Higgs-Boson Physics
Lectures I and II

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CTEQ Summer School, Pittsburgh, July 2013

Outline

- Understanding the Electroweak Symmetry Breaking (EWSB) as a first step towards a more fundamental theory of particle physics.

- Living through a new era, a particle with properties of the Standard-Model Higgs boson has been discovered at the LHC:

  $\longrightarrow$ study the newly discovered particle: Higgs precision physics
  $\longrightarrow$ look for more evidence of new physics beyond the SM

- Setting the scene:

  $\longrightarrow$ The Higgs mechanism and EWSB in the Standard Model.
  $\longrightarrow$ SM Higgs-boson production cross sections and decay branching ratios
  $\longrightarrow$ understanding hadronic environment and experimental measurements
  $\longrightarrow$ understanding and refining theoretical predictions

- Looking for a SM Higgs boson at hadron colliders:

  $\longrightarrow$ Tevatron Higgs-physics program
  $\longrightarrow$ LHC Higgs-physics program

- The Higgs paves the way ...

  $\longrightarrow$ theoretical implications of a light scalar
  $\longrightarrow$ where do we go from here?

Breaking the Electroweak Symmetry: Why and How?

- The gauge symmetry of the Standard Model (SM) forbids gauge boson mass terms, but:

$$M_{W^\pm} = 80.385 \pm 0.015\,\text{GeV} \quad \text{and} \quad M_Z = 91.1875 \pm 0.0021\,\text{GeV}$$

$$\Downarrow$$

Electroweak Symmetry Breaking (EWSB)

- Broad spectrum of ideas proposed to explain the EWSB:
  - ▷ Weakly coupled dynamics embedded into some more fundamental theory at a scale Λ (probably $\simeq$ TeV):

    $\Longrightarrow$ Higgs Mechanism in the SM or its extensions (MSSM, etc.)

    $\longrightarrow$ Little Higgs models
  - ▷ Strongly coupled dynamics at the TeV scale:

    $\longrightarrow$ Technicolor in its multiple realizations.
  - ▷ Extra dimensions beyond the 3+1 space-time dimensions

Different but related .....

- Explicit fermion mass terms also violate the gauge symmetry of the SM:

  $\longrightarrow$ introduced through new gauge invariant interactions, as dictated by the mechanism of EWSB

  $\longrightarrow$ intimately related to flavor mixing and the origin of CP-violation: new experimental evidence on this side will give further insight.

The story begins in 1964 . . .

with Englert and Brout; Higgs; Hagen, Guralnik and Kibble

VOLUME 13, NUMBER 9 PHYSICAL REVIEW LETTERS 31 AUGUST 1964

BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
(Received 26 June 1964)

VOLUME 13, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

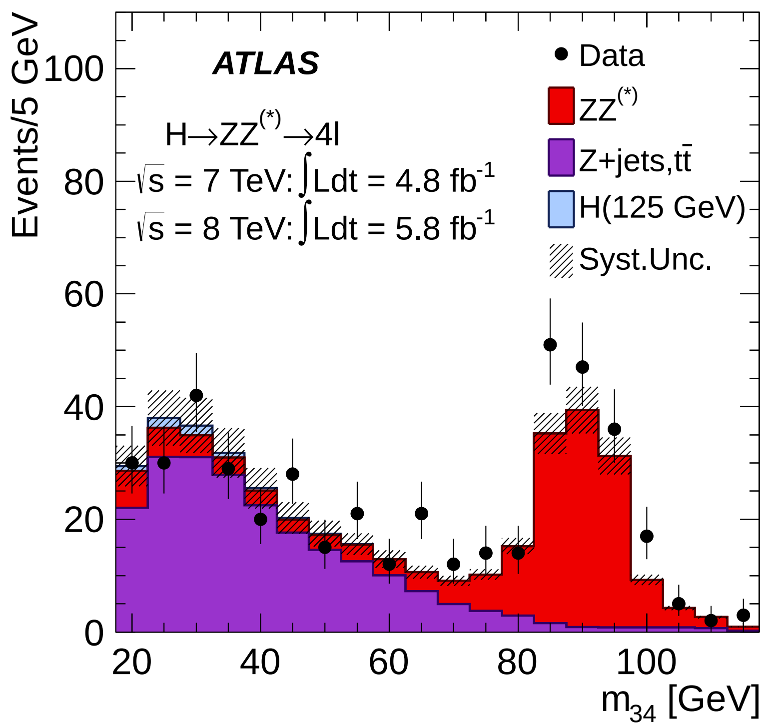
Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

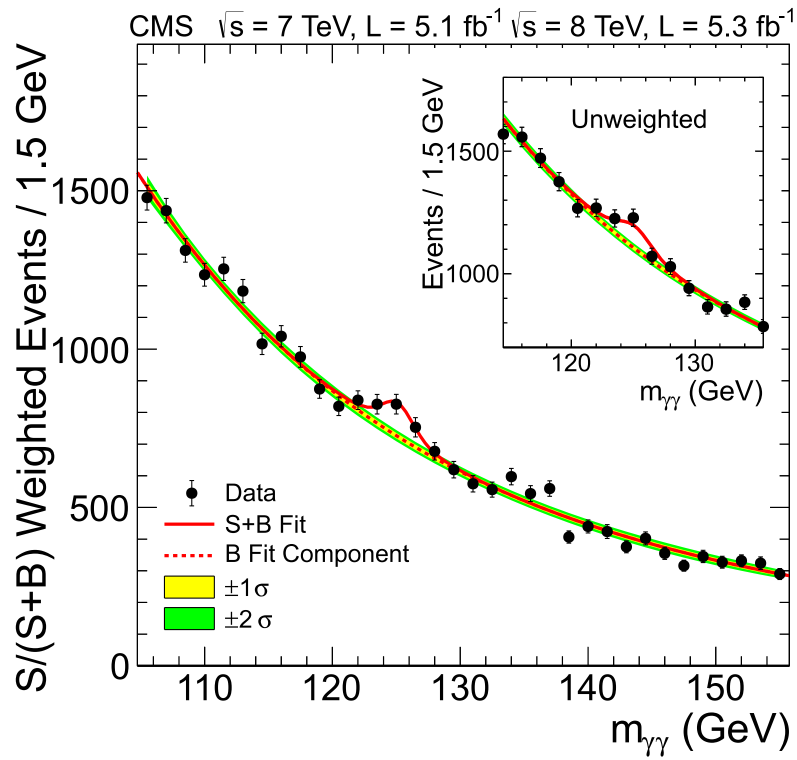
VOLUME 13, NUMBER 20 PHYSICAL REVIEW LETTERS 16 NOVEMBER 1964

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

G. S. Guralnik,† C. R. Hagen,‡ and T. W. B. Kibble
Department of Physics, Imperial College, London, England
(Received 12 October 1964)

… and comes to the discovery of a particle in 2012 …

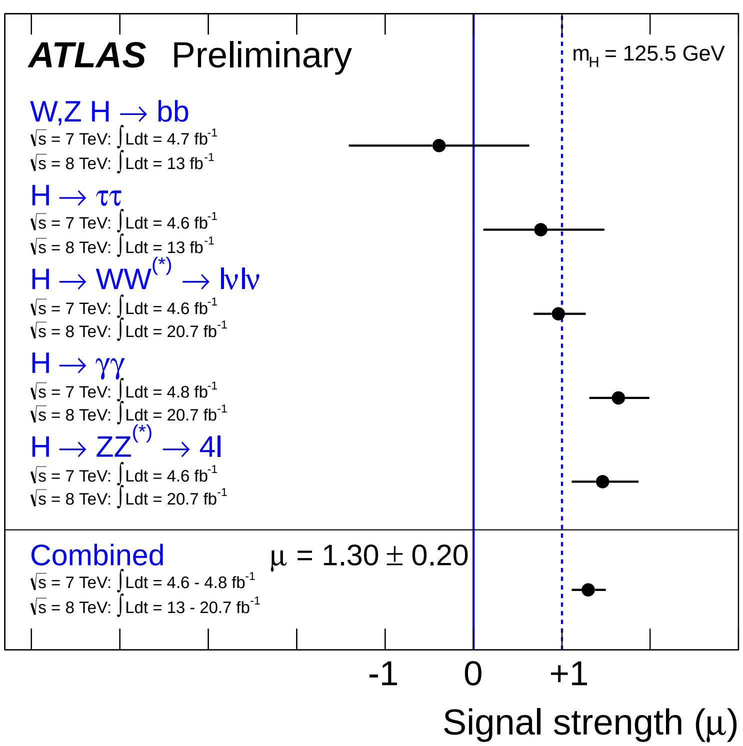


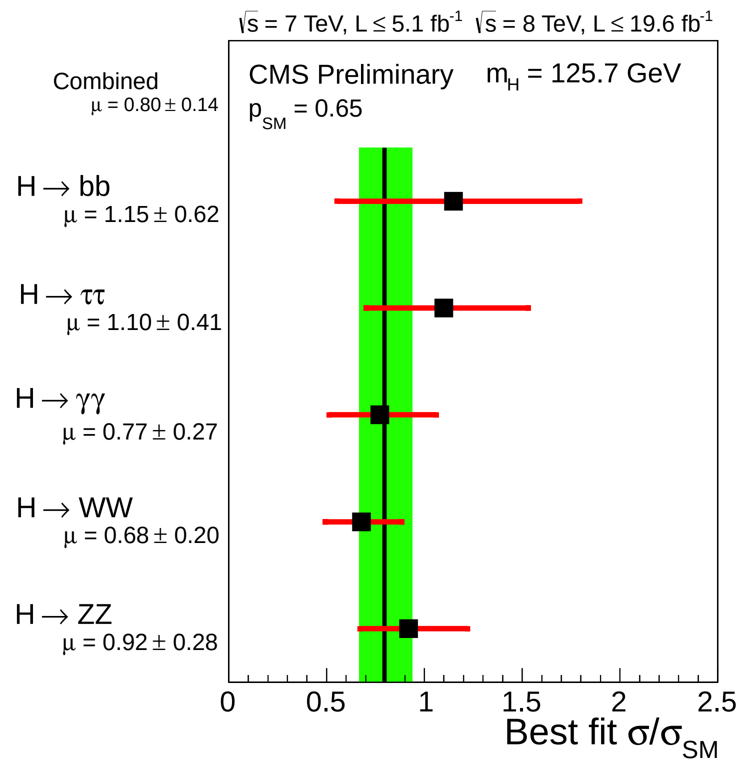


at

$$m_H = \begin{cases} 125.5 \pm 0.2\,(\text{stat})^{+0.5}_{-0.6}\,(\text{syst}) & \text{ATLAS} \\ 125.7 \pm 0.3\,(\text{stat}) \pm 0.3\,(\text{syst}) & \text{CMS} \end{cases}$$

… that very much fit the SM Higgs boson …





The Higgs mechanism and the breaking of the Electroweak Symmetry in the Standard Model

- ▷ Toy model: breaking of an abelian gauge symmetry.
- ▷ Quantum effects in spontaneously broken gauge theories.
- ▷ The Standard Model: breaking of the $SU(2)_L \times U(1)_Y$ symmetry.
- ▷ Fermion masses through Yukawa-like couplings to the Higgs field.

Spontaneous Breaking of a Gauge Symmetry

Abelian Higgs mechanism: one vector field $A^\mu(x)$ and one complex scalar field $\phi(x)$:

$$\boxed{\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi}$$

where

$$\mathcal{L}_A = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

and $(D^\mu = \partial^\mu + igA^\mu)$

$$\mathcal{L}_\phi = (D^\mu\phi)^* D_\mu\phi - V(\phi) = (D^\mu\phi)^* D_\mu\phi - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

$\mathcal{L}$ invariant under local phase transformation, or local $U(1)$ symmetry:

$$\begin{aligned} \phi(x) &\rightarrow e^{i\alpha(x)}\phi(x) \\ A^\mu(x) &\rightarrow A^\mu(x) + \frac{1}{g}\partial^\mu\alpha(x) \end{aligned}$$

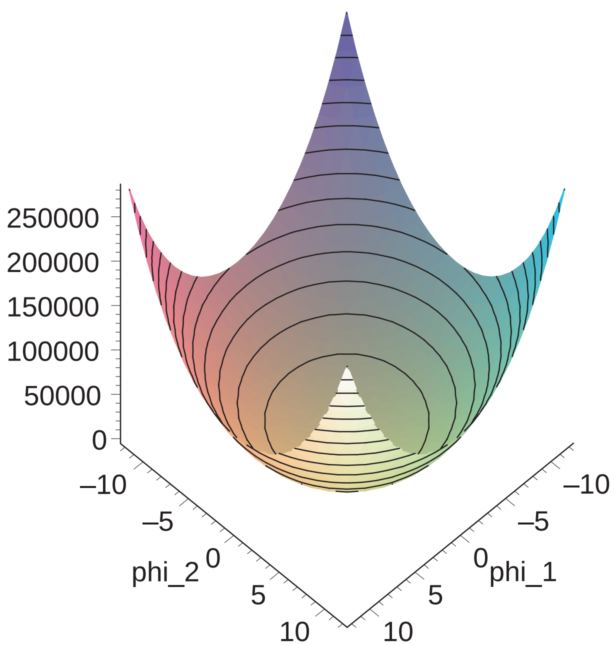
Mass term for A^μ breaks the $U(1)$ gauge invariance.

Can we build a gauge invariant massive theory? Yes.

Consider the potential of the scalar field:

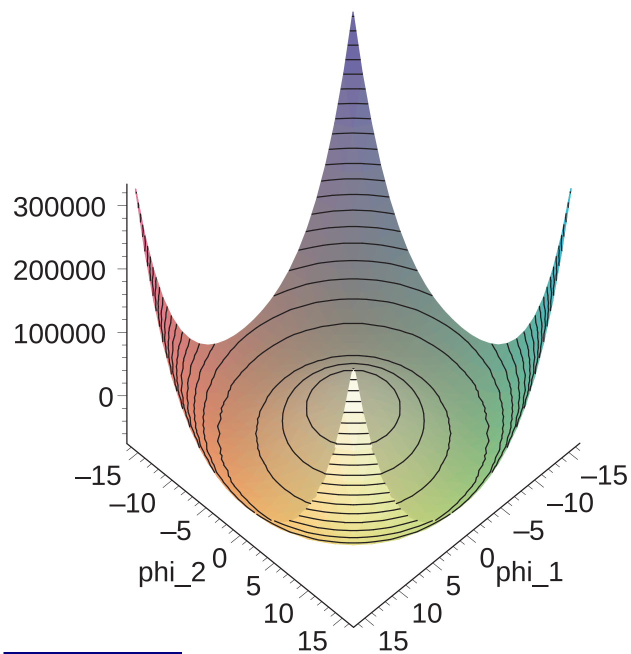
$$V(\phi) = \mu^2 \phi^* \phi + \lambda(\phi^* \phi)^2$$

where $\lambda > 0$ (to be bounded from below), and observe that:



$\boxed{\mu^2 > 0}$ $\rightarrow$ unique minimum:

$$\phi^* \phi = 0$$



$\boxed{\mu^2 < 0}$ $\rightarrow$ degeneracy of minima:

$$\phi^* \phi = \frac{-\mu^2}{2\lambda}$$

- $\mu^2 > 0 \longrightarrow$ electrodynamics of a massless photon and a massive scalar field of mass μ ($g = -e$).

