2}YB_\mu$$

acting on the left/right fields as

$$\begin{aligned} D_{\mu,L} &= \partial_\mu - \frac{ig}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} gW_\mu^3 - g'YB_\mu & 0 \\ 0 & -gW_\mu^3 - g'YB_\mu \end{pmatrix} \\ D_{\mu,R} &= \partial_\mu + ig'\frac{1}{2}YB_\mu \end{aligned}$$

where $W_\mu^\pm$ are defined as:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

Separating kinetic and interaction (current) terms, $\mathcal{L}_{EW}^{\text{ferm}}$ can be written as

$$\mathcal{L}_{EW}^{\text{ferm}} = \mathcal{L}_{kin}^{\text{ferm}} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

where

$$\begin{aligned}
\mathcal{L}_{kin}^{\text{ferm}} &= \bar{L}_L (i\partial\!\!\!/) L_L + \bar{e}_R (i\partial\!\!\!/) e_R + \dots \\
\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_{eL} \gamma^\mu e_L + W_\mu^- \bar{e}_L \gamma^\mu \nu_{eL} + \dots \\
\mathcal{L}_{NC} &= \frac{g}{2} W_\mu^3 \left[\bar{\nu}_{eL} \gamma^\mu \nu_{eL} - \bar{e}_L \gamma^\mu e_L\right] + \frac{g'}{2} B_\mu \left[Y(L)(\bar{\nu}_{eL} \gamma^\mu \nu_{eL} + \bar{e}_L \gamma^\mu e_L)\right. \\
&+ \left. Y(e_R) \bar{\nu}_{eR} \gamma^\mu \nu_{eR} + Y(e_R) \bar{e}_R \gamma^\mu e_R\right] + \dots
\end{aligned}$$

$W_\mu^\pm$ → mediators of charged currents

W_μ^3, B_μ → mediators of neutral currents

However, neither W_3^μ nor B_μ can be identified with the photon field (A_μ) because they couple to neutral fermions

Rotate W_μ^3 and B_μ introducing a ***weak mixing angle*** $\boldsymbol{\theta_W}$ *(a.k.a.* ***Weinberg angle****)*

$$W_\mu^3 = \sin\theta_W A_\mu + \cos\theta_W Z_\mu$$
$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$$

Such that the kinetic term is still diagonal and the neutral current Lagrangian becomes ($\psi = \nu_{eL}, e_L, e_R$)

$$\mathcal{L}_{NC} = \bar{\psi}\gamma^\mu \left(g\sin\theta_W T^3 + g'\cos\theta_W \frac{Y}{2}\right)\psi A_\mu + \bar{\psi}\gamma^\mu \left(g\cos\theta_W T^3 - g'\sin\theta_W \frac{Y}{2}\right)\psi Z_\mu$$

$$eQ = g\sin\theta_W T^3 + g'\cos\theta_W \frac{Y}{2}$$

Q **identified as the e.m. charge**

Applying Q to any fermion field gives

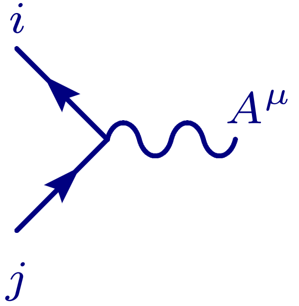
$$g\sin\theta_W = g'\cos\theta_W = e$$

Notice:

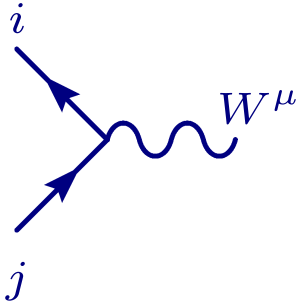
- Charged and neutral current **violate P** (couple differently to L- and R-handed fields)
- Neutral currents are **universal** (same for all fermion generations) ↔ $SU(2)$ gauge symmetry
- **No tree-level flavor-changing neutral currents** $(across\ generations)$

Feynman rules: obtained from tree-level 2- and 3-point correlation functions

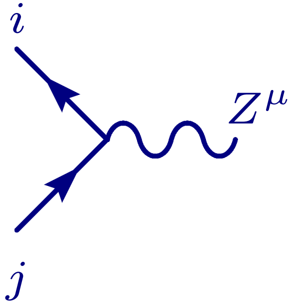
You can calculate them!



$$= \quad -ieQ_f\gamma^\mu$$



$$= \quad \frac{ie}{2\sqrt{2}s_w}\gamma^\mu(1-\gamma_5)$$



$$= \quad ie\gamma^\mu(v_f - a_f\gamma_5)$$

$$v_f = -\frac{s_W}{c_W}Q_f + \frac{T_f^3}{2s_W c_W}$$ (Vector coupling)

$$a_f = \frac{T_f^3}{2s_W c_W}$$ (Axial vector coupling)

2 - Lagrangian of the gauge fields

The $SU(2)_L \times U(1)_Y$ gauge-field Lagrangian is initially written as:

$$\mathcal{L}_{EW}^{\text{gauge}} = -\frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$\begin{cases} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W_{\mu\nu}^a &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc} W_\mu^b W_\nu^c \end{cases}$$

and then expressed in terms of the physical charged- and neutral-current mediators ($W_\mu^\pm, Z_\mu, A_\mu$) obtaining:

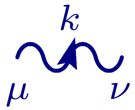
$$\mathcal{L}_{EW}^{\text{gauge}} = \mathcal{L}_{kin}^{\text{gauge}} + \mathcal{L}_{EW}^{3V} + \mathcal{L}_{EW}^{4V}$$

where

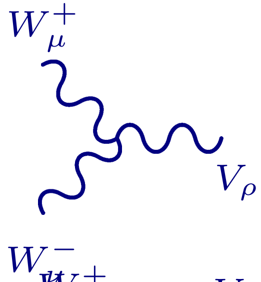
$$\begin{aligned} \mathcal{L}_{kin}^{\text{gauge}} &= -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) \\ &- \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ \mathcal{L}_{EW}^{3V} &= (\text{3-gauge-boson vertices involving } ZW^+W^- \text{ and } AW^+W^-) \\ \mathcal{L}_{EW}^{4V} &= (\text{4-gauge-boson vertices involving } ZZW^+W^-,\ AAW^+W^-, \\ & AZW^+W^-, \text{ and } W^+W^-W^+W^-) \end{aligned}$$

Feynman rules: obtained from tree-level 2-, 3-, and 4-point correlation

In a particular gauge choice
(more details discussed later)

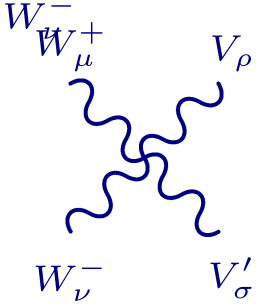


$$= \frac{-i}{k^2 - M_V^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_V^2}\right)$$



$$= ieC_V\left[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\right]$$

$$\begin{cases} C_\gamma = 1 \\ C_Z = -\dfrac{c_W}{s_W} \end{cases}$$



$$= ie^2 C_{VV'}\left(2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right)$$

$$\begin{cases} C_{\gamma\gamma} = -1 \qquad C_{\gamma Z} = \dfrac{c_W}{s_W} \\ C_{ZZ} = -\dfrac{c_W^2}{s_W^2} \qquad C_{WW} = \dfrac{1}{s_W^2} \end{cases}$$

3 - Lagrangian of the scalar field

The SSB of he EW SM gauge symmetry $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$

See Lecture 1

Introduce one complex scalar doublet of $SU(2)_L$with $Y = 1/2$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \longleftrightarrow \quad \mathcal{L}_{EW}^{SSB} = (D^\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

where: $$D_\mu \phi = (\partial_\mu - igW_\mu^a T^a - ig' Y_\phi B_\mu) \;\; (T^a = \sigma^a/2, \; a = 1, 2, 3)$$

The EW SM gauge symmetry is realized as a spontaneously broken symmetry by choosing a particular vacuum expectation value (vev) of the field ϕ that minimizes the scalar potential, e.g.

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{with} \quad v = \left(\frac{-\mu^2}{\lambda} \right)^{1/2} \quad (\mu^2 < 0, \; \lambda > 0)$$

Notice: all $SU(2)_L \times U(1)_Y$ generators are broken for this choice of vacuum configuration, but $Q = T^3 + Y$ is not

The weak gauge boson mass terms arise from

$$(D^\mu\phi)^\dagger D_\mu\phi \quad\longrightarrow\quad \cdots + \frac{1}{8}(0\ \ v)\left(gW_\mu^a\sigma^a + g'B_\mu\right)\left(gW^{b\mu}\sigma^b + g'B^\mu\right)\begin{pmatrix}0\\ v\end{pmatrix} + \cdots$$

$$\longrightarrow\quad \cdots + \frac{1}{2}\frac{v^2}{4}\left[g^2(W_\mu^1)^2 + g^2(W_\mu^2)^2 + (-gW_\mu^3 + g'B_\mu)^2\right] + \cdots$$

where we can read that

$$\begin{cases} W_\mu^\pm = \dfrac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2) \longrightarrow \boxed{M_W = g\frac{v}{2}} \\ Z_\mu = \dfrac{1}{\sqrt{g^2+g'^2}}(gW_\mu^3 - g'B_\mu) \longrightarrow \boxed{M_Z = \sqrt{g^2+g'^2}\frac{v}{2}} \end{cases}$$

Notice: M_W and M_Z are function of gauge couplings (g, g') and $v = \sqrt{\mu^2/\lambda}$, as expected for a SSB theory

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \quad , \quad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \quad\longrightarrow\quad \frac{M_W}{M_Z} = \cos\theta_W$$

while the linear combination orthogonal to Z_μ remains massless and corresponds to the photon field

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'W_\mu^3 + gB_\mu) \quad\longrightarrow\quad \boxed{M_A = 0}$$

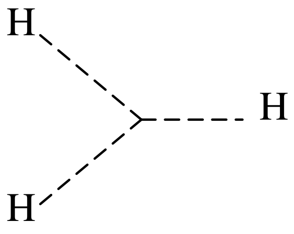
To identify the physical scalar field of the SM, work in unitary gauge

$$\phi(x) = \frac{e^{\frac{i}{v}\vec{\chi}(x)\cdot\vec{\tau}}}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \xrightarrow{SU(2)} \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

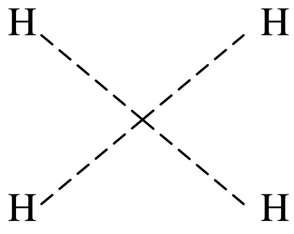
such that **the Lagrangian only depends on the field *H*** (the $\chi_a(x)$ degrees of freedom having been *traded* for the longitudinal component of the massive gauge bosons)

$$\mathcal{L} = \mu^2 H^2 - \lambda v H^3 - \frac{1}{4} H^4 = -\frac{1}{2} M_H^2 H^2 - \underbrace{\sqrt{\frac{\lambda}{2}} M_H H^3 - \frac{1}{4} \lambda H^4}$$

H **→ SM Higgs boson** with **mass** $M_H^2 = -2\mu^2 = 2\lambda v^2$ and **self-couplings**



$$= -3i\frac{M_H^2}{v}$$



$$= -3i\frac{M_H^2}{v^2}$$

Note on gauge choice: R_ξ gauges

Quantization of gauge theories implies choosing a gauge-fixing condition

- Abelian gauge case, for simplicity

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^* D_\mu\phi - V(\phi)$$

- Upon SSB

$$\phi(x) = \frac{1}{\sqrt{2}}((v + \phi_1(x)) + i\phi_2(x))$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial^\mu\phi_1 + gA^\mu\phi_2)^2 + \frac{1}{2}\underbrace{(\partial^\mu\phi_2 - gA^\mu(v + \phi_1))^2}_{} - V(\phi)$$

Eliminate momentum-dependent ϕ_2 contributions to A_μ propagator by cleverly choosing the gauge condition

$$G = \frac{1}{\sqrt{\xi}}(\partial_\mu A^\mu + \xi g v \phi_2)$$

The generating functional of the quantum theory becomes:

$$Z = C \int \mathcal{D}A\mathcal{D}\phi_1\mathcal{D}\phi_2 \exp\left[\int d^4x \left(\mathcal{L} - \frac{1}{2}G^2\right)\right] \det\left(\frac{\delta G}{\delta \alpha}\right)$$

where:

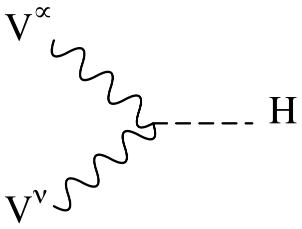
$$\begin{aligned}
\mathcal{L} - \frac{1}{2}G^2 &= -\frac{1}{2}A_\mu\left(-g^{\mu\nu}\partial^2 + \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu - (gv)^2 g^{\mu\nu}\right)A_\nu \\
&\quad \frac{1}{2}(\partial_\mu\phi_1)^2 - \frac{1}{2}m_{\phi_1}^2\phi_1^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 - \frac{\xi}{2}(gv)^2\phi_2^2 + \cdots \\
&+ \\
\mathcal{L}_{ghost} &= \bar{c}\left[-\partial^2 - \xi(gv)^2\left(1+\frac{\phi_1}{v}\right)\right]c \quad \longleftarrow \quad \det\left(\frac{\delta G}{\delta \alpha}\right)
\end{aligned}$$

Gauge and scalar propagators in the generic R_ξ gauge:

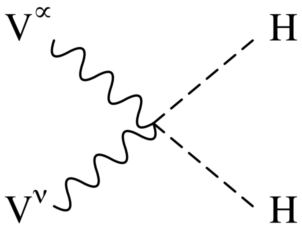
$$\begin{aligned}
\langle A^\mu(k)A^\nu(-k)\rangle &= \frac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) + \frac{-i\xi}{k^2 - \xi m_A^2}\left(\frac{k^\mu k^\nu}{k^2}\right) \\
\langle \phi_1(k)\phi_1(-k)\rangle &= \frac{-i}{k^2 - m_{\phi_1}^2} \\
\langle \phi_2(k)\phi_2(-k)\rangle &= \langle c(k)\bar{c}(-k)\rangle = \frac{-i}{k^2 - \xi m_A^2}
\end{aligned}$$

Notice: ϕ_2 directly talks to the longitudinal component of A^μ, same mass!