- $\mu^2 < 0 \longrightarrow$ when we choose a minimum, the original $U(1)$ symmetry is spontaneously broken or hidden.

$$\phi_0 = \left(-\frac{\mu^2}{2\lambda}\right)^{1/2} = \frac{v}{\sqrt{2}} \quad \longrightarrow \quad \phi(x) = \phi_0 + \frac{1}{\sqrt{2}}\left(\phi_1(x) + i\phi_2(x)\right)$$

$$\Downarrow$$

$$\mathcal{L} = \underbrace{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}g^2v^2A^\mu A_\mu}_{\text{massive vector field}} + \underbrace{\frac{1}{2}(\partial^\mu\phi_1)^2 + \mu^2\phi_1^2}_{\text{massive scalar field}} + \underbrace{\frac{1}{2}(\partial^\mu\phi_2)^2 + gvA_\mu\partial^\mu\phi_2}_{\text{Goldstone boson}} + \ldots$$

Side remark: The ϕ_2 field actually generates the correct transverse structure for the mass term of the (now massive) A^μ field propagator:

$$\langle A^\mu(k)A^\nu(-k)\rangle = \frac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) + \cdots$$

More convenient parameterization (unitary gauge):

$$\phi(x) = \frac{e^{i\frac{\chi(x)}{v}}}{\sqrt{2}}(v + H(x)) \quad \xrightarrow{U(1)} \quad \frac{1}{\sqrt{2}}(v + H(x))$$

The $\chi(x)$ degree of freedom (Goldstone boson) is rotated away using gauge invariance, while the original Lagrangian becomes:

$$\mathcal{L} = \mathcal{L}_A + \frac{g^2v^2}{2}A^\mu A_\mu + \frac{1}{2}\left(\partial^\mu H \partial_\mu H + 2\mu^2 H^2\right) + \ldots$$

which describes now the dynamics of a system made of:

- a massive vector field A^μ with $m_A^2 = g^2v^2$;
- a real scalar field H of mass $m_H^2 = -2\mu^2 = 2\lambda v^2$: the Higgs field.

$$\Downarrow$$

Total number of degrees of freedom is balanced

Non-Abelian Higgs mechanism: several vector fields $A_\mu^a(x)$ and several (real) scalar field $\phi_i(x)$:

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi \quad , \quad \mathcal{L}_\phi = \frac{1}{2}(D^\mu\phi)^2 - V(\phi) \quad , \quad V(\phi) = \mu^2\phi^2 + \frac{\lambda}{2}\phi^4$$

($\mu^2 < 0$, $\lambda > 0$) invariant under a non-Abelian symmetry group G:

$$\phi_i \longrightarrow (1 + i\alpha^a t^a)_{ij}\phi_j \overset{t^a = iT^a}{\longrightarrow} (1 - \alpha^a T^a)_{ij}\phi_j$$

(s.t. $D_\mu = \partial_\mu + gA_\mu^a T^a$). In analogy to the Abelian case:

$$\frac{1}{2}(D_\mu\phi)^2 \quad \longrightarrow \quad \ldots + \frac{1}{2}g^2(T^a\phi)_i(T^b\phi)_i A_\mu^a A^{b\mu} + \ldots$$

$$\overset{\phi_{min}=\phi_0}{\longrightarrow} \quad \ldots + \frac{1}{2}\underbrace{g^2(T^a\phi_0)_i(T^b\phi_0)_i}_{m_{ab}^2} A_\mu^a A^{b\mu} + \ldots =$$

$\boxed{T^a\phi_0 \neq 0}$ $\longrightarrow$ massive vector boson + (Goldstone boson)

$\boxed{T^a\phi_0 = 0}$ $\longrightarrow$ massless vector boson + massive scalar field

Classical $\longrightarrow$ Quantum : $\boxed{V(\phi) \longrightarrow V_{eff}(\varphi_{cl})}$

The stable vacuum configurations of the theory are now determined by the extrema of the Effective Potential:

$$V_{eff}(\varphi_{cl}) = -\frac{1}{VT}\Gamma_{eff}[\phi_{cl}] \ \ , \ \ \phi_{cl} = \text{constant} = \varphi_{cl}$$

where

$$\Gamma_{eff}[\phi_{cl}] = W[J] - \int d^4y J(y)\phi_{cl}(y) \quad , \quad \phi_{cl}(x) = \frac{\delta W[J]}{\delta J(x)} = \langle 0|\phi(x)|0\rangle_J$$

$W[J] \longrightarrow$ generating functional of connected correlation functions
$\Gamma_{eff}[\phi_{cl}] \longrightarrow$ generating functional of 1PI connected correlation functions

$V_{eff}(\varphi_{cl})$ can be organized as a loop expansion (expansion in $\hbar$), s.t.:

$$V_{eff}(\varphi_{cl}) = V(\varphi_{cl}) + \text{loop effects}$$

SSB $\longrightarrow$ non trivial vacuum configurations

Gauge fixing : the R_ξ gauges. Consider the abelian case:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^* D_\mu\phi - V(\phi)$$

upon SSB:

$$\phi(x) = \frac{1}{\sqrt{2}}((v + \phi_1(x)) + i\phi_2(x))$$

$$\Downarrow$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial^\mu\phi_1 + gA^\mu\phi_2)^2 + \frac{1}{2}(\partial^\mu\phi_2 - gA^\mu(v + \phi_1))^2 - V(\phi)$$

Quantizing using the gauge fixing condition:

$$G = \frac{1}{\sqrt{\xi}}(\partial_\mu A^\mu + \xi g v \phi_2)$$

in the generating functional

$$Z = C\int \mathcal{D}A\mathcal{D}\phi_1\mathcal{D}\phi_2 \exp\left[\int d^4x \left(\mathcal{L} - \frac{1}{2}G^2\right)\right] \det\left(\frac{\delta G}{\delta\alpha}\right)$$

($\alpha \longrightarrow$ gauge transformation parameter)

$$
\begin{aligned}
\mathcal{L} - \frac{1}{2}G^2 &= -\frac{1}{2}A_\mu\left(-g^{\mu\nu}\partial^2 + \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu - (gv)^2 g^{\mu\nu}\right)A_\nu \\
& \quad \frac{1}{2}(\partial_\mu\phi_1)^2 - \frac{1}{2}m_{\phi_1}^2\phi_1^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 - \frac{\xi}{2}(gv)^2\phi_2^2 + \cdots \\
&+ \\
\mathcal{L}_{ghost} &= \bar{c}\left[-\partial^2 - \xi(gv)^2\left(1 + \frac{\phi_1}{v}\right)\right]c
\end{aligned}
$$

such that:

$$
\begin{aligned}
\langle A^\mu(k)A^\nu(-k)\rangle &= \frac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) + \frac{-i\xi}{k^2 - \xi m_A^2}\left(\frac{k^\mu k^\nu}{k^2}\right) \\
\langle\phi_1(k)\phi_1(-k)\rangle &= \frac{-i}{k^2 - m_{\phi_1}^2} \\
\langle\phi_2(k)\phi_2(-k)\rangle &= \langle c(k)\bar{c}(-k)\rangle = \frac{-i}{k^2 - \xi m_A^2}
\end{aligned}
$$

$$
\boxed{\text{Goldtone boson } \phi_2, \iff \text{longitudinal gauge bosons}}
$$

The Higgs sector of the Standard Model :

$SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_Q$

Introduce one complex scalar doublet of $SU(2)_L$ with $Y=1/2$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \longleftrightarrow \quad \mathcal{L} = (D^\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

where $D_\mu \phi = (\partial_\mu - igA^a_\mu \tau^a - ig' Y_\phi B_\mu)$, $(\tau^a = \sigma^a/2,\ a=1,2,3)$.

The SM symmetry is spontaneously broken when $\langle \phi \rangle$ is chosen to be (e.g.):

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{with} \quad v = \left(\frac{-\mu^2}{\lambda} \right)^{1/2} \quad (\mu^2 < 0,\ \lambda > 0)$$

The gauge boson mass terms arise from:

$$(D^\mu \phi)^\dagger D_\mu \phi \quad \longrightarrow \quad \cdots + \frac{1}{8} (0 \;\; v) \left(g A^a_\mu \sigma^a + g' B_\mu \right) \left(g A^{b\mu} \sigma^b + g' B^\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} + \cdots$$

$$\longrightarrow \quad \cdots + \frac{1}{2} \frac{v^2}{4} \left[g^2 (A^1_\mu)^2 + g^2 (A^2_\mu)^2 + (-g A^3_\mu + g' B_\mu)^2 \right] + \cdots$$

And correspond to the weak gauge bosons:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \pm A_\mu^2) \longrightarrow \boxed{M_W = g\frac{v}{2}}$$

$$Z_\mu^0 = \frac{1}{\sqrt{g^2+g'^2}}(gA_\mu^3 - g'B_\mu) \longrightarrow \boxed{M_Z = \sqrt{g^2+g'^2}\frac{v}{2}}$$

while the linear combination orthogonal to Z_μ^0 remains massless and corresponds to the photon field:

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'A_\mu^3 + gB_\mu) \longrightarrow \boxed{M_A = 0}$$

Notice: using the definition of the weak mixing angle, θ_w:

$$\cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}} \quad , \quad \sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}$$

the W and Z masses are related by: $\boxed{M_W = M_Z\cos\theta_w}$

The scalar sector becomes more transparent in the unitary gauge:

$$\phi(x) = \frac{e^{\frac{i}{v}\vec{\chi}(x)\cdot\vec{\tau}}}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \xrightarrow{SU(2)} \quad \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

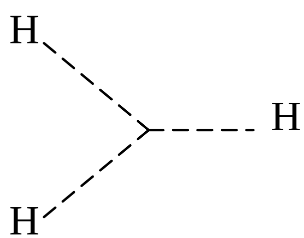
after which the Lagrangian becomes

$$\mathcal{L} = \mu^2 H^2 - \lambda v H^3 - \frac{1}{4} H^4 = -\frac{1}{2} M_H^2 H^2 - \sqrt{\frac{\lambda}{2}} M_H H^3 - \frac{1}{4}\lambda H^4$$

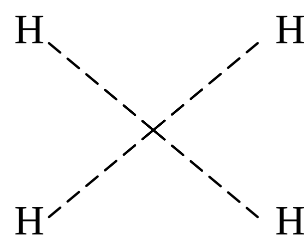
Three degrees of freedom, the $\chi^a(x)$ Goldstone bosons, have been reabsorbed into the longitudinal components of the $W_\mu^\pm$ and Z_μ^0 weak gauge bosons. One real scalar field remains:

the Higgs boson, H, with mass $\boxed{M_H^2 = -2\mu^2 = 2\lambda v^2}$

and self-couplings:

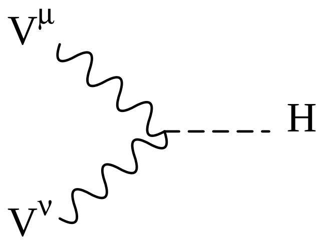


$= -3i\frac{M_H^2}{v}$

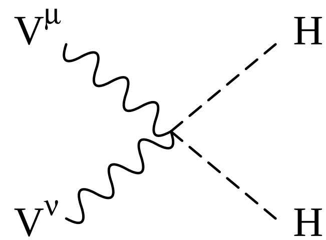


$= -3i\frac{M_H^2}{v^2}$

From $(D^\mu\phi)^\dagger D_\mu\phi \longrightarrow$ Higgs-Gauge boson couplings:



$= 2i\frac{M_V^2}{v}g^{\mu\nu}$



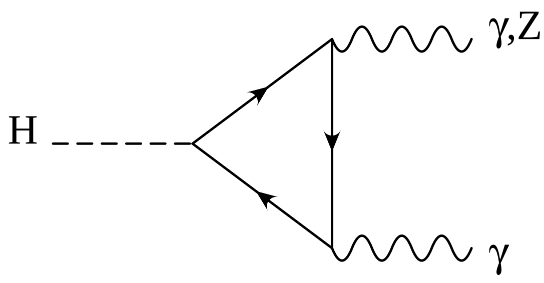
$= 2i\frac{M_V^2}{v^2}g^{\mu\nu}$

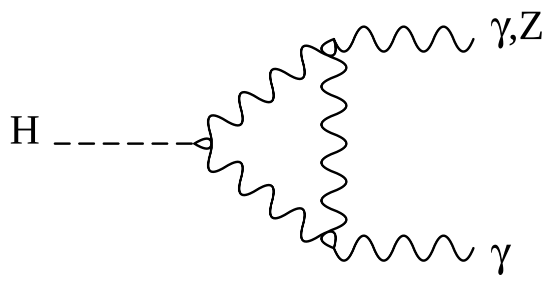
Notice: The entire Higgs sector depends on only two parameters, e.g.

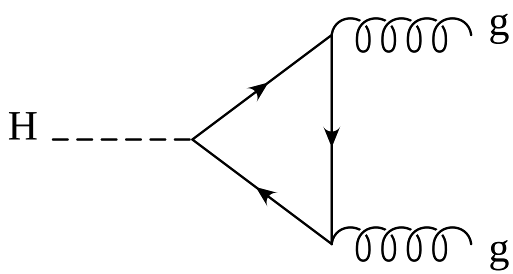
M_H and v

v measured in μ-decay:
$v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV $\longrightarrow$ **SM Higgs Physics depends on M_H**

Also: remember Higgs-gauge boson loop-induced couplings:







They will be discussed in the context of Higgs boson decays.

Finally: Higgs boson couplings to quarks and leptons

The gauge symmetry of the SM also forbids fermion mass terms ($m_{Q_i} Q^i_L u^i_R, \ldots$), but all fermions are massive.

$$\Downarrow$$

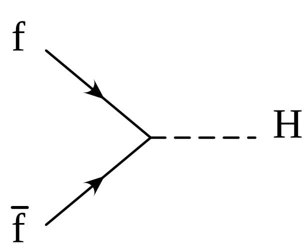
Fermion masses are generated via gauge invariant Yukawa couplings:

$$\mathcal{L}_{Yukawa} = -\Gamma^{ij}_u \bar{Q}^i_L \phi^c u^j_R - \Gamma^{ij}_d \bar{Q}^i_L \phi d^j_R - \Gamma^{ij}_e \bar{L}^i_L \phi l^j_R + h.c.$$

such that, upon spontaneous symmetry breaking:

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \longrightarrow \boxed{m_f = \Gamma_f \frac{v}{\sqrt{2}}}$$

and



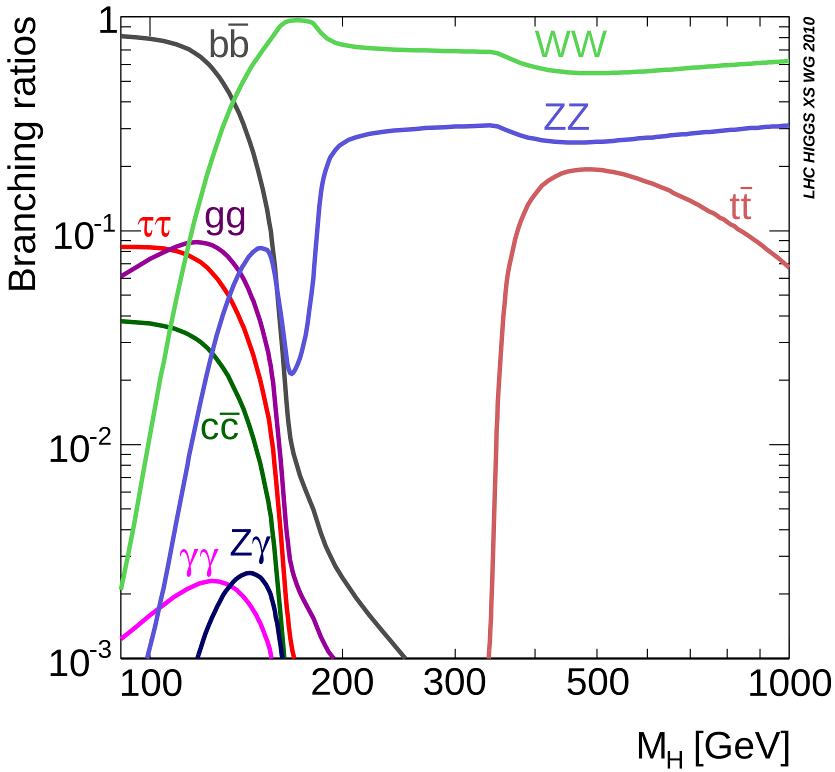
$$= -i\frac{m_f}{v} = -iy_t$$

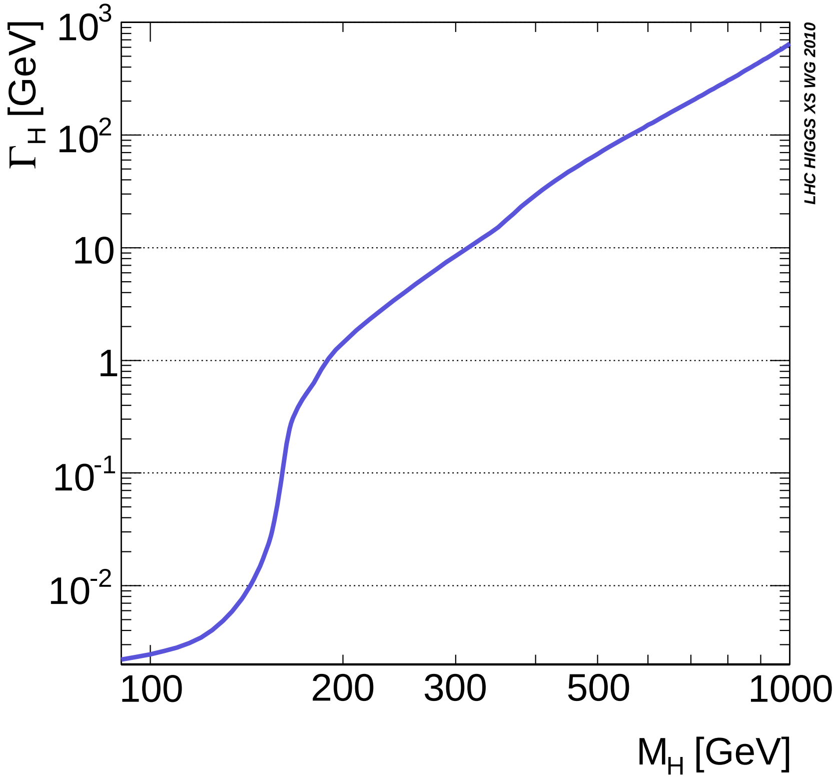
Essential building blocks to understand the Tevatron and LHC Higgs-physics program:

- ▷ decay branching ratios
- ▷ hadronic production cross sections

including quantum corrections due to strong and EW interactions

SM Higgs boson decay branching ratios and width





Observe difference between light and heavy Higgs

These curves include: tree level + QCD and EW loop corrections

Tree level decays: $H \to f\bar{f}$ and $H \to VV$ $(V = W, Z)$

At lowest order:

$$\Gamma(H \to f\bar{f}) = \frac{G_F M_H}{4\sqrt{2}\pi} N_{cf} m_f^2 \beta_f^3$$

$$\Gamma(H \to VV) = \frac{G_F M_H^3}{16\sqrt{2}\pi} \delta_V \left(1 - \tau_V + \frac{3}{4}\tau_V^2\right) \beta_V$$