From $\mathcal{L}_{EW}^{SSB} \rightarrow (D^\mu \phi)^\dagger (D_\mu \phi) \rightarrow$ couplings to gauge bosons



$= 2i\frac{M_V^2}{v}g^{\mu\nu}$

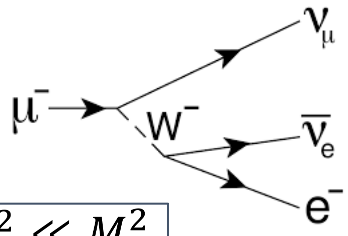


$= 2i\frac{M_V^2}{v^2}g^{\mu\nu}$

Notice: the entire scalar sector depends only on two parameters → (μ^2, λ) or (v, λ) or (M_H, v)

Very constrained paradigm: precision measurements of M_H, v, and Higgs-boson couplings are the ultimate test of the SM

- M_H, Higgs boson couplings → LHC experiments
- $v \rightarrow \mu$ decay



$q^2 \ll M_W^2$

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} f(m_e^2/m_\mu^2)(1+\delta_{RC})$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2} \longrightarrow v = (2G_F)^{-1/2}$$

4 – Yukawa Lagrangian (scalar-fermion interaction)

Fermion masses are generated via **gauge-invariant Yukawa-like couplings**

$$\mathcal{L}_{EW}^{Yukawa} = -\Gamma_u^{ij}\bar{Q}_L^i\phi^c u_R^j - \Gamma_d^{ij}\bar{Q}_L^i\phi d_R^j - \Gamma_e^{ij}\bar{L}_L^i\phi l_R^j + \text{h.c.}$$

Such that upon SSB

$$\begin{aligned}\mathcal{L}_{EW}^{Yukawa} &= -\Gamma_u^{ij}\bar{u}_L^i\frac{v+H}{\sqrt{2}}u_R^j - \Gamma_d^{ij}\bar{d}_L^i\frac{v+H}{\sqrt{2}}d_R^j - \Gamma_e^{ij}\bar{l}_L^i\frac{v+H}{\sqrt{2}}l_R^j + \text{h.c.}\\ &= -\sum_{f,i,j}\bar{f}_L^i M_f^{ij} f_R^j\left(1+\frac{H}{v}\right) + \text{h.c.}\end{aligned}$$

non-diagonal "*mass*" matrix $\quad M_f^{ij} = \Gamma_f^{ij}\frac{v}{\sqrt{2}} \xrightarrow{U_L^f,\,U_R^f} M_f^D = (U_L^f)^\dagger M_f U_R^f$ Diagonal mass matrix

$$f_L'^i = (U_L^f)_{ij} f_L^j \quad \text{and} \quad f_R'^i = (U_R^f)_{ij} f_R^j$$

Mass eigenstates

Rotating to the "*mass*" basis

$$\mathcal{L}_{EW}^{Yukawa} = \sum_{f,i,j} \bar{f}_L'^{\,i} [(U_L^f)^\dagger M_f U_R^f] f_R'^{\,j} \left(1 + \frac{H}{v}\right) + \text{h.c.}$$

$$= \sum_{f,i,j} m_f \left(\bar{f}_L' f_R' + \bar{f}_R' f_L'\right) \left(1 + \frac{H}{v}\right)$$

modifies the charge-currents Lagrangian by a **matrix of flavor-mixing couplings**:

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \bar{u}_L'^{\,i} [(U_L^u)^\dagger U_L^d] \gamma^\mu d_L^j W_\mu^+ + \text{h.c.}$$

$$V_{CKM} = (U_L^u)^\dagger U_L^d$$

Cabibbo-Kobayashi-Maskawa matrix

➢ Why there is no CKM for leptons in the SM?

Very intriguing: flavor physics has its origin in the scalar sector of the SM, and follows from the mechanism that generates fermion masses.

Testing the EW SM consistency

- Including quantum corrections.
- Global fits of EW precision observables.

Standard Model – Quantum corrections

The SM Lagrangian is made of renormalizable dim=4 structures (all of them!)

$$\begin{aligned}\mathcal{L}_{SM} &= \mathcal{L}_{QCD} + \mathcal{L}_{EW} \\ &= \mathcal{L}_{EW}^{\text{ferm}} + \mathcal{L}_{EW}^{\text{gauge}} + \mathcal{L}_{EW}^{SSB} + \mathcal{L}_{EW}^{Yukawa}\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{QCD} &\rightarrow \bar{\psi}(\partial\!\!\!/ - m)\psi\,,\ \bar{\psi}A\!\!\!/\psi\,,\ \frac{1}{4}G^{a,\mu\nu}G^a_{\mu\nu} \\ \mathcal{L}_{EW}^{\text{ferm}} &\rightarrow \bar{\psi}_L(\partial\!\!\!/)\psi_L\,,\ \bar{\psi}_L V\!\!\!/\psi_L \\ \mathcal{L}_{EW}^{\text{gauge}} &\rightarrow \frac{1}{4}F^{a,\mu\nu}F^a_{\mu\nu}\,,\ \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ \mathcal{L}_{EW}^{SSB} &\rightarrow \partial^\mu\phi\partial_\mu\phi,\ \mu^2\phi^2,\ \phi^4 \\ \mathcal{L}_{EW}^{Yukawa} &\rightarrow \bar{\psi}_L H\psi_R\end{aligned}$$

See Lecture 1

The systematic procedure outlined in these lectures will apply with extra constraints imposed by the presence of a partially spontaneously broken gauge symmetry.

The set of fundamental parameters of the SM Lagrangian is:

$$g_{s,0}\ ,\ g_0\ ,\ g'_0\ ,\ \mu_0\ ,\ \lambda_0\ ,\ y_{f,0}\ ,\ V_0^{ij}$$

here taken as **bare parameters**. Thanks to relations induced by the symmetries of the theory, e.g.

$$e = g\sin\theta_W = g'\cos\theta_W \quad \to \quad e = \frac{gg'}{\sqrt{g^2+g'^2}}$$

"natural" relation: they will be finite, but **corrections depend on input parameters ($m_t, M_H, \ldots$)**

$$M_W = \frac{gv}{2}\ ,\quad M_Z = \frac{v\sqrt{g^2+g'^2}}{2} \quad \to \quad \frac{M_W}{M_Z} = \frac{g}{\sqrt{g^2+g'^2}} = \frac{e}{g'} = \cos\theta_W$$

we can trade them for other or "better" sets of input parameters (more precisely measured), for example:

$$g_{s,0}\ ,\ e_0\ ,\ M_{W,0}\ ,\ M_{Z,0}\ ,\ M_{H,0}\ ,\ m_{f,0}\ ,\ V_0^{ij}$$

and **switch to** the corresponding set of **renormalized or physical parameters** upon **imposing suitable renormalization conditions**.

Renormalization conditions

- **QCD**: in the absence of a mass scale, use MS scheme or minimal subtraction scheme, i.e. subtract just pole parts of each divergent proper vertex.

- **EW**: use procedure illustrated in **Lecture 1** for a scalar $\lambda\phi^4$ toy model → **on-shell subtraction scheme**.
  - Mass/coupling renormalization

$$M^2_{W,0} = M^2_W + \delta M^2_W \, , \, \ldots \, , \, m_{f,0} = m_f + \delta m_f \, , \, V_0^{ij} = V^{ij} + \delta V^{ij}$$

  - Field renormalization

$$W_0^\pm = \sqrt{Z_W} W^\pm \, , \, \begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{ZZ}} & \sqrt{Z_{ZA}} \\ \sqrt{Z_{AZ}} & \sqrt{Z_{AA}} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix} \ldots$$

  - Impose renormalization conditions (traditionally) of the form ("on-shell" conditions):

$$\delta M^2_W = \mathrm{Re}[\Sigma^{\mathrm{W}}_{\mathrm{T}}(\mathrm{M}^2_{\mathrm{W}})] \, , \, \delta Z_W = -\mathrm{Re}[\Sigma^{W\prime}_T(M^2_W)] \, , \, \ldots \quad \alpha(0) = \frac{e^2}{4\pi}$$

Thomson limit $q^2 \to 0$

Once expressed in terms of the renormalized parameters and fields, any physical observable is finite and can be calculated at the proper perturbative order in QCD+EW and compared with experimental results.

Global fits of precision measurements

- The symmetry structure of the Standard Model defines specific relations among couplings and masses, such that a minimal set of parameters can be identified.

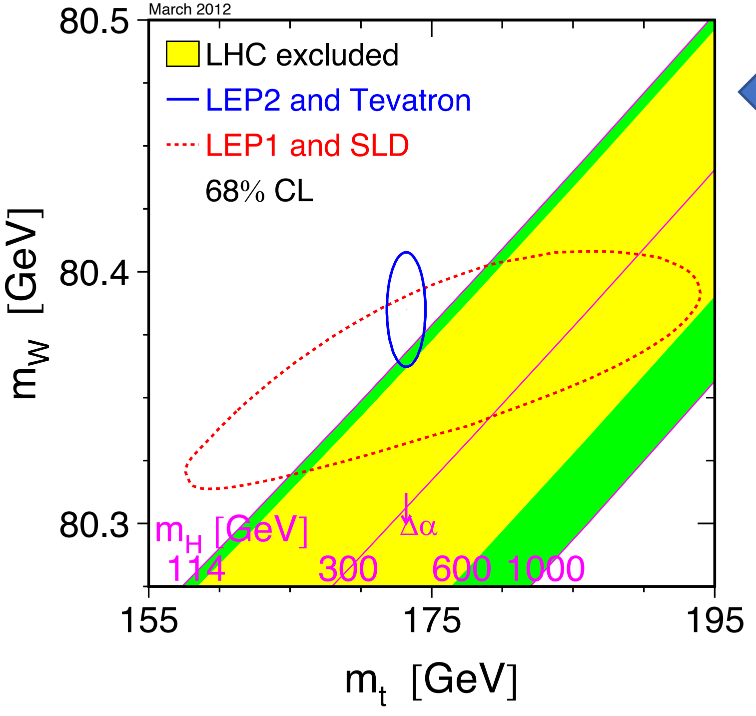
- The renormalizability of the theory assures that tree-level relations are modified by finite calculable corrections.

- **Precision measurements** of masses and couplings via multiple observables:
  - Test the consistency of the theory at the quantum level
  - Indirectly probe new physics via virtual effects

Very successful history!

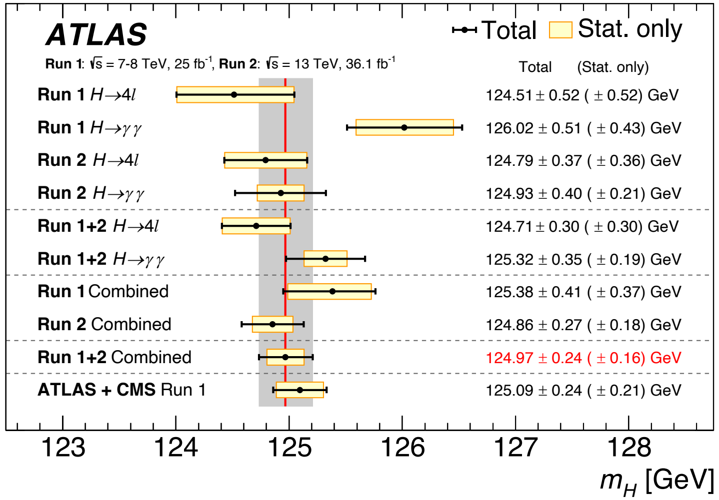
The last successful story

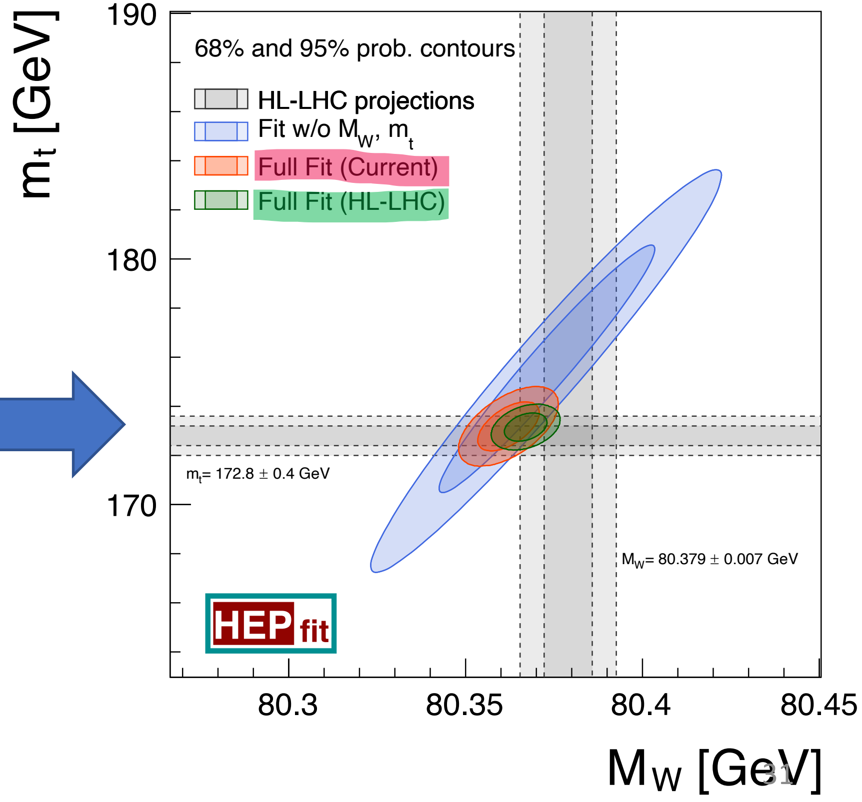
Global fits of precision EW observables gave us strong indications of where to find the SM Higgs boson and we now use its mass as one of the EW precision observables of the EW global fit to constrain new physics.



$$M_H = 94^{+29}_{-24}\ \text{GeV}$$

$$M_H < 152\,(171)\ \text{GeV}$$





Global fits of EW precision observables – general strategy

- **Pick a minimal set of input parameters to SM predictions,** e.g.
  - $\alpha, G_F, M_{Z,} M_H, m_t, m_f, V_{CKM}, \alpha_s \rightarrow \alpha$–scheme
  - $M_W, G_F, M_{Z,} M_H, m_t, m_f, V_{CKM}, \alpha_s \rightarrow M_W$–scheme

- The best measured ones (α, G_F) are fixed, the others are floated.

- **Compute EW precision observables** (EWPO), **including all known higher-order quantum corrections**
  - Z-pole observables (LEP/SLD):$\Gamma_Z, \sin^2\theta_{eff}, A_l, A_{FB}, \ldots$
  - W-observables (LEP II, Tevatron, LHC): M_W, Γ_W
  - $m_t, M_H, \sin^2\theta_{eff}$ (Tevatron/LHC)

- **Perform best fit to EW precision data** (EWPD) through different fitting procedures and compare with experimental measurements.

- Beyond SM: parametrize new physics effects on EWPO and constrain deviations from SM in terms of chosen parameters. Examples:
  - **Oblique parameters**: S,T,U, …
  - **SM effective field theory** (SMEFT) → Wilson coefficients

EW Observables: Theoretical parametrization

- **Analytic** theoretical predictions of **Z and W boson observables**.
  - Ex: Z-pole observables:

$$sin^2\theta_{eff,l} = \frac{1}{4}\left(1 - \frac{g_{V,l}}{g_{A,l}}\right)$$

$$A_f = \frac{2\left(\frac{g_{V,f}}{g_{A,f}}\right)}{1 + \left(\frac{g_{V,f}}{g_{A,f}}\right)^2} \qquad A_{FB,f} = \frac{3}{4}A_e A_f$$

$$\Gamma_{Z,f} = N_f \frac{G_F M_Z^3}{24\sqrt{2}\pi} 4\left[(g_{V,f})^2 + (g_{A,f})^2\right] \qquad \sigma^0_{had} = \frac{12\pi}{M_Z^2}\frac{\Gamma_e \Gamma_{had}}{\Gamma_Z^2} \qquad R^0_e = \frac{\Gamma_{had}}{\Gamma_e} \qquad R^0_{q,\nu} = \frac{\Gamma_{q,\nu}}{\Gamma_{had}}$$

all in terms of Z-fermion couplings

- **Functions of all the parameters of the model** (masses, couplings) **through SM quantum corrections**