$(\beta_i = \sqrt{1-\tau_i},\ \tau_i = 4m_i^2/M_H^2,\ \delta_{W,Z} = 2, 1,\ (N_c)_{l,q} = 1, 3)$

Ex.1: **Higher order corrections to $H \to q\bar{q}$**

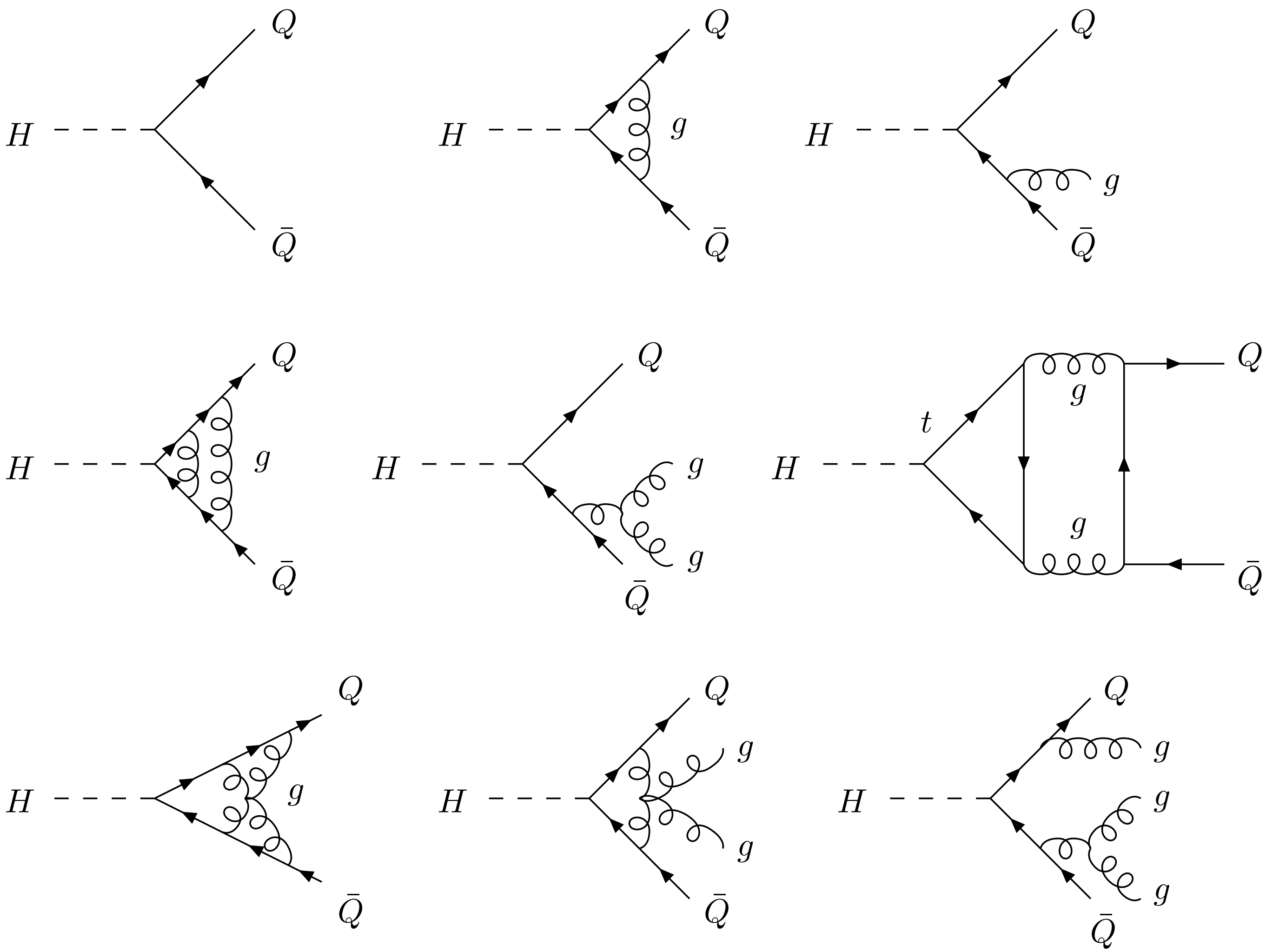
QCD corrections dominant:

$$\Gamma(H \to q\bar{q})_{\rm QCD} = \frac{3 G_F M_H}{4\sqrt{2}\pi} \bar{m}_q^2(M_H) \beta_q^3 \left[\Delta_{QCD} + \Delta_t\right]$$

$$\Delta_{QCD} = 1 + 5.67 \frac{\alpha_s(M_H)}{\pi} + (35.94 - 1.36 N_F) \left(\frac{\alpha_s(M_H)}{\pi}\right)^2 + \cdots$$

$$\Delta_t = \left(\frac{\alpha_s(M_H)}{\pi}\right)^2 \left[1.57 - \frac{2}{3} \ln \frac{M_H^2}{m_t^2} + \frac{1}{9} \ln^2 \frac{\bar{m}_q^2(M_H)}{M_H^2}\right] + \cdots$$

Consist of both virtual and real corrections, e.g.:



- Large Logs absorbed into $\overline{MS}$ quark mass

$$\text{Leading Order}: \quad \bar{m}_Q(\mu) = \bar{m}_Q(m_Q)\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{\frac{2b_0}{\gamma_0}}$$

$$\text{Higher order}: \quad \bar{m}_Q(\mu) = \bar{m}_Q(m_Q)\frac{f\left(\alpha_s(\mu)/\pi\right)}{f\left(\alpha_s(m_Q)/\pi\right)}$$

where (from renormalization group equation)

$$\begin{aligned}
f(x) &= \left(\frac{25}{6}x\right)^{\frac{12}{25}}[1 + 1.014x + \ldots] \quad \text{for} \quad m_c < \mu < m_b \\
f(x) &= \left(\frac{23}{6}x\right)^{\frac{12}{23}}[1 + 1.175x + \ldots] \quad \text{for} \quad m_b < \mu < m_t \\
f(x) &= \left(\frac{7}{2}x\right)^{\frac{4}{7}}[1 + 1.398x + \ldots] \quad \text{for} \quad \mu > m_t
\end{aligned}$$

- Large corrections, when $M_H \gg m_Q$

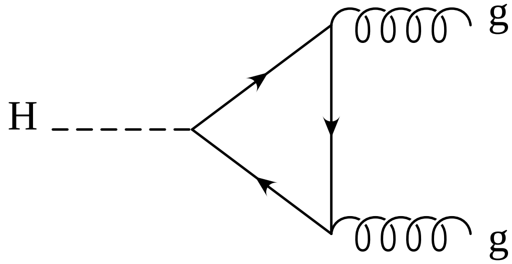
$$m_b(m_b) \simeq 4.2\,\text{GeV} \longrightarrow \bar{m}_b(M_h \simeq 100\,\text{GeV}) \simeq 3\,\text{GeV}$$

  Branching ratio smaller by almost a factor 2.

- Main uncertainties: $\alpha_s(M_Z)$, pole masses: $m_c(m_c)$, $m_b(m_b)$.

Loop-induced decays: $H \to gg$, $H \to \gamma\gamma$, and $H \to Z\gamma$

Start from lowest order:



$$\Gamma(H \to gg) = \frac{G_F \alpha_s^2 M_H^3}{36\sqrt{2}\pi^3} \left| \sum_q A_q^H(\tau_q) \right|$$

where $\tau_q = 4m_q^2/M_H^2$ and

$$A_q^H(\tau) = \frac{3}{2}\tau \left[1 + (1-\tau) f(\tau)\right]$$

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4}\left[\ln \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & \tau < 1 \end{cases}$$

Main contribution from top quark $\longrightarrow$ optimal situation to use Low Energy Theorems to add higher order corrections.

Low-energy theorems, in a nutshell.

- Observing that:
  In the $p_H \to 0$ limit: the interactions of a Higgs boson with the SM particles arise by substituting

$$M_i \longrightarrow M_i \left(1 + \frac{H}{v}\right) \quad (i = f, W, Z)$$

  In practice: Higgs taken on shell ($p_H^2 = M_H^2$), and limit $p_H \to 0$ is limit of small Higgs masses (e.g.: $M_H^2 \ll 4m_t^2$).

- Then

$$\lim_{p_H \to 0} \mathcal{A}(X \to Y + H) = \frac{1}{v} \sum_i M_i \frac{\partial}{\partial M_i} \mathcal{A}(X \to Y)$$

  very convenient!

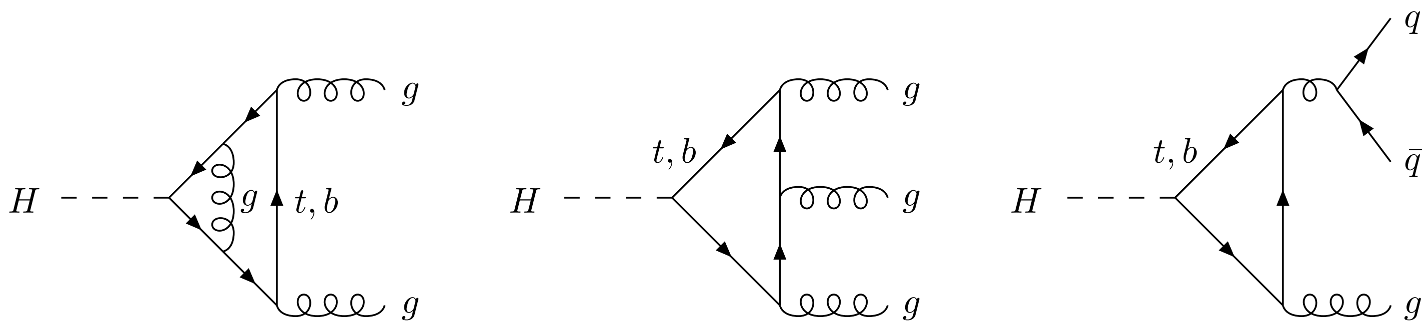
- Equivalent to an Effective Theory described by:

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} \frac{H}{v} (1 + O(\alpha_s))$$

  including higher order QCD corrections.

Ex. 2: Higher order corrections to $H \to gg$

QCD corrections dominant:



Difficult task since decay is already a loop effect.

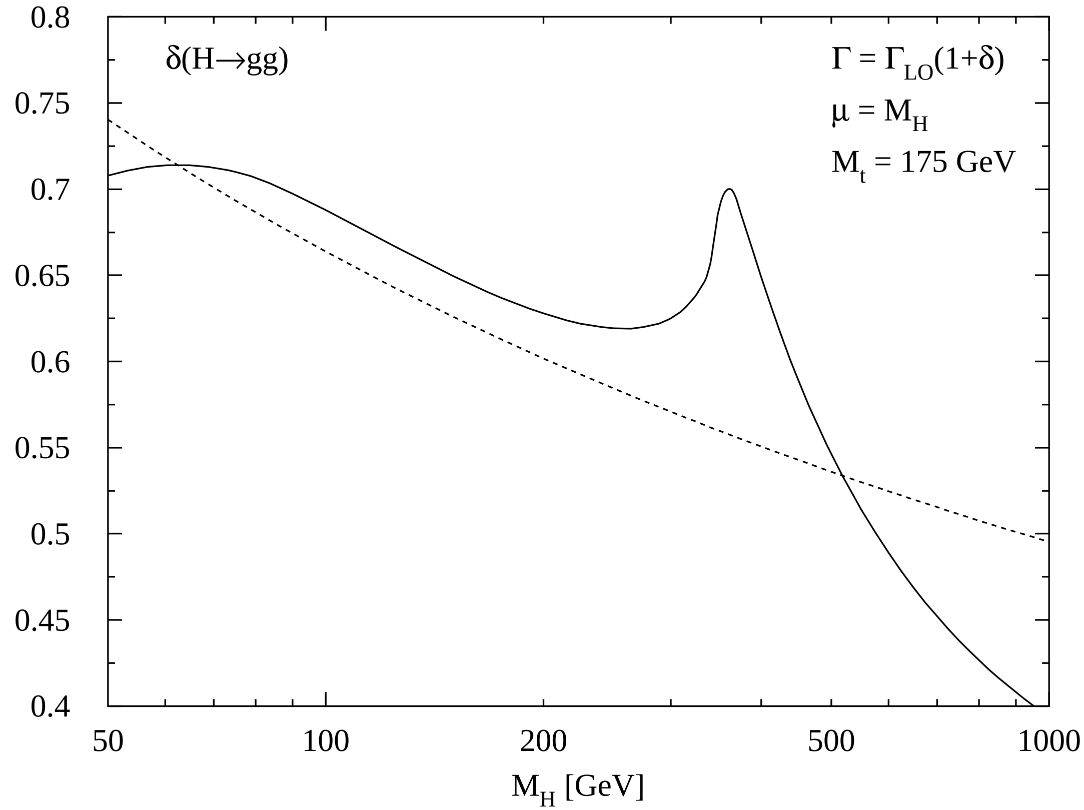
However, full massive calculation of $\Gamma(H \to gg(q), q\bar{q}g)$ agrees with $m_t \gg M_H$ result at 10%

$$\Gamma(H \to gg(q), q\bar{q}g) = \Gamma_{LO}(\alpha_s^{(N_L)}(M_H)) \left[1 + E^{(N_L)} \frac{\alpha_s^{(N_L)}}{\pi}\right]$$

$$E^{(N_L)} \xrightarrow{M_H^2 \ll 4m_q^2} \frac{95}{4} - \frac{7}{6} N_L$$

Dominant soft/collinear radiation do not resolve the Higgs boson coupling to gluons $\longrightarrow$ QCD corrections are just a (big) rescaling factor

NLO QCD corrections almost $60 - 70\%$ of LO result in the low mass region:



solid line $\longrightarrow$ full massive NLO calculation

dashed line $\longrightarrow$ heavy top limit $(M_H^2 \ll 4m_t^2)$

NNLO corrections calculated in the heavy top limit: add 20%

$\longrightarrow$ perturbative stabilization. Residual theoretical uncertainty $\simeq 10\%$.

For completeness:

$$\Gamma(H\to\gamma\gamma)=\frac{G_F\alpha^2M_H^3}{128\sqrt{2}\pi^3}\left|\sum_f N_{cf}e_f^2A_f^H(\tau_f)+A_W^H(\tau_W)\right|^2$$

where ($f(\tau)$ as in $H\to gg$):

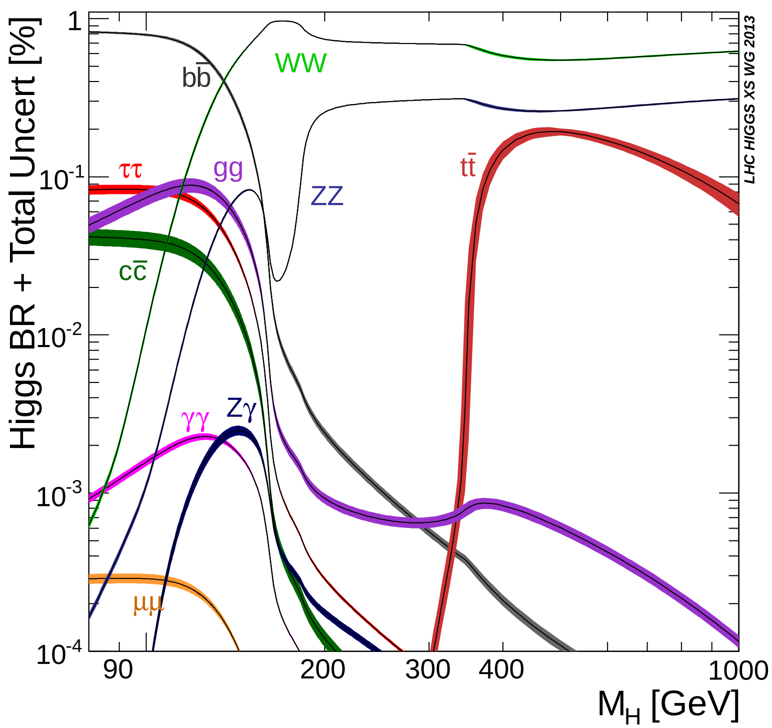
$$\begin{aligned} A_f^H &= 2\tau\left[1+(1-\tau)f(\tau)\right] \\ A_W^H(\tau) &= -\left[2+3\tau+3\tau(2-\tau)f(\tau)\right] \end{aligned}$$

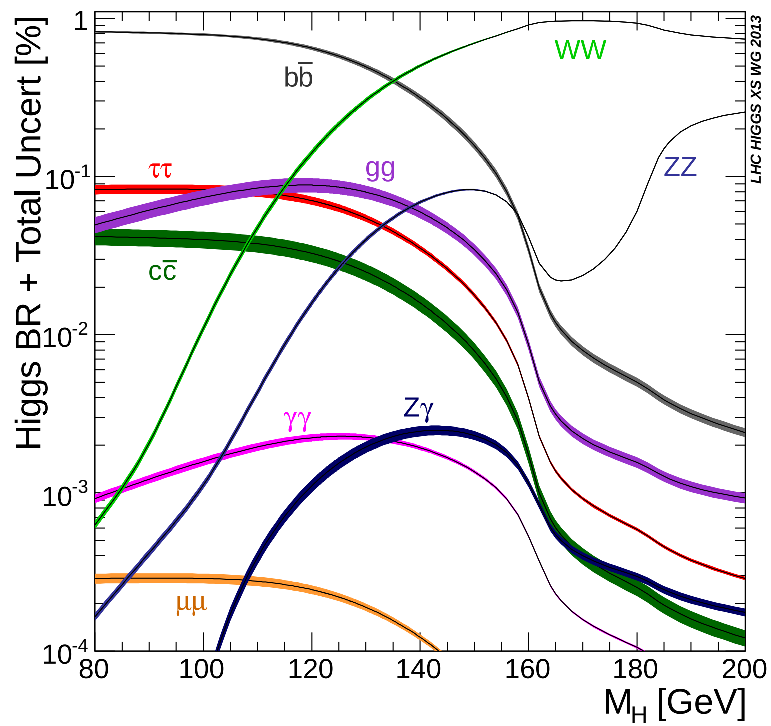
$$\Gamma(H\to Z\gamma)=\frac{G_F^2M_W^2\alpha M_H^3}{64\pi^4}\left(1-\frac{M_Z^2}{M_H^2}\right)^3\left|\sum_f A_f^H(\tau_f,\lambda_f)+A_W^H(\tau_W,\lambda_W)\right|^2$$

where the form factors $A_f^H(\tau,\lambda)$ and $A_W^H(\tau,\lambda)$ can be found in the literature (see, e.g., M. Spira, hep-ph/9705337).