Global fit of EW observables – last update

For M_W we combine:

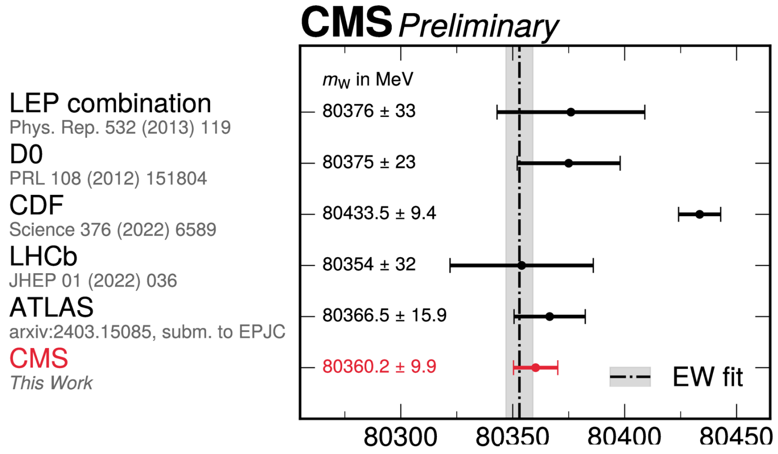
- ❑ All LEP 2 measurements
- ❑ Previous Tevatron average
- ❑ ATLAS and LHCb early measurements
- ❑ CDF [M_W=(80.4335±0.0094) GeV]
- ❑ ATLAS [M_W=(80.3665±0.016) GeV]
- ❑ CMS [M_W=(80.3602±0.010) GeV]

M_W = 80.366 ± 0.0080 GeV (without CDF)
80.356 ± 0.0045 GeV (from fit)

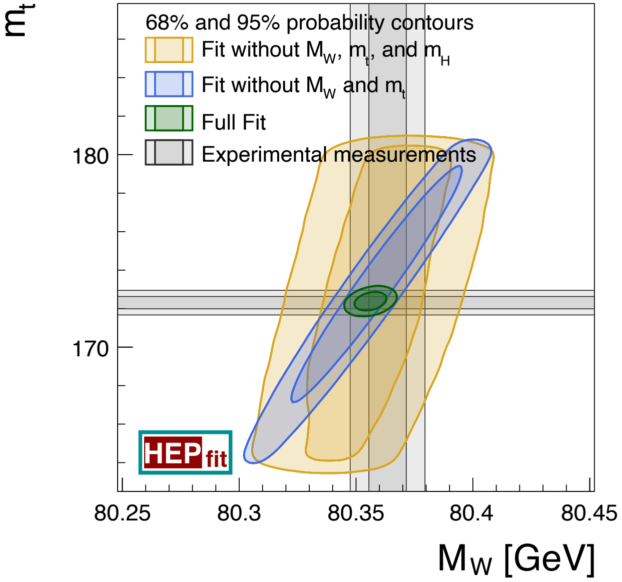
For m_t we combine:

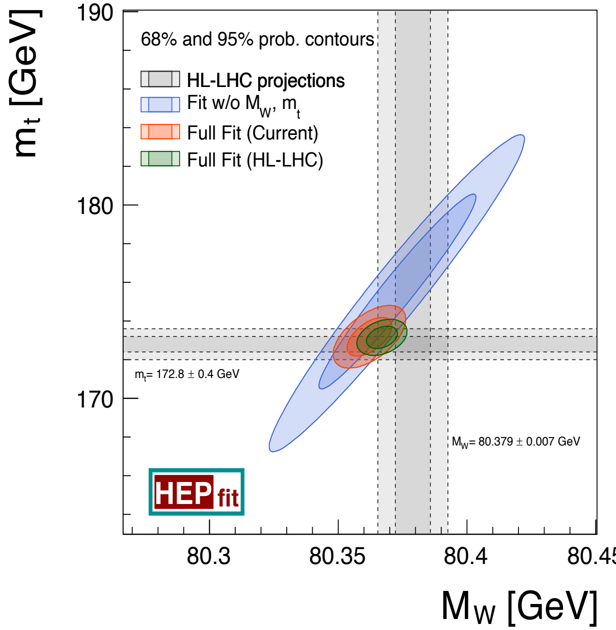
- ❑ 2016 Tevatron combination
- ❑ ATLAS Run 1 and early Run2 results
- ❑ CMS Run 1 and early Run 2 results
- ❑ CMS l+j [m_t=(171.77±0.38) GeV]
- ❑ CMS l+j boosted [m_t=(173.06±0.83) GeV]
- ❑ ATLAS l+j boosted [mt=172.95±0.53) GeV

m_t = 172.31 ± 0.32 GeV
172.38 ± 0.31 GeV (from fit)



With HL precision





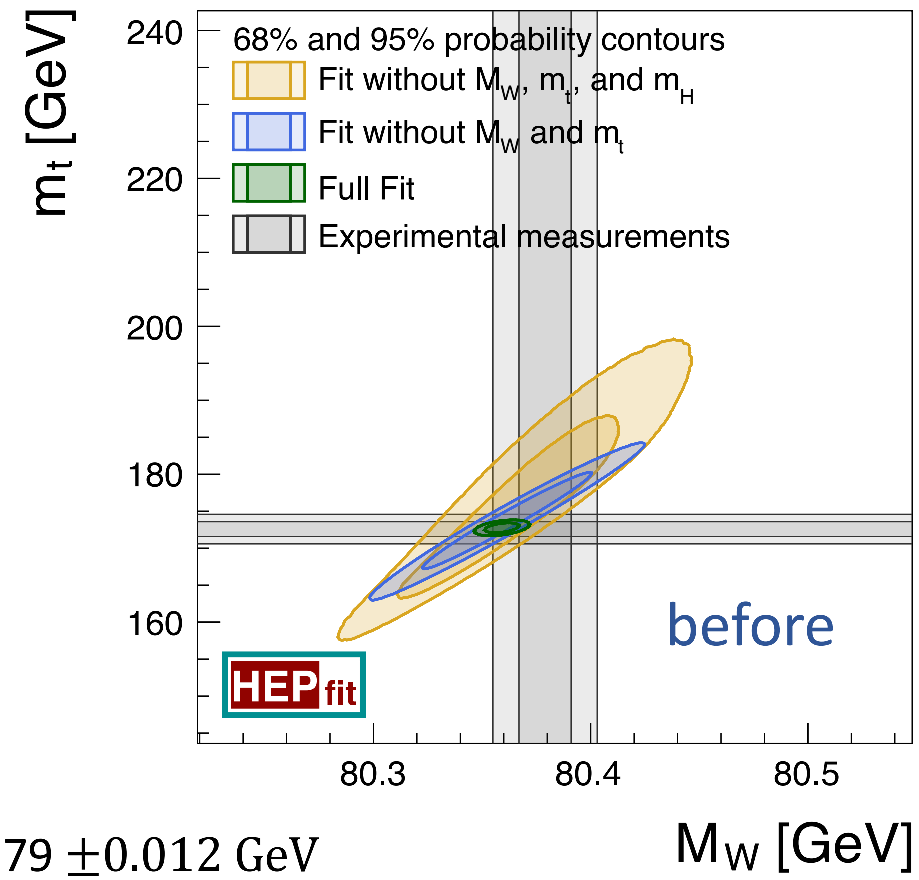
J. de Blas et al. 2204.04204, **updated**