For both decays, both QCD and EW corrections are very small ($\simeq 1-3\%$).

Including parametric/systematic uncertainties ...



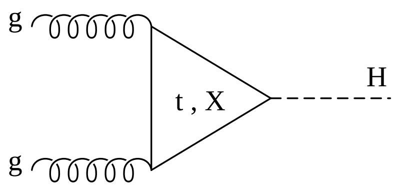


taking into account

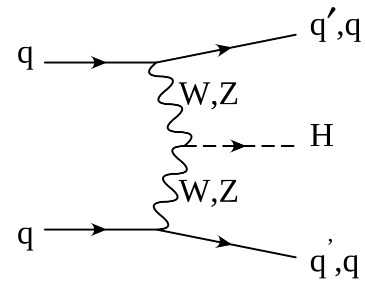
- ▷ known higher order effects (included into HDECAY)
- ▷ full decay of $W/Z \to f\bar{f}$ with higher order effects (PROPHECY4f)
- ▷ errors from input parametes, missing higher order corrections (few % in low mass region)

$p\bar{p}, pp$ colliders: SM Higgs production modes

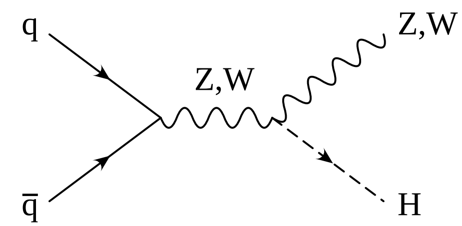
$gg \to H$



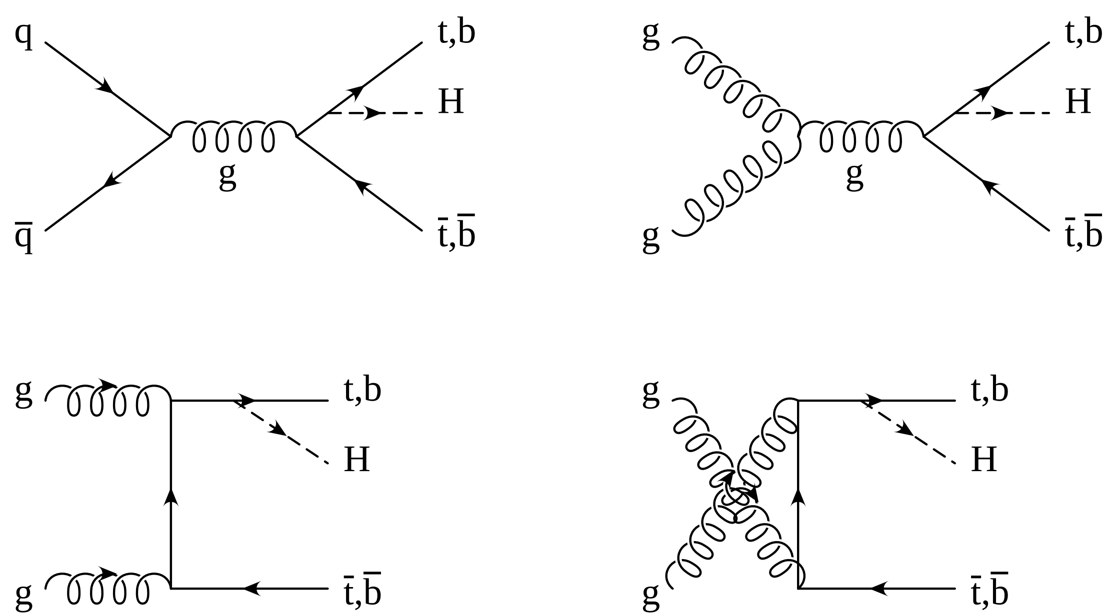
$qq \to qqH$



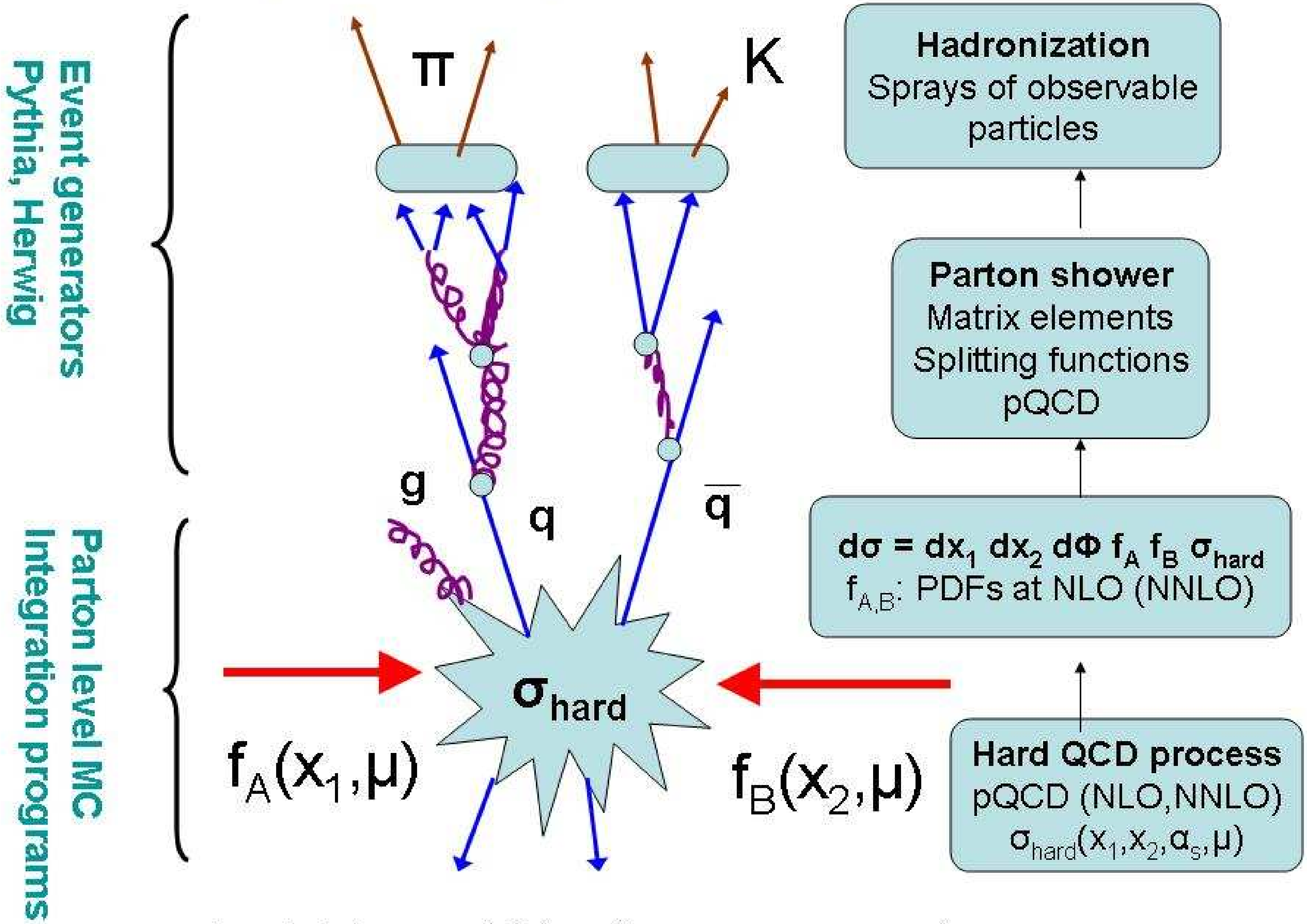
$qq \to WH, ZH$



$q\bar{q}, gg \to t\bar{t}H, b\bar{b}H$



Anatomy of a QCD prediction at hadron colliders



+ underlying event, interactions among remnants

Schematically ...

The hard cross section is calculated perturbatively

$$\hat{\sigma}(ij \to X) = \alpha_s^k \sum_{m=0}^{n} \hat{\sigma}_{ij}^{(m)} \alpha_s^m$$

n=0 : Leading Order (LO), or tree level or Born level
n=1 : Next to Leading Order (NLO), include $O(\alpha_s)$ corrections
......

and convoluted with initial state parton densities at the same order.

Renormalization and factorization scale dependence left at any fixed order.
Setting $\boxed{\mu_R = \mu_F = \mu}$:

$$\sigma(pp, p\bar{p} \to X) = \sum_{ij} \int dx_1 dx_2 f_i^p(x_1, \mu) f_j^{p,\bar{p}}(x_2, \mu) \sum_{m=0}^{n} \hat{\sigma}_{ij}^{(m)}(\mu, Q^2) \alpha_s^{m+k}(\mu)$$

Systematic theoretical error from:

- PDF and $\alpha_s(\mu)$;
- left over scale dependence;
- input parameters.

Hard cross sections: pushing the loop order, why?

- Stability and predictivity of theoretical results, since less sensitivity to unphysical renormalization/factorization scales. First reliable normalization of total cross-sections and distributions.

- Physics richness: more channels and more partons in final state, i.e. more structure to better model (in perturbative region):

  $\longrightarrow$ differential cross-sections, exclusive observables;
  $\longrightarrow$ jet formation/merging and hadronization;
  $\longrightarrow$ initial state radiation.

- First step towards matching with algorithms that resum particular sets of large corrections in the perturbative expansion:

  $\longrightarrow$ resummed calculations (e.g. soft/collinear logs, kinematic logs);
  $\longrightarrow$ parton shower Monte Carlo programs (e.g. PYTHIA, HERWIG).

NLO: challenges have largely been faced and enormous progress has been made

- several independent codes based on traditional FD's approach
- several NLO processes collected and viable in MFCM ($\rightarrow$ interfaced with FROOT) [Campbell, Ellis]
- Enormous progress towards automation:
  - $\rightarrow$ Virtual corrections: new techniques based on unitarity methods and recursion relations
    - ▷ BlackHat [Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre]
    - ▷ Rocket [Ellis, Giele, Kunszt, Melnikov, Zanderighi]
    - ▷ HELAC+CutTools,Samurai [Bevilacqua, Czakon, van Harmeren, Papadopoulos, Pittau,Worek; Mastrolia, Ossola, Reiter, Tramontano]
  - $\rightarrow$ Real corrections: based on Catani-Seymour Dipole subtraction or FKS subtraction
    - ▷ Sherpa [Gleisberg, Krauss]
    - ▷ Madgraph (AutoDipole) [Hasegawa, Moch, Uwer]
    - ▷ Madgraph (MadDipole) [Frederix, Gehrmann, Greiner]
    - ▷ Madgraph (MadFKS) [Frederix,Frixione, Maltoni, Stelzer]

- virtual+real:
  - ▷ MadLoop+MadFKS [Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau]

- interface to parton shower well advanced:
  - ▷ MC@NLO [Frixione, Webber, Nason, Frederix, Maltoni, Stelzer]
  - ▷ POWHEG [Nason, Oleari, Alioli, Re]

When is NLO not enough?

- When NLO corrections are large, to tests the convergence of the perturbative expansion. This may happen when:
  - → processes involve multiple scales, leading to large logarithms of the ratio(s) of scales;
  - → new parton level subprocesses first appear at NLO;
  - → new dynamics first appear at NLO;
  - → ...

- When truly high precision is needed (very often the case!).

- When a really reliable error estimate is needed.

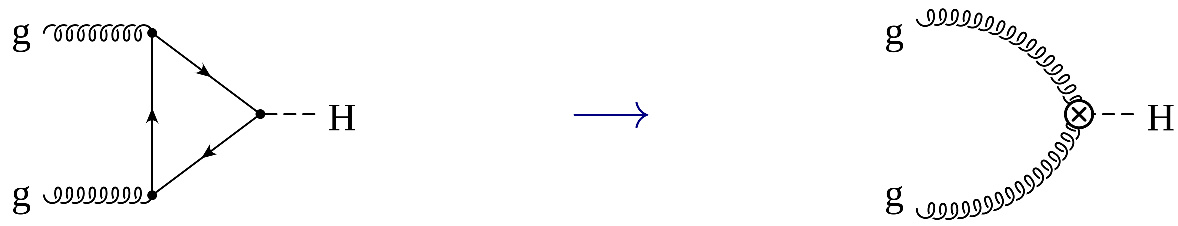
A tutorial: $gg \to H$, main production mode

... large K-factors, scale dependence, resummations, and more.

NLO QCD corrections calculated exactly and in the $m_t \to \infty$ limit: perfect agreement even for $M_H >> m_t$.

$$\Downarrow$$

Dominant soft dynamics do not resolve the Higgs boson coupling to gluons



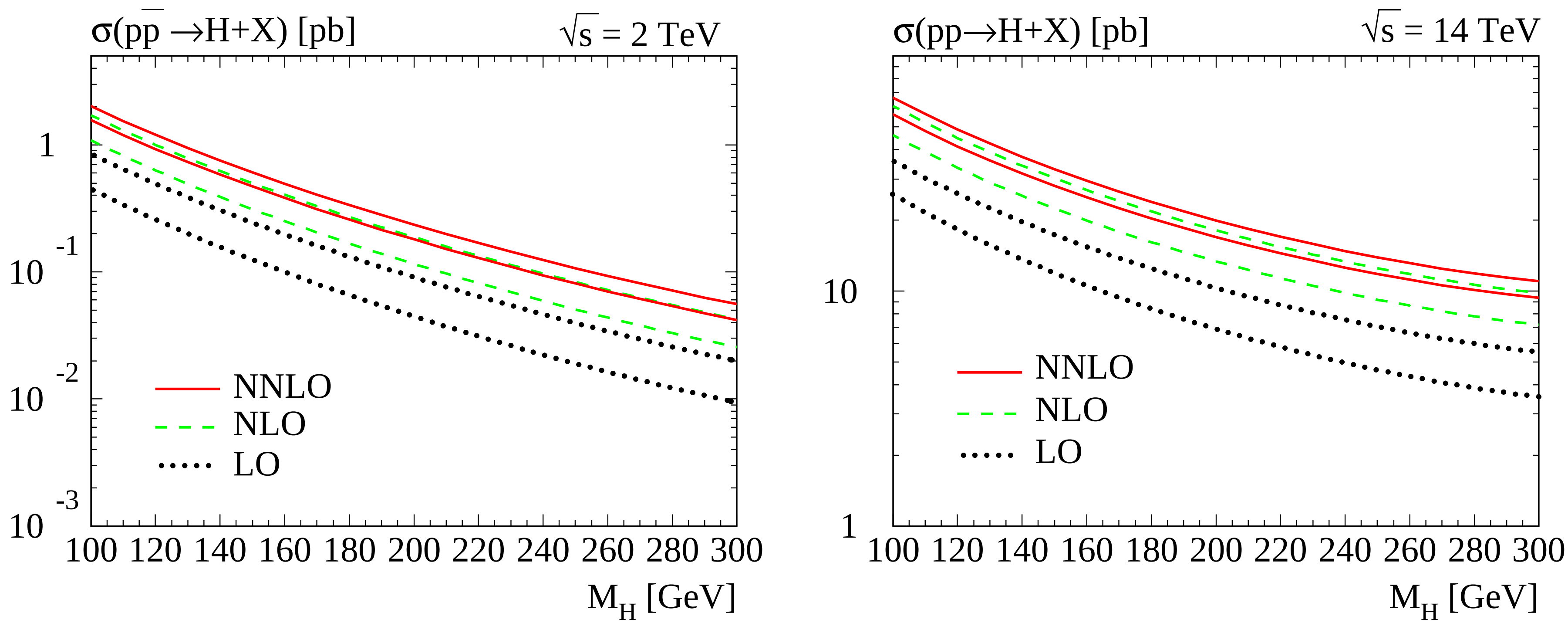
$$\Downarrow$$

$$\mathcal{L}_{eff} = \frac{H}{4v} C(\alpha_s) G^{a\mu\nu} G^a_{\mu\nu}$$

where, including NLO and NNLO QCD corrections:

$$C(\alpha_s) = \frac{1}{3}\frac{\alpha_s}{\pi}\left[1 + c_1 \frac{\alpha_s}{\pi} + c_2 \left(\frac{\alpha_s}{\pi}\right)^2 + \cdots\right]$$