J. de Blas et al. 1902.04070
HL/HE-LHC Report

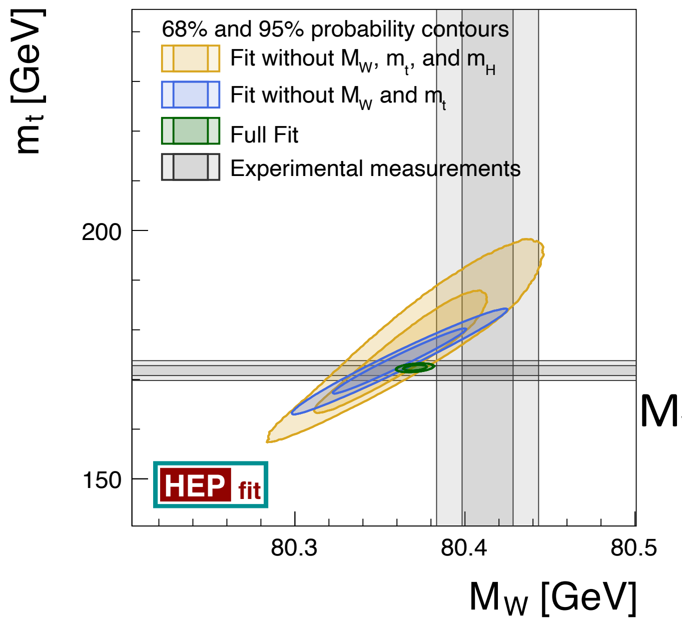
Highlighting sensitivity to anomalies

A recent challenge: CDF new M_W measurement

De Blas et al.
[2204.04204]

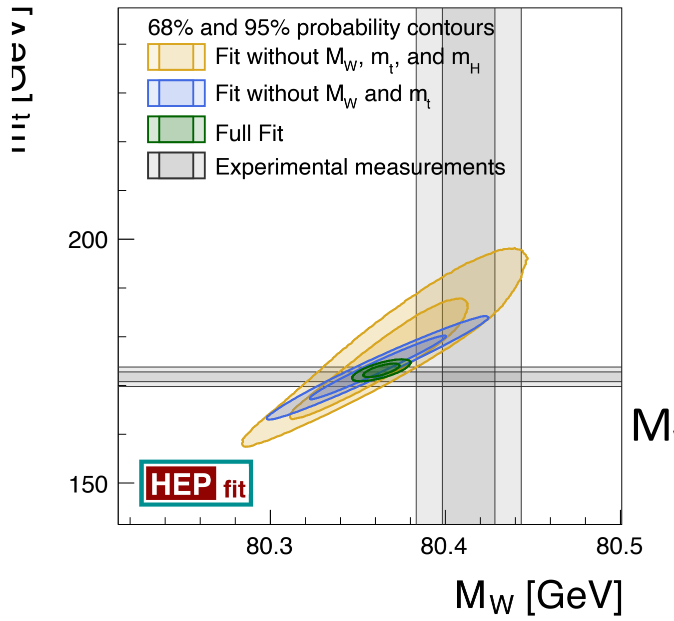


M_W=80.379 ±0.012 GeV



after "standard"

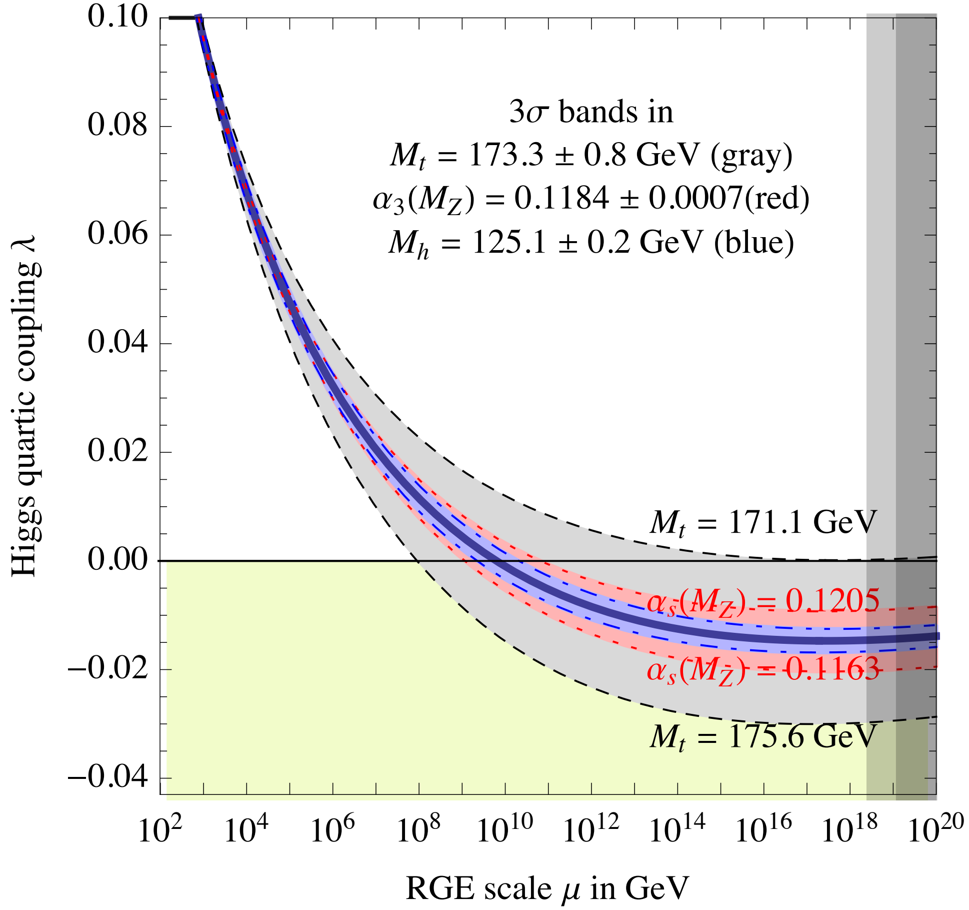
M_W=80.4133 ±0.0088 GeV



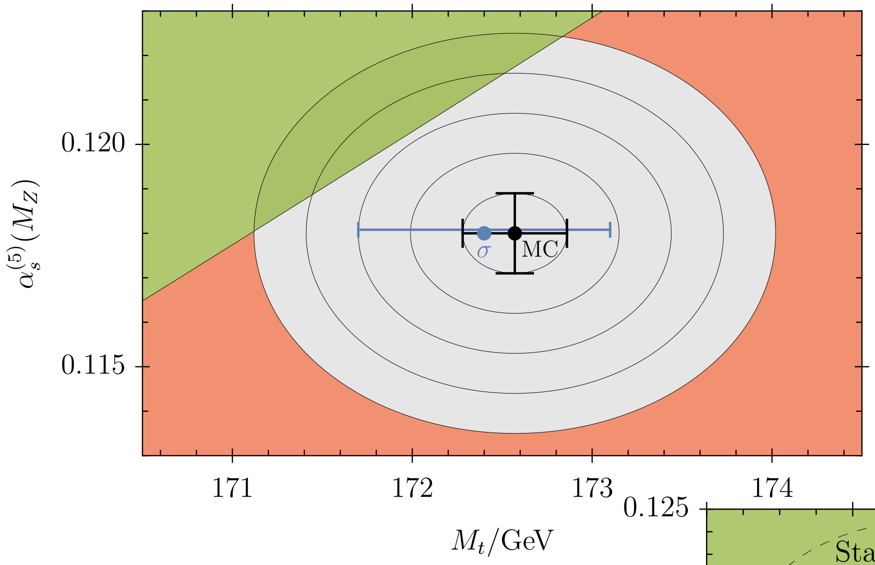
after "conservative"

M_W=80.4133 ±0.015 GeV

Far-reaching effects of EW precision fits: SM vacuum stability

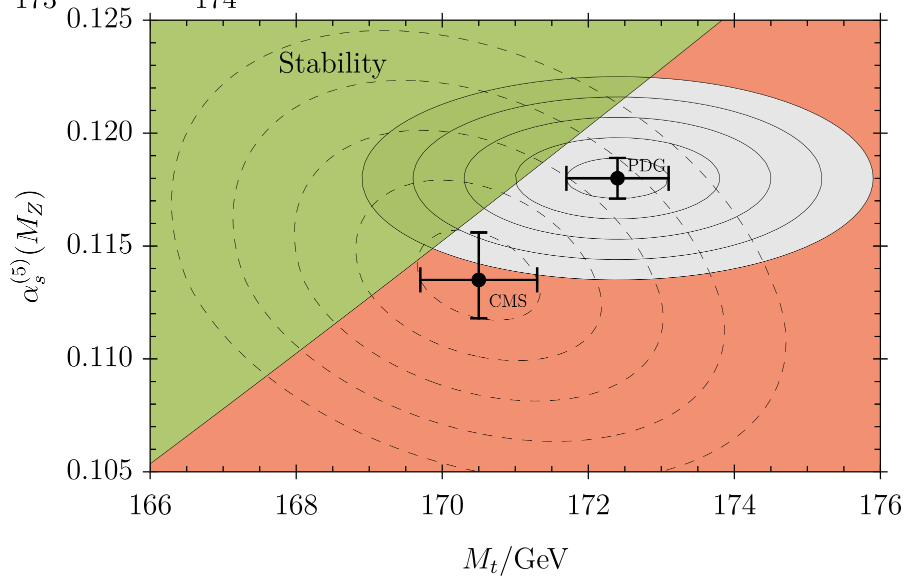


Buttazzo et al., arXiv:1307.3536



Uncertainty dominated by central values and errors for **top-quark mass** and **strong coupling constant**

G Hiller et al. arXiv:2401.08811



Criticality (**λ → 0**) condition reached for $\Lambda \approx 10^{10}$–10^{12} GeV.
Is this a signal of NP below the Planck scale?

CMS, 1904:05237:
Combined fit of M_t and α_s:
effect of **correlations**

Beyond the SM: {S,T,U}

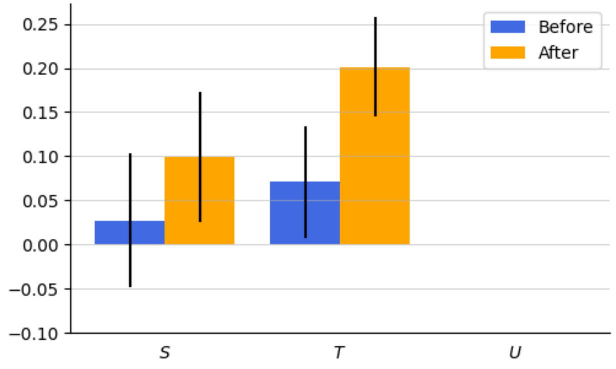
$$S = -16\pi \Pi'^{NP}_{30}(0) = 16\pi[\Pi'^{NP}_{33}(0) - \Pi'^{NP}_{3Q}(0)]$$

$$T = \frac{4}{s_W^2 c_W^2 M_Z^2}[\Pi^{NP}_{11}(0) - \Pi^{NP}_{33}(0)]$$

$$U = 16\pi[\Pi^{NP\prime}_{11} - \Pi'^{NP}_{33}(0)]$$

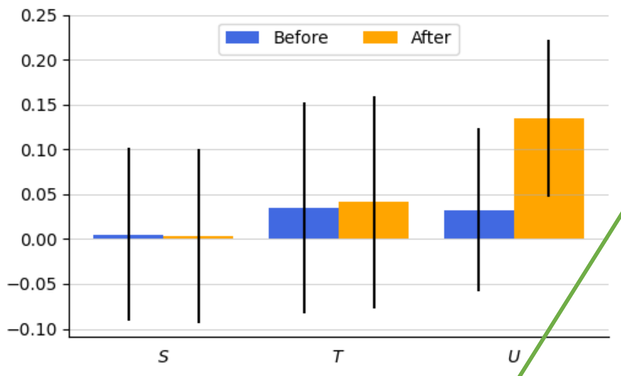
$$g_{SM} + \Delta g \begin{cases} \Delta g^{Zf\bar{f}} \propto (S,T) \\ \Delta g^{Wf'\bar{f}} \propto (S,T,U) \end{cases}$$

$$\mathcal{O} = \mathcal{O}_{SM} + \Delta\mathcal{O}_{NP}(S,T,U)$$



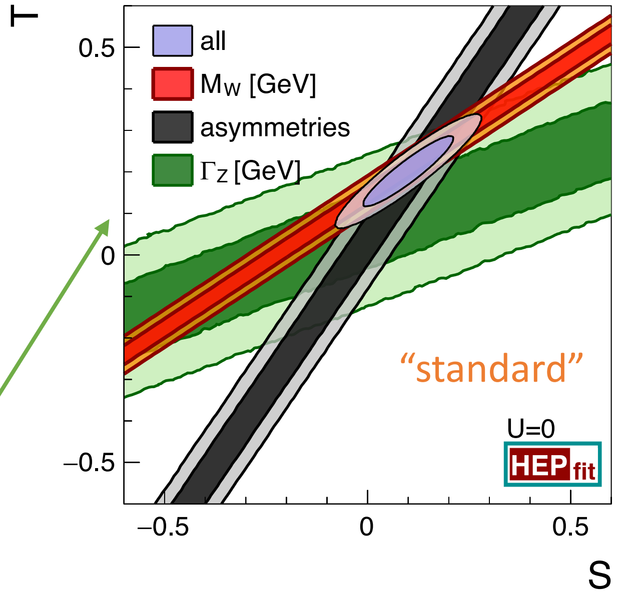
U=0, (S,T) reabsorb impact of M_W

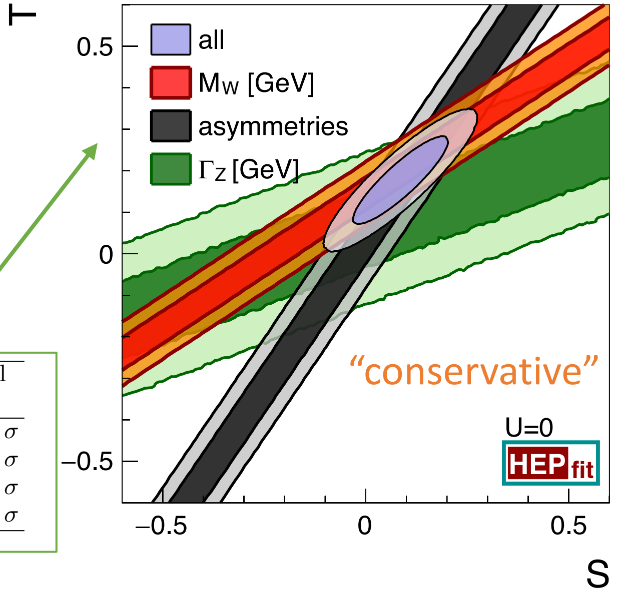
	Result	Correlation		Result	Correlation		
	$(IC_{ST}/IC_{SM} = 25.0/80.2)$			$(IC_{STU}/IC_{SM} = 25.3/80.2)$			
S	0.100 ± 0.073	1.00		0.005 ± 0.096	1.00		
T	0.202 ± 0.056	0.93	1.00	0.040 ± 0.120	0.91	1.00	
U	–	–	–	0.134 ± 0.087	−0.65	−0.88	1.00