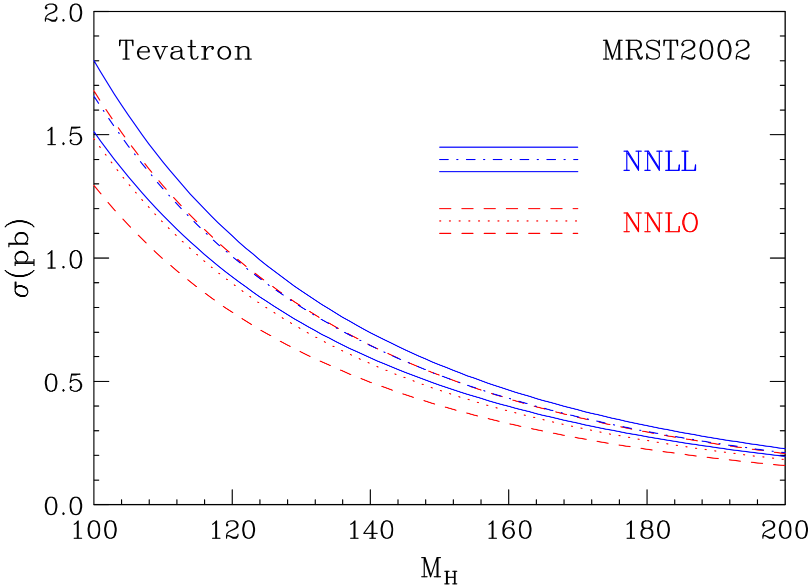
Fixed order NNLO:

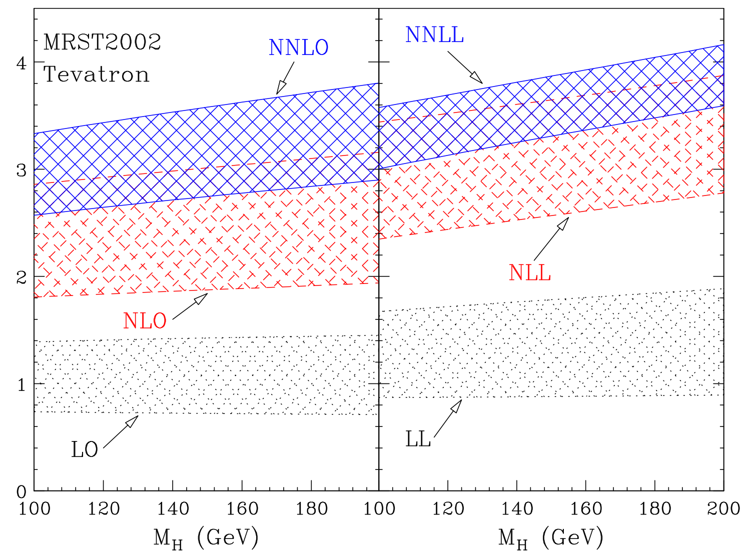


[Harlander,Kilgore (02)]

- very large corrections in going LO → NLO (K=1.7-1.9) → NNLO (K=2-2.2);
- perturbative convergence LO → NLO (70%) → NNLO (30%): residual 15% theoretical uncertainty.
- Tevatron case: still some tension.

Resumming effects of soft radiation ...





[Catani,de Florian,Grazzini,Nason(03)]

Theoretical uncertainty reduced to:

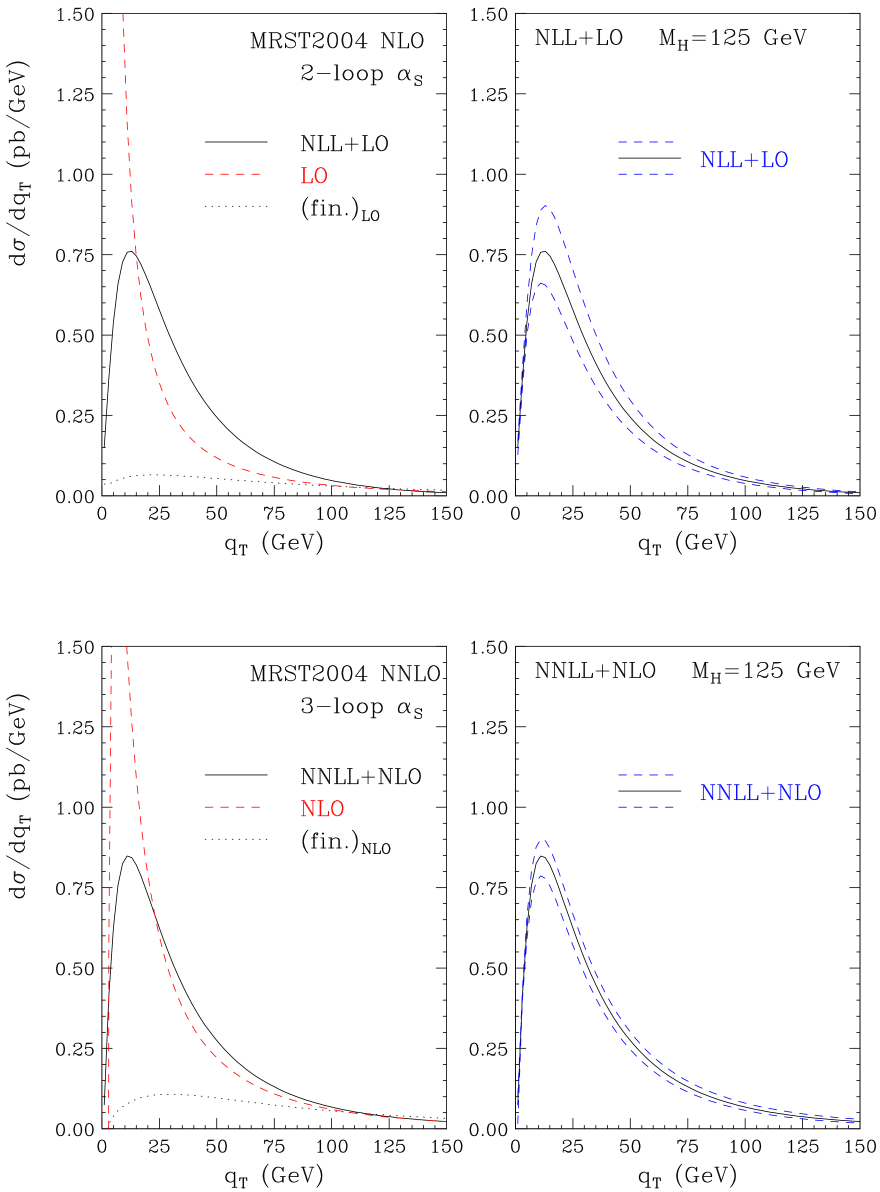
$\longrightarrow$ $\simeq$ 10% perturbative uncertainty, including the $m_t \to \infty$ approximation.

$\longrightarrow$ $\simeq$ 10% (estimated) from NNLO PDF's (now existing!).

But ... <u>let us remember</u> that: going from MRST2002 to MSTW2008 greatly affected the Tevatron/LHC cross section: from 9%/30% ($M_H = 115$ GeV) to -9%/+9% ($M_H = 200/300$ GeV) !

[De Florian,Grazzini (09)]

Resumming effects of soft radiation for q_T^H spectrum ...



large q_T $\xrightarrow{q_T > M_H}$

perturbative expansion in $\alpha_s(\mu)$

small q_T $\xrightarrow{q_T \ll M_H}$

need to resum large $\ln(M_H^2/q_T^2)$

residual uncertainty:

LO-NLL: 15-20%

NLO-NNLL: 8-20%

[Bozzi, Catani, De Florian, Grazzini (04-08)]

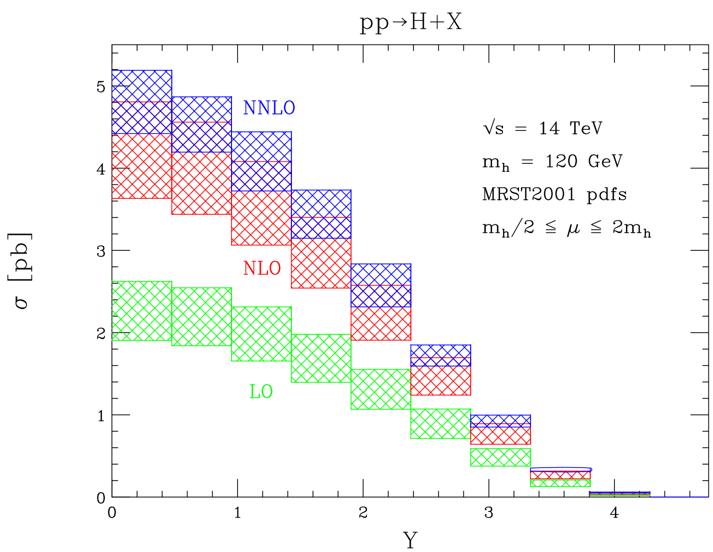
Exclusive NNLO results: $gg \to H$, $H \to \gamma\gamma, WW, ZZ$

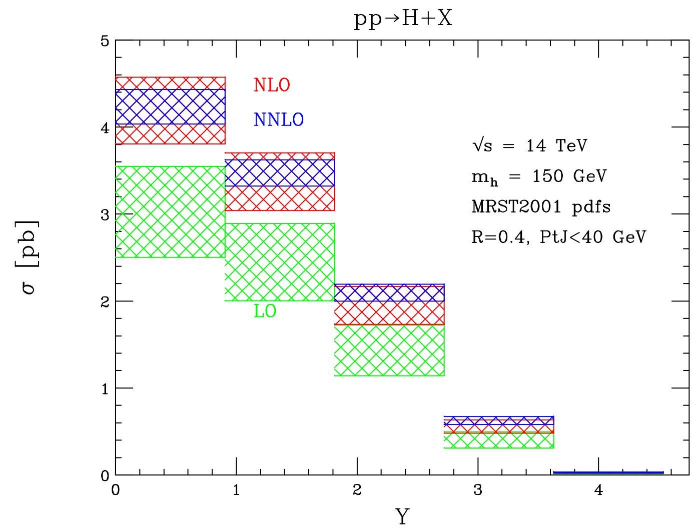
Extension of (IR safe) subtraction method to NNLO

$\longrightarrow$ HNNLO[Catani,Grazzini (05)]

$\longrightarrow$ FEHiP [Anastasiou,Melnikov,Petriello (05)]

Essential tools to reliably implement experimental cuts/vetos.



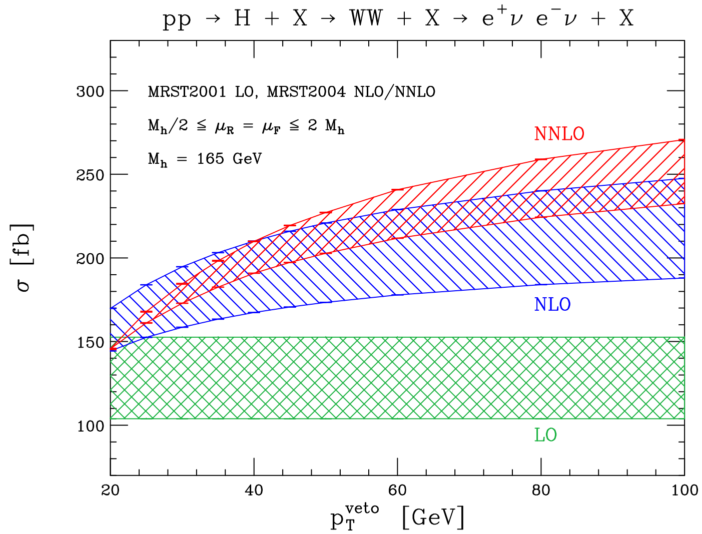


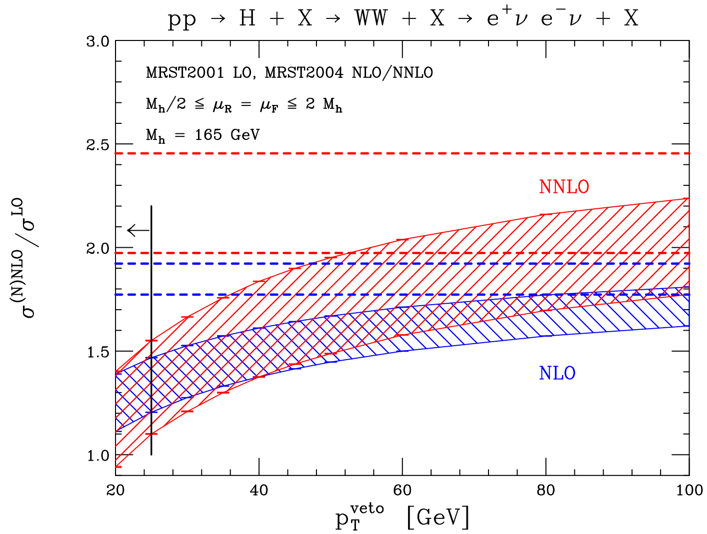
[Anastasiou,Melnikov,Petriello (05)]

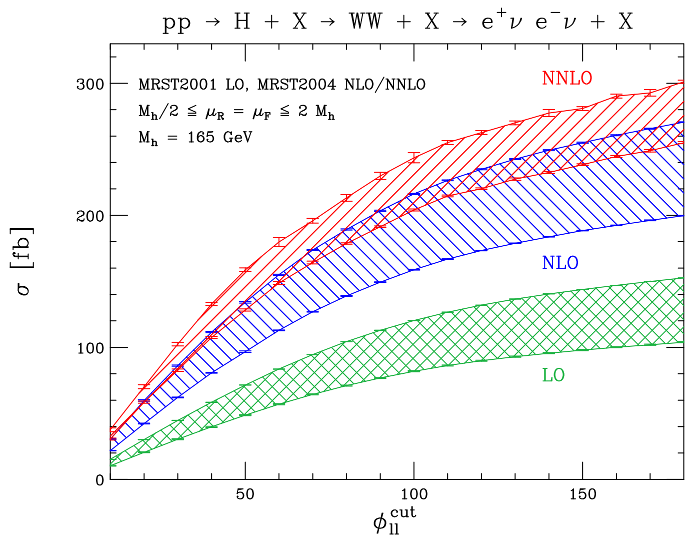
jet veto (to enhance $H \to WW$ signal with respect to $t\bar{t}$ background) seems to improve perturbative stability of y-distribution $\longrightarrow$ jet veto is removing non-NNLO contributions.

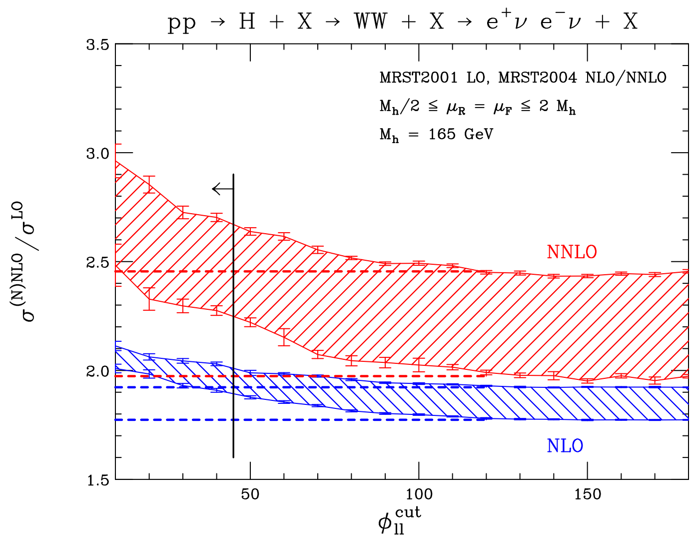
Full fledged $(gg \to) H \to W^+W^- \to l^+\nu l^-\bar{\nu}$

The magnitude of higher order corrections varies significantly with the signal selection cuts.