U≠0, U reabsorb impact of M_W

Model	Pred. M_W [GeV] *standard average*	Pull	Pred. M_W [GeV] *conservative average*	Pull
SM	80.3499 ± 0.0056	$6.5\,\sigma$	80.3505 ± 0.0077	$3.7\,\sigma$
ST	80.366 ± 0.029	$1.6\,\sigma$	80.367 ± 0.029	$1.4\,\sigma$
STU	80.32 ± 0.54	$0.2\,\sigma$	80.32 ± 0.54	$0.2\,\sigma$
SMEFT	80.66 ± 1.68	$-0.1\,\sigma$	80.66 ± 1.68	$-0.1\,\sigma$





Beyond the SM: SMEFT (d=6)

$$\mathcal{O}_{\phi l}^{(1)} = (\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{l}_L \gamma^\mu l_L) \ ,$$
$$\mathcal{O}_{\phi l}^{(3)} = (\phi^\dagger i \overleftrightarrow{D}^i_\mu \phi)(\bar{l}_L \, \sigma_i \gamma^\mu l_L) \ ,$$
$$\mathcal{O}_{\phi e} = (\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{e}_R \gamma^\mu e_R) \ ,$$
$$\mathcal{O}_{\phi q}^{(1)} = (\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{q}_L \gamma^\mu q_L) \ ,$$
$$\mathcal{O}_{\phi q}^{(3)} = (\phi^\dagger i \overleftrightarrow{D}^i_\mu \phi)(\bar{q}_L \, \sigma_i \gamma^\mu q_L)$$
$$\mathcal{O}_{\phi u} = (\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{u}_R \gamma^\mu u_R) \ ,$$
$$\mathcal{O}_{\phi d} = (\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{d}_R \gamma^\mu d_R) \ ,$$

Zff/Wff vertex corrections

$$\mathcal{O}_{\phi WB} = (\phi^\dagger \sigma_i \phi) \, W^i_{\mu\nu} B^{\mu\nu} \ ,$$
$$\mathcal{O}_{\phi D} = (\phi^\dagger D^\mu \phi)^* \, (\phi^\dagger D_\mu \phi) \ ,$$

W/Z propagators ⟷ S,T

$$\mathcal{O}_{ll} = (\bar{l}_L \gamma^\mu l_L)(\bar{l}_L \gamma^\mu l_L)$$

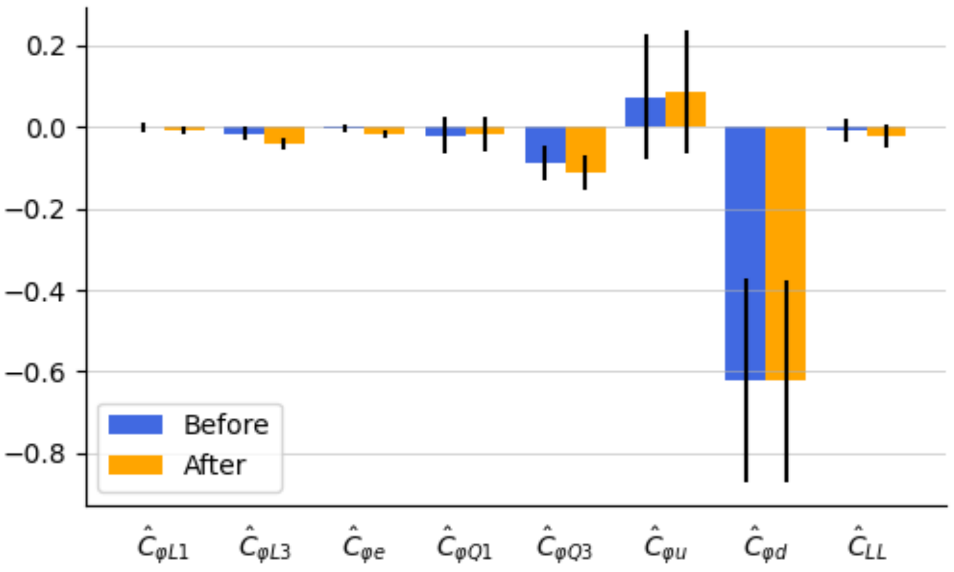
→ G_F

Only 8 independent combinations enter EWPO

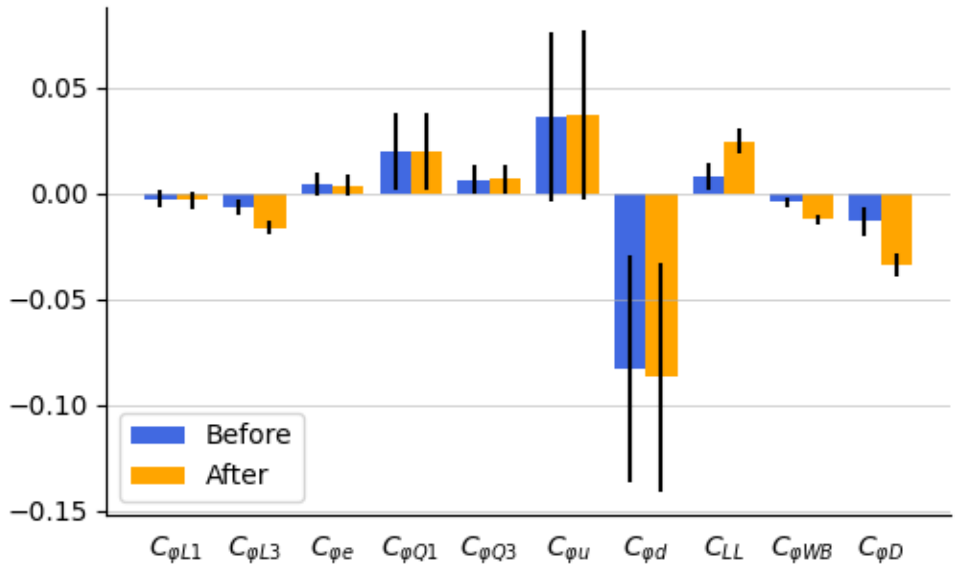
$$\hat{C}^{(1)}_{\varphi f} = C^{(1)}_{\varphi f} - \frac{Y_f}{2} C_{\varphi D}, \quad f = l, q, e, u, d,$$
$$\hat{C}^{(3)}_{\varphi f} = C^{(3)}_{\varphi f} + \frac{c_w^2}{4 s_w^2} C_{\varphi D} + \frac{c_w}{s_w} C_{\varphi WB}, \quad f = l, q,$$
$$\hat{C}_{ll} = \frac{1}{2}\left((C_{ll})_{1221} + (C_{ll})_{2112}\right) = (C_{ll})_{1221},$$

	$C_{\varphi D}$	$C_{\varphi WB}$	$C^{(3)}_{\varphi L}$	C_{LL}	$C^{(1)}_{\varphi L}$	$C_{\varphi e}$	$C^{(1)}_{\varphi Q}$	$C^{(3)}_{\varphi Q}$	$C_{\varphi u}$	$C_{\varphi d}$
M_W	✓	✓	✓	✓						
$\sin^2\theta_{eff,l}$	✓	✓	✓	✓	✓	✓				
Γ_W	✓	✓	✓	✓	✓		✓			✓
⋮										
Γ_Z	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Global fit of all coefficients



Fit of individual coefficients



No substantial impact of new mt and MW measurements, within uncertainty of the fit.

Adding Higgs and top observables will lift the degeneracy

↓

All 10 coefficients constrained independently by the global fit

	$C_{\varphi D}$	$C_{\varphi WB}$	$C_{\varphi L}^{(3)}$	C_{LL}	$C_{\varphi L}^{(1)}$	$C_{\varphi e}$	$C_{\varphi Q}^{(1)}$	$C_{\varphi Q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
M_W	✓	✓	✓	✓						
$\sin^2\theta_{eff,l}$	✓	✓	✓	✓	✓	✓				
Γ_W	✓	✓	✓	✓	✓		✓			✓
⋮										
Γ_Z	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