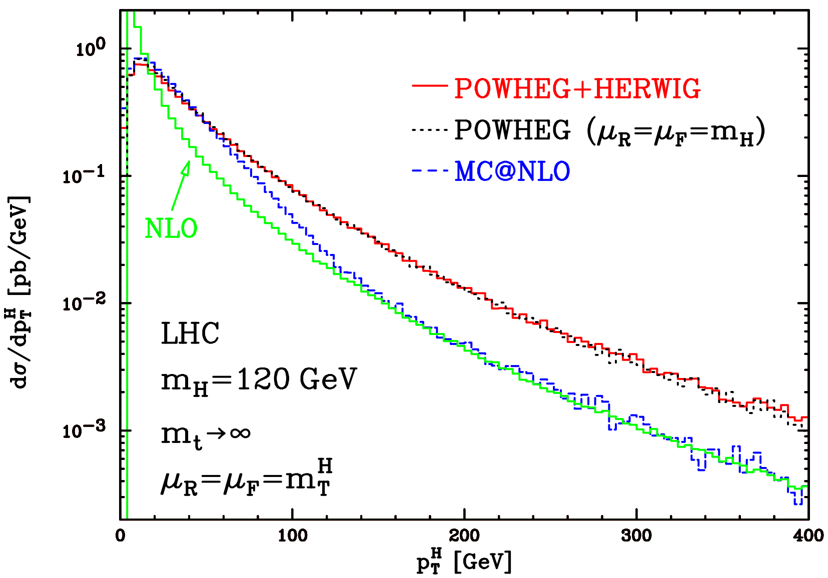


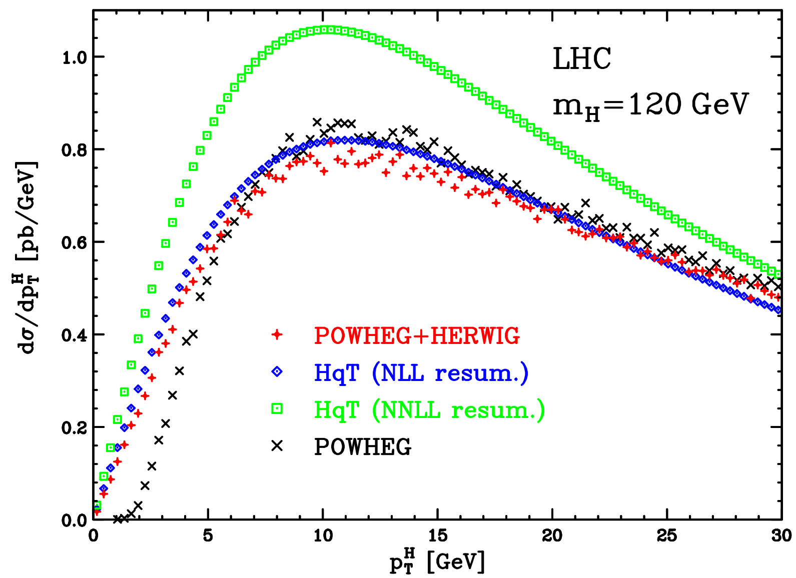




[Anastasiou,Dissertori,Stöckli (07)]

$gg \to H$ implemented in MC@NLO and POWHEG





[Nason, Oleari, Alioli, Re]

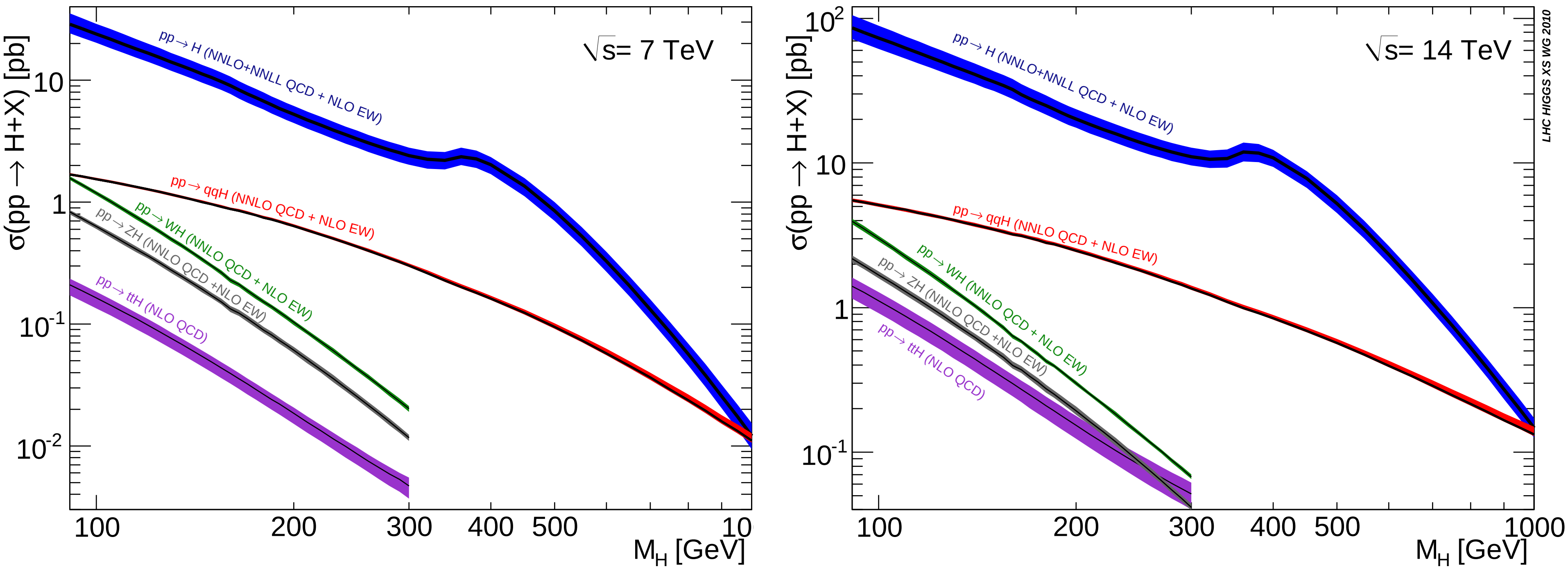
→ general good agreement with PYTHIA;

→ comparison MC@NLO vs POWHEG understood;

→ comparison with resummed NLL results under control.

→ rescale effects using NNLL/NLL knowledge.

Inclusive SM Higgs Production at the LHC: theoretical predictions and their uncertainty



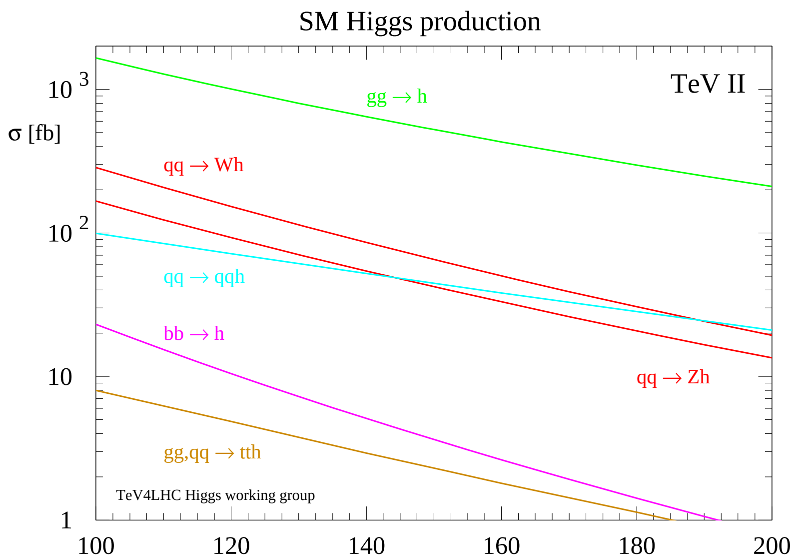
(LHC Higgs Cross Sections Working Group, arXiv:1101.0593 → CERN Yellow Book)

- all orders of calculated higher orders corrections included (tested with all existing calculations);
- theory errors (scales, PDF, α_s, ...) combined according to common recipe.
- Updates: arXiv:1201.3084 and arXiv:1307.1347 (including fine scan of the 125-126 GeV region).

Looking for a SM Higgs boson at hadron colliders:

- ▷ Tevatron Higgs-physics program
- ▷ LHC Higgs-physics program

Tevatron: pioneering the way for a light SM-like Higgs boson discovery



Lower mass region:

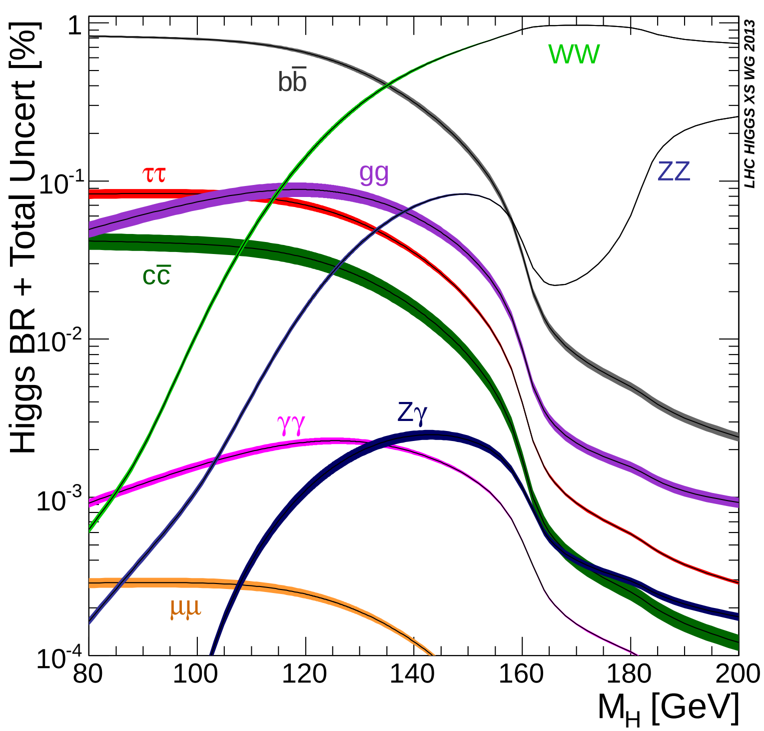
$$q\bar{q}' \to WH, H \to b\bar{b}$$

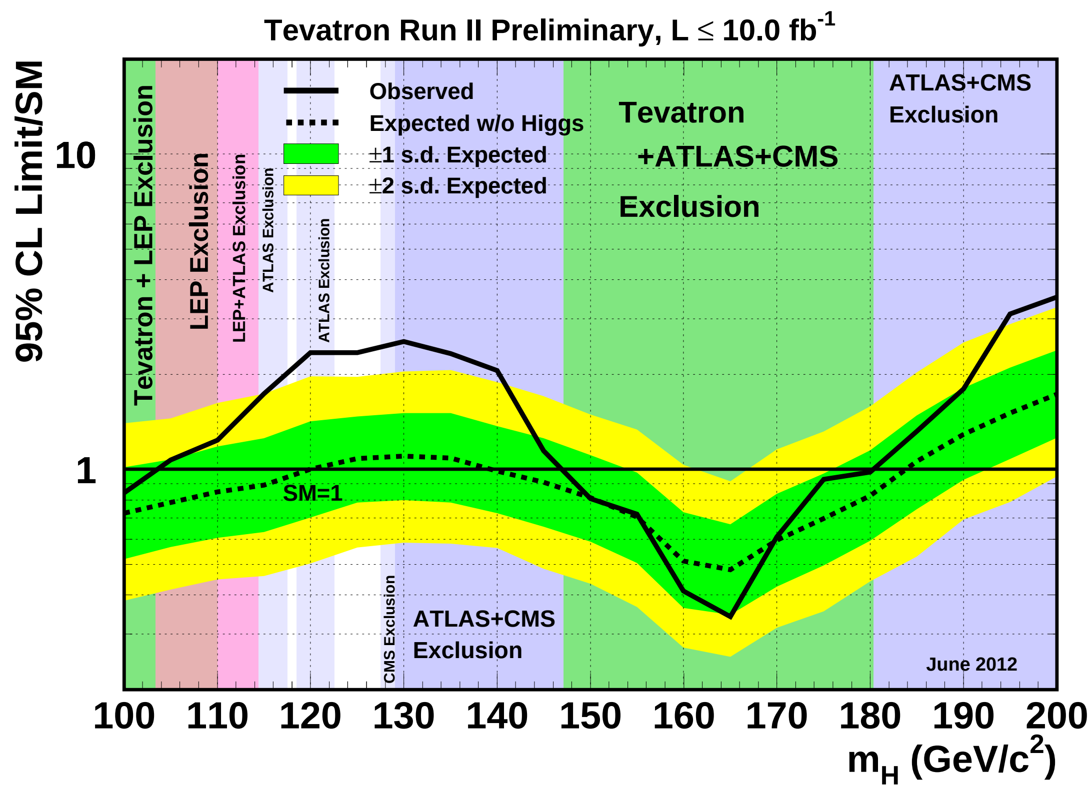
Higher mass region:

$$gg \to H, H \to W^+W^-$$

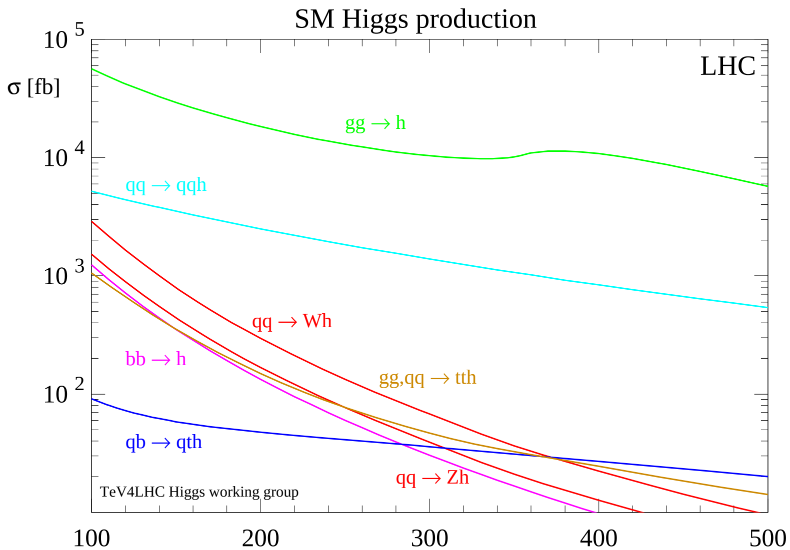
(smaller impact:

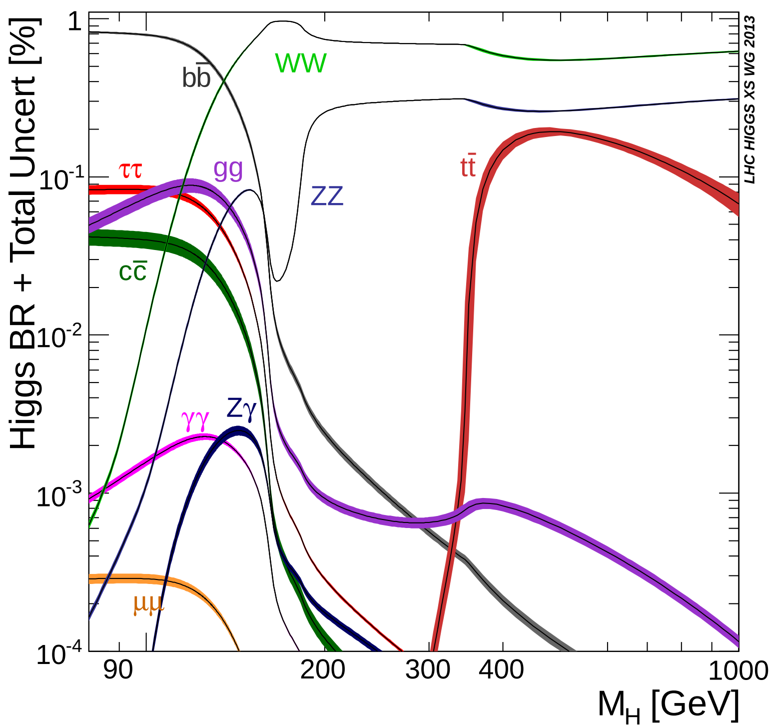
$q\bar{q} \to q'\bar{q}'H$, $q\bar{q}, gg \to t\bar{t}H$)





LHC@7 and 8 TeV: discovery of a SM-like Higgs boson





Many channels have been studied:

$$gg \to H \;,\; H \to \gamma\gamma, WW, ZZ$$
$$qq \to qqH \;,\; H \to \gamma\gamma, WW, ZZ, \tau\tau$$
$$q\bar{q}' \to WH \;,\; H \to \gamma\gamma, b\bar{b}$$
$$q\bar{q}, gg \to t\bar{t}H \;,\; H \to \gamma\gamma, \tau\tau, b\bar{b}$$

Discovery reached with:

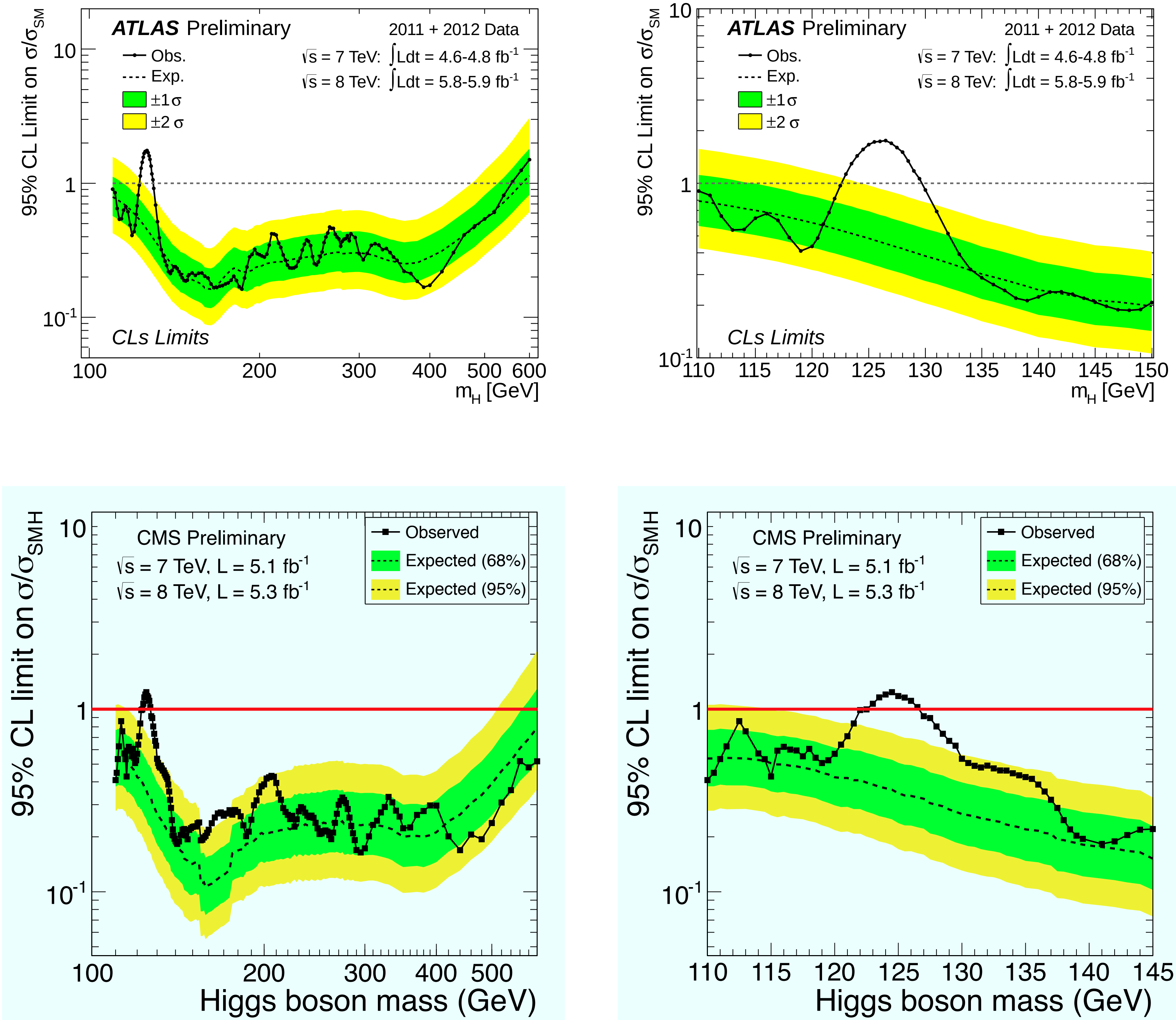
$H \to \gamma\gamma$ (untagged,VBF)

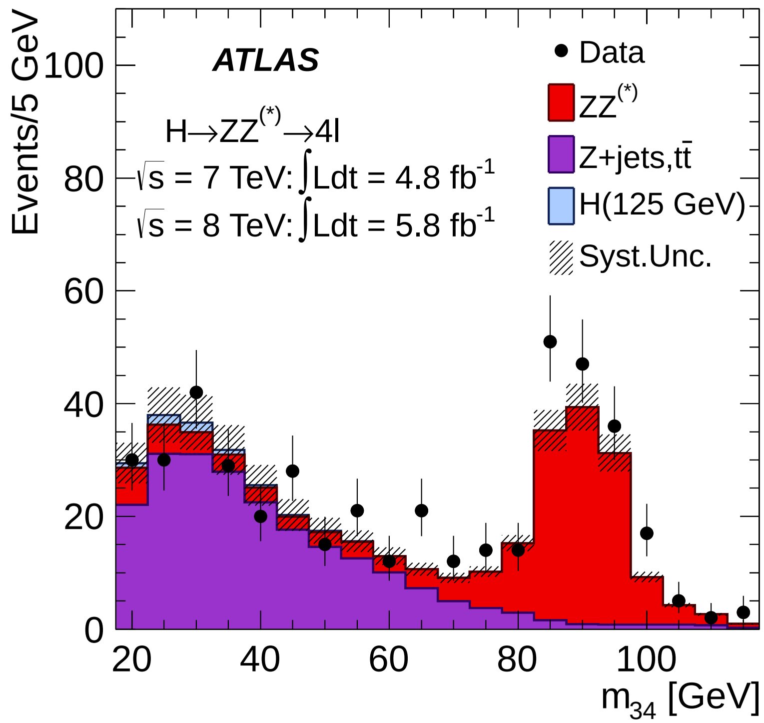
$H \to ZZ$ (untagged)

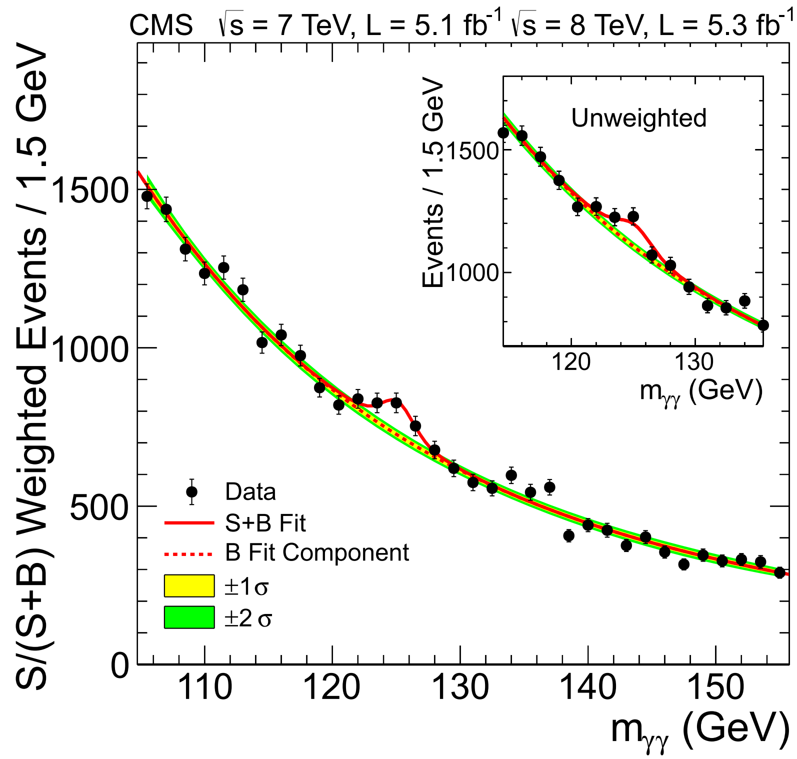
$H \to WW$ (untagged, VBF)

$H \to \tau\tau$ (untagged, VBF)

$H \to b\bar{b}$ (VH, $V = W, Z$)


ATLAS Preliminary
2011 + 2012 Data
√s = 7 TeV: ∫Ldt = 4.6-4.8 fb-1
√s = 8 TeV: ∫Ldt = 5.8-5.9 fb-1
Obs.
Exp.
±1σ
±2 σ
CLs Limits
95% CL Limit on σ/σSM
mH [GeV]
ATLAS Preliminary
2011 + 2012 Data
√s = 7 TeV: ∫Ldt = 4.6-4.8 fb-1
√s = 8 TeV: ∫Ldt = 5.8-5.9 fb-1
Obs.
Exp.
±1σ
±2 σ
CLs Limits
95% CL Limit on σ/σSM
mH [GeV]
CMS Preliminary
√s = 7 TeV, L = 5.1 fb-1
√s = 8 TeV, L = 5.3 fb-1
Observed
Expected (68%)
Expected (95%)
95% CL limit on σ/σSMH
Higgs boson mass (GeV)
CMS Preliminary
√s = 7 TeV, L = 5.1 fb-1
√s = 8 TeV, L = 5.3 fb-1
Observed
Expected (68%)
Expected (95%)
95% CL limit on σ/σSMH
Higgs boson mass (GeV)

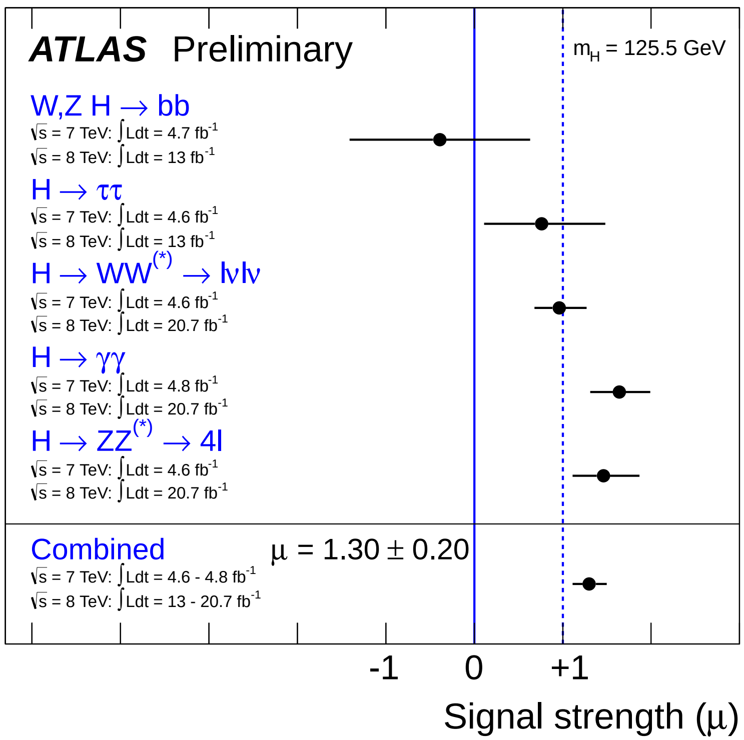


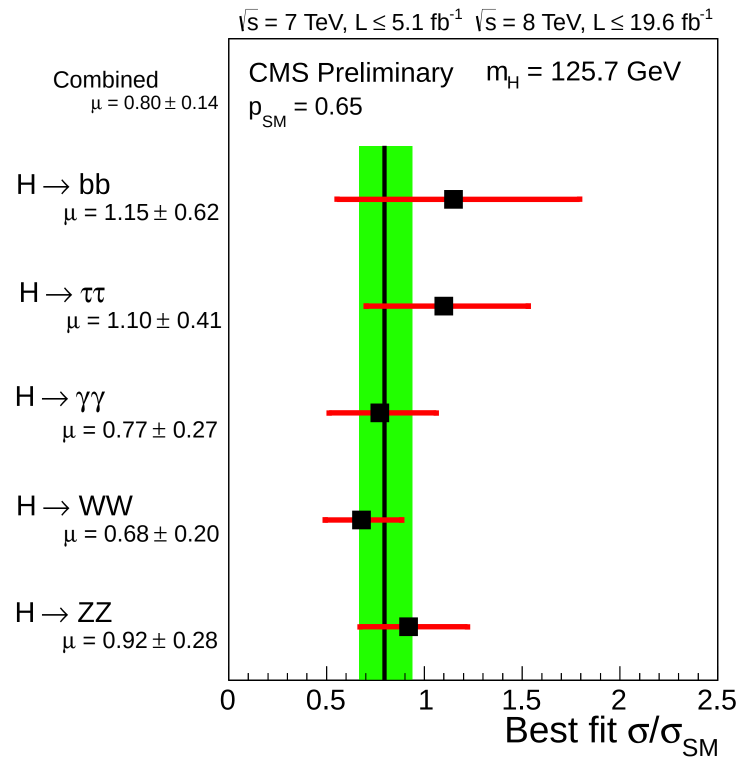


resonance peak measured at:

$$m_H = \begin{cases} 125.5 \pm 0.2\,(\mathrm{stat})^{+0.5}_{-0.6}\,(\mathrm{syst}) & \mathrm{ATLAS} \\ 125.7 \pm 0.3\,(\mathrm{stat}) \pm 0.3\,(\mathrm{syst}) & \mathrm{CMS} \end{cases}$$

with properties compatible with a SM-like Higgs





Is it really the SM Higgs boson?

- ▷ measure couplings, spin, parity, CP
- ▷ look for indirect/direct signals of new physics
- ▷ many possible scenarios

The Higgs paved the way (we hope) for many exciting discoveries to come.

Couplings

Gradual approach to a very complex problem assumes:

- only one underlying Higgs boson resonance at $M_H = 125$ GeV
- zero-width approximation, i.e.

$$(\sigma \cdot BR)(ii \to H \to ff) = \frac{\sigma_{ii} \cdot \Gamma_{ff}}{\Gamma_H}$$

- no specific assumption on any other state of new physics
- modification of coupling strength only (same tensor structure as SM) by overall rescaling factor. e.g.

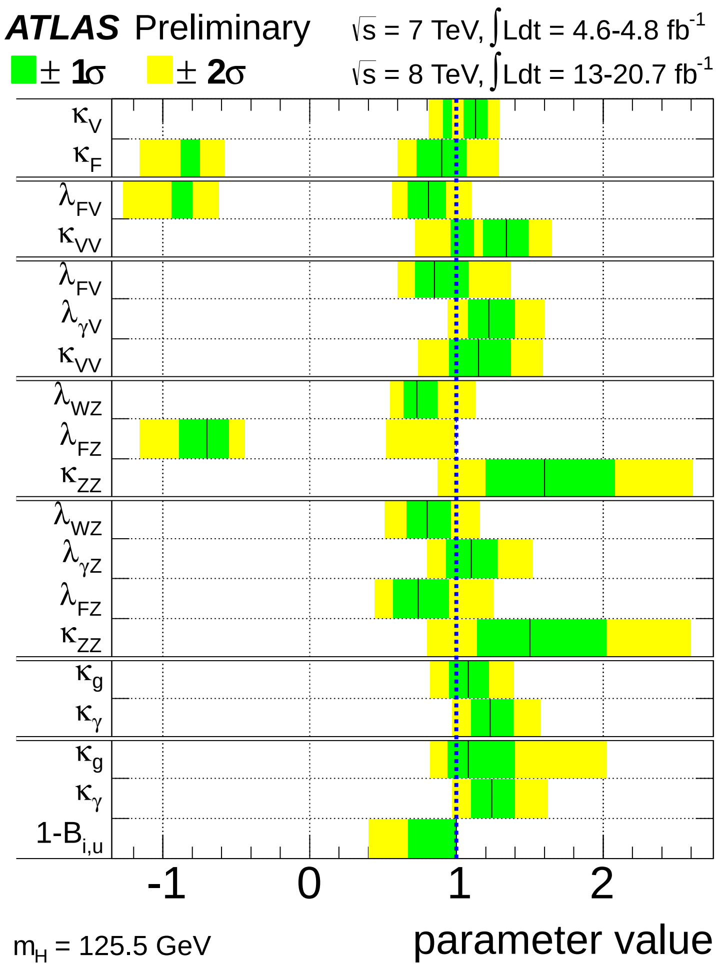
$$(\sigma \cdot BR)(ii \to H \to ff) = \sigma_{SM}(ii \to H) \cdot BR_{SM}(H \to ff) \frac{\kappa_i^2 \cdot \kappa_f^2}{\kappa_H^2}$$

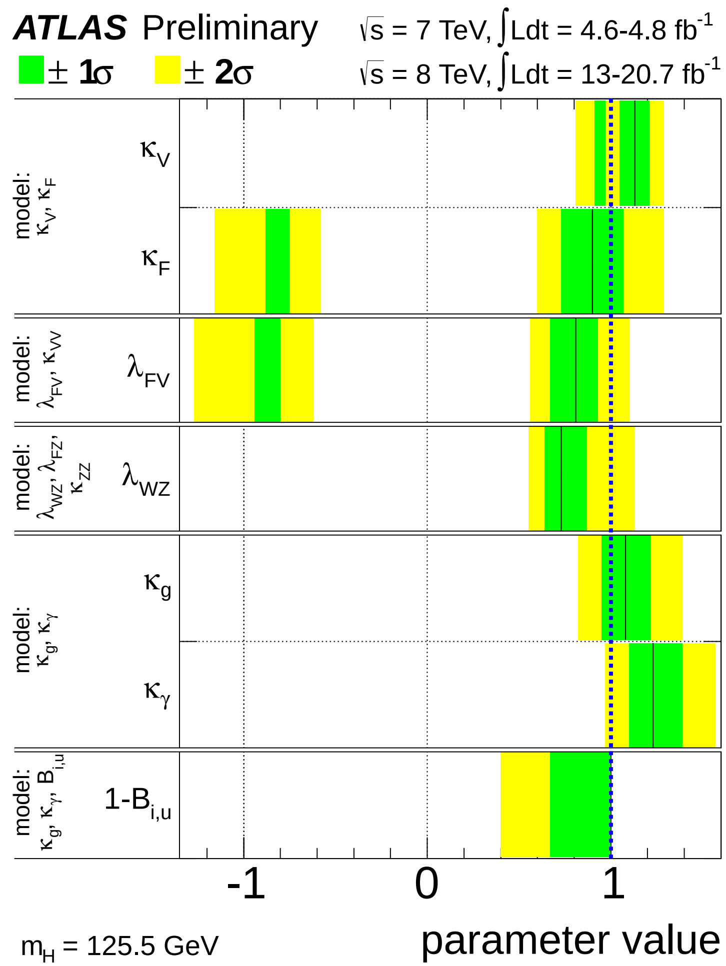
- QCD corrections factorize w.r.t. coupling rescaling.
- functional dependence among κ_i^2 vs κ_i^2 free parameters

Explore various scenarios

- one common scale factor
- different scaling for vector-boson and fermion couplings
- $\kappa_w \neq \kappa_Z$ (probe custodial symmetry)
- $\kappa_u \neq \kappa_d$ (probe fermion sector)
- κ_g and κ_γ free parameters, while all others $\kappa_i = 1$ (SM)
- ...

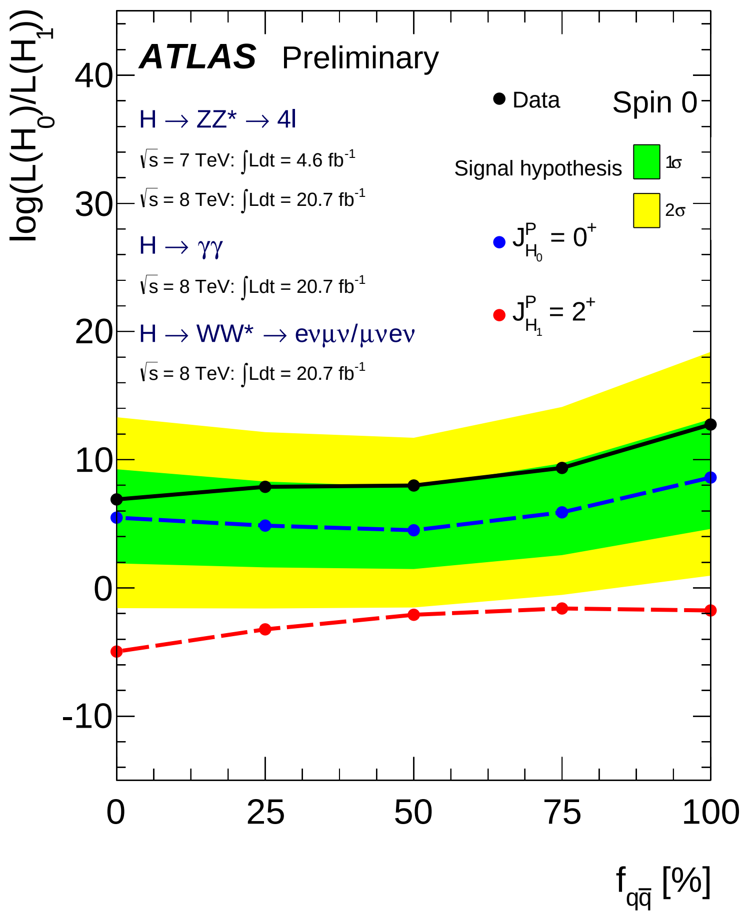
In each case κ_H can be treated as a free parameter or not.

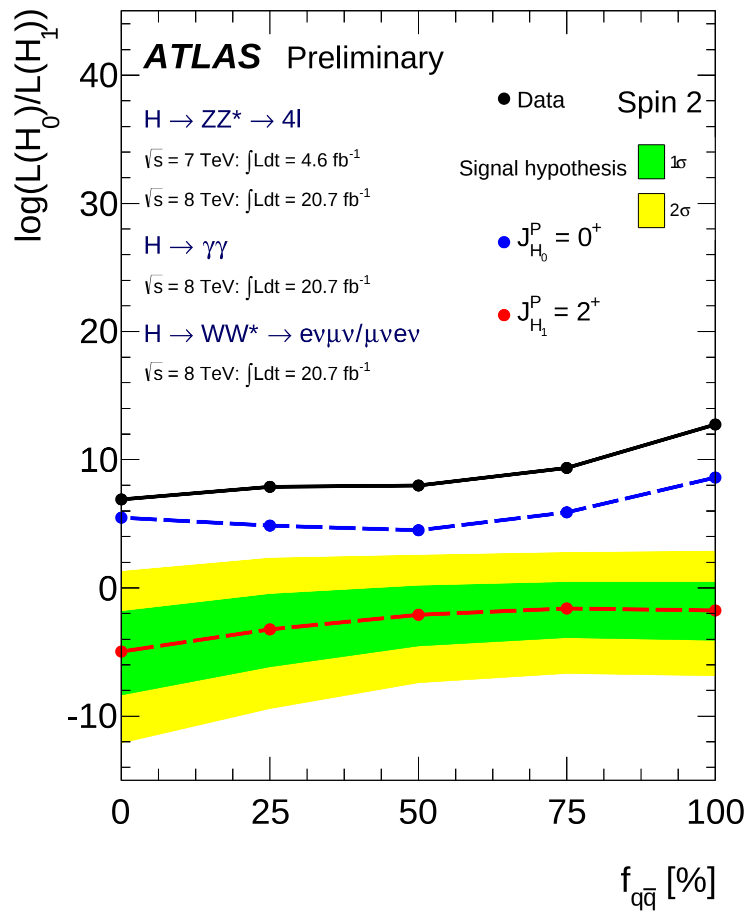




Compatibility with SM expectations between 5% and 10%.

Spin, parity, CP





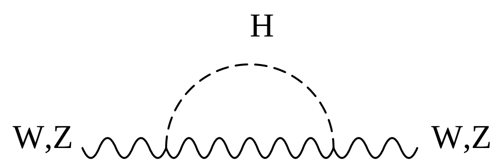
More theoretical constraints . . .

EW precision fits: perturbatively calculate observables in terms of few parameters:

$$M_Z, G_F, \alpha(M_Z), M_W, m_f, (\alpha_s(M_Z))$$

extracted from experiments with high accuracy. Only SM unknown: M_H.

- SM needs Higgs boson to cancel infinities, e.g.

$$M_W, M_Z \longrightarrow$$ 

- Finite logarithmic contributions survive, e.g. radiative corrections to $\rho = M_W^2/(M_Z^2 \cos^2\theta_W)$:

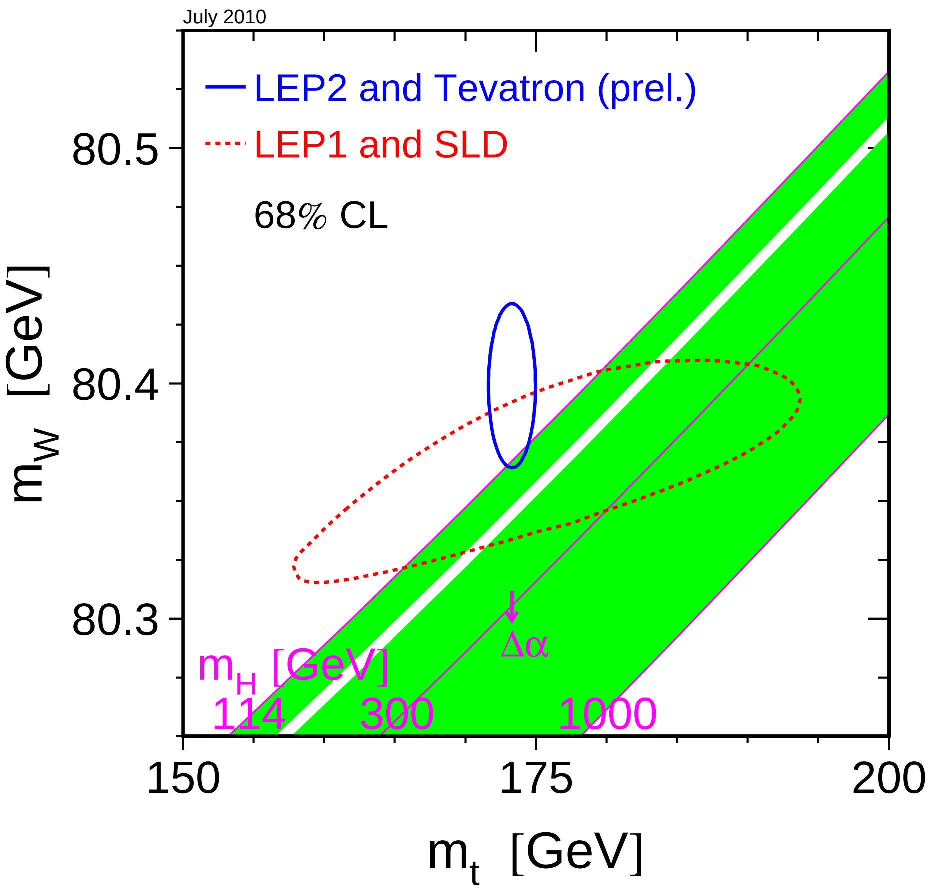
$$\rho = 1 - \frac{11 g^2}{96\pi^2 \tan^2\theta_W} \ln\left(\frac{M_H}{M_W}\right)$$

  Main effects in oblique radiative corrections (S,T-parameters)

- Same constraints apply to any model of new physics.

SM Higgs-boson mass range: constrained by EW precision fits

Increasing precision will continue to provide an invaluable tool to test the consistency of the SM and its extensions.



$m_W = 80.399 \pm 0.023$ GeV

$m_t = 173.3 \pm 1.1$ GeV

$\Downarrow$

$M_H = 89^{+35}_{-26}$ GeV

$M_H < 158\,(185)$ GeV

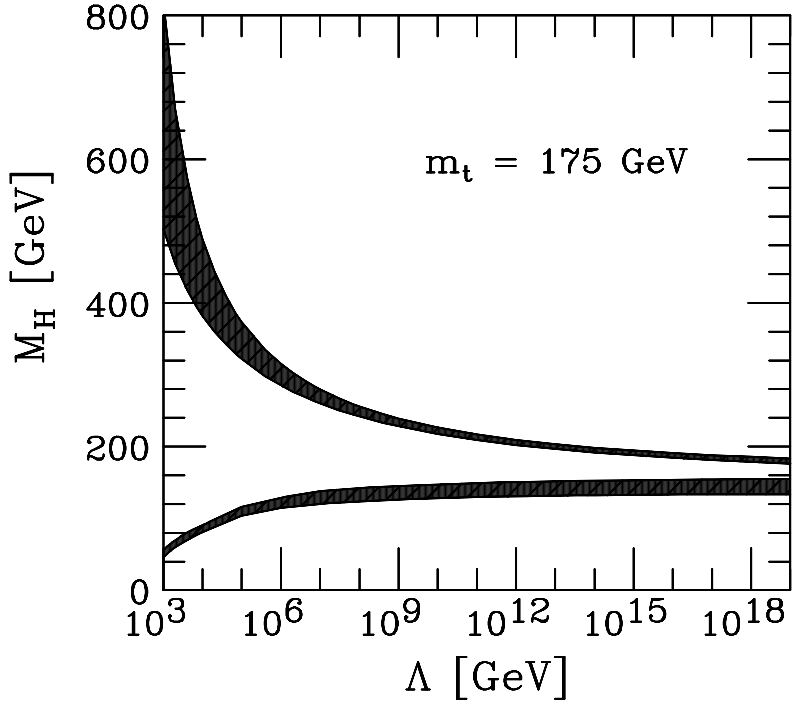
plus exclusion limits (95% c.l.):

$M_H > 114.4$ GeV (LEP)

$M_H \neq 158 - 175$ GeV (Tevatron)

Other theoretical constraints on M_H in the Standard Model

SM as an effective theory valid up to a scale Λ. The Higgs sector of the SM actually contains two unknowns: $\boxed{M_H \text{ and } \Lambda}$.



Bounds given by:

$\longrightarrow$ unitarity

$\longrightarrow$ triviality

$\longrightarrow$ vacuum stability

$\longrightarrow$ fine tuning

$\boxed{M_H^2 = 2\lambda v^2}$ $\longrightarrow$ M_H determines the weak/strong coupling behavior of the theory, i.e. the limit of validity of the perturbative approach.

Unitarity: longitudinal gauge boson scattering cross section at high energy grows with M_H.

Electroweak Equivalence Theorem:

in the high energy limit ($s \gg M_V^2$)

$$\mathcal{A}(V_L^1 \dots V_L^n \to V_L^1 \dots V_L^m) = (i)^n(-i)^m \mathcal{A}(\omega^1 \dots \omega^n \to \omega^1 \dots \omega^m) + O\left(\frac{M_V^2}{s}\right)$$

(V_L^i=longitudinal weak gauge boson; ω^i=associated Goldstone boson).

Example: $\boxed{W_L^+ W_L^- \to W_L^+ W_L^-}$

$$\mathcal{A}(W_L^+ W_L^- \to W_L^+ W_L^-) \sim -\frac{1}{v^2}\left(-s - t + \frac{s^2}{s - M_H^2} + \frac{t^2}{t - M_H^2}\right)$$

$$\mathcal{A}(\omega^+ \omega^- \to \omega^+ \omega^-) = -\frac{M_H^2}{v^2}\left(\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2}\right)$$

$$\Downarrow$$

$$\mathcal{A}(W_L^+ W_L^- \to W_L^+ W_L^-) = \mathcal{A}(\omega^+ \omega^- \to \omega^+ \omega^-) + O\left(\frac{M_W^2}{s}\right)$$

Using partial wave decomposition:

$$\mathcal{A} = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|\mathcal{A}^2| \quad \longrightarrow \quad \sigma = \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)|a_l|^2 = \frac{1}{s}\mathrm{Im}\left[\mathcal{A}(\theta=0)\right]$$

$$\Downarrow$$

$$\boxed{|a_l|^2 = \mathrm{Im}(a_l)} \quad \longrightarrow \quad \boxed{|\mathrm{Re}(a_l)| \leq \tfrac{1}{2}}$$

Most constraining condition for $W_L^+ W_L^- \to W_L^+ W_L^-$ from

$$a_0(\omega^+\omega^- \to \omega^+\omega^-) = -\frac{M_H^2}{16\pi v^2}\left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s}\log\left(1 + \frac{s}{M_H^2}\right)\right) \overset{s \gg M_H^2}{\longrightarrow} -\frac{M_H^2}{8\pi v^2}$$

$$\boxed{|\mathrm{Re}(a_0)| < \tfrac{1}{2}} \quad \longrightarrow \quad \boxed{M_H < 870 \text{ GeV}}$$

Best constraint from coupled channels ($2W_L^+ W_L^- + Z_L Z_L$):

$$a_0 \overset{s \gg M_H^2}{\longrightarrow} -\frac{5M_H^2}{32\pi v^2} \quad \longrightarrow \quad \boxed{M_H < 780 \text{ GeV}}$$

Observe that: if there is no Higgs boson, i.e. $M_H \gg s$:

$$a_0(\omega^+\omega^- \to \omega^+\omega^-) \stackrel{M_H^2 \gg s}{\longrightarrow} -\frac{s}{32\pi v^2}$$

Imposing the unitarity constraint $\longrightarrow$ $\boxed{\sqrt{s_c} < 1.8 \text{ TeV}}$

Most restrictive constraint $\longrightarrow$ $\boxed{\sqrt{s_c} < 1.2 \text{ TeV}}$

$\Downarrow$

New physics expected at the TeV scale

Exciting !!

this is the range of energies of both Tevatron and LHC

Triviality: a $\lambda\phi^4$ theory cannot be perturbative at all scales unless $\lambda = 0$.

In the SM the scale evolution of λ is more complicated:

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 24y_t^4 + \cdots$$

($t = \ln(Q^2/Q_0^2)$, $y_t = m_t/v$ $\rightarrow$ top quark Yukawa coupling).

Still, for large λ ($\leftrightarrow$ large M_H) the first term dominates and (at 1-loop):

$$\lambda(Q) = \frac{\lambda(Q_0)}{1 - \frac{3}{4\pi^2}\lambda(Q_0)\ln\left(\frac{Q^2}{Q_0^2}\right)}$$

when Q grows $\longrightarrow$ $\lambda(Q)$ hits a pole $\rightarrow$ triviality

Imposing that $\lambda(Q)$ is finite, gives a scale dependent bound on M_H:

$$\frac{1}{\lambda(\Lambda)} > 0 \quad \longrightarrow \quad M_H^2 < \frac{8\pi^2 v^2}{3\log\left(\frac{\Lambda^2}{v^2}\right)}$$

where we have set $Q \rightarrow \Lambda$ and $Q_0 \rightarrow v$.

Vacuum stability: $\boxed{\lambda(Q) > 0}$

For small λ ($\leftrightarrow$ small M_H) the last term in $d\lambda/dt = \ldots$ dominates and:

$$\lambda(\Lambda) = \lambda(v) - \frac{3}{4\pi^2} y_t^4 \log\left(\frac{\Lambda^2}{v^2}\right)$$

from where a first rough lower bound is derived:

$$\boxed{\lambda(\Lambda) > 0 \quad \longrightarrow \quad M_H^2 > \tfrac{3v^2}{2\pi^2} y_t^4 \log\left(\tfrac{\Lambda^2}{v^2}\right)}$$

More accurate analyses use 2-loop renormalization group improved V_{eff}.

Fine-tuning: M_H is unstable to ultraviolet corrections

$$M_H^2 = (M_H^0)^2 + \frac{g^2}{16\pi^2}\Lambda^2 \cdot \text{constant} + \text{ higher orders}$$

$M_H^0 \rightarrow$ fundamental parameter of the SM
$\Lambda \rightarrow$ UV-cutoff scale

Unless $\Lambda \simeq$ EW-scale, fine-tuning is required to get $M_H \simeq$ EW-scale.

More generally, the all order calculation of V_{eff} would give:

$$\bar{\mu}^2 = \mu^2 + \Lambda^2 \sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/Q)$$

Veltman condition: the absence of large quadratic corrections is guaranteed by:

$$\sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/M_H) = 0 \quad \text{or better} \sum_{n=0}^{n_{max}} c_n(\lambda_i) \log^n(\Lambda/M_H) < \frac{v^2}{\Lambda^2}$$

where: $n_{max} = 0, 1, 2 \longrightarrow \Lambda \simeq 2, 15, 50$ TeV.

Conclusions and Outlook

- The discovery of a SM-like Higgs boson has given the first crucial hint to break the EWSB code and access the UV completion of the Standard Model.
- Two complementary paths: precision measurement of couplings and discovery of new resonances.
- We haven't discussed: complementarity of hadron/lepton colliders