The Standar Model of Particle Physics
Lecture I

Aspects and properties of the fundamental theory of particle interactions at the electroweak scale

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Outline of Lecture I

- Approaching the study of particles and their interactions:
  - $\longrightarrow$ global and local symmetry principles;
  - $\longrightarrow$ consequences of broken or hidden symmetries.
- Towards the Standard Model of Particle Physics:
  - $\longrightarrow$ main experimental evidence;
  - $\longrightarrow$ possible theoretical scenarios.
- The Standard Model of Particle Physics:
  - $\longrightarrow$ Lagrangian: building blocks and symmetries;
  - $\longrightarrow$ strong interactions: Quantum Chromodynamics;
  - $\longrightarrow$ electroweak interactions, the Glashow-Weinberg-Salam theory:
    - properties of charged and neutral currents;
    - breaking the electroweak symmetry: the SM Higgs mechanism.

Particles and forces are a realization of fundamental symmetries of nature

Very old story: Noether's theorem in *classical mechanics*

$$L(q_i, \dot{q}_i) \text{ such that } \frac{\partial L}{\partial q_i} = 0 \longrightarrow p_i = \frac{\partial L}{\partial \dot{q}_i} \text{ conserved}$$

to any symmetry of the Lagrangian is associated a conserved physical quantity:

- ▷ $q_i = x_i \quad \longrightarrow \quad p_i$ linear momentum;
- ▷ $q_i = \theta_i \quad \longrightarrow \quad p_i$ angular momentum.

Generalized to the case of a *relativistic quantum theory* at multiple levels:

- ▷ $q_i \to \phi_j(x)$ coordinates become "fields" $\leftrightarrow$ "particles"
- ▷ $\mathcal{L}(\phi_j(x), \partial_\mu \phi_j(x))$ can be symmetric under many transformations.
- ▷ To any continuous symmetry of the Lagrangian we can associate a conserved current

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)} \delta\phi_j \quad \text{such that} \quad \partial_\mu J^\mu = 0$$

The symmetries that make the world as we know it . . .

- *translations*:
  conservation of energy and momentum;
- *Lorentz transformations* (rotations and boosts):
  conservation of angular momentum (orbital and spin);
- *discrete transformations* (P,T,C,CP,. . .):
  conservation of corresponding quantum numbers;
- *global transformations of internal degrees of freedom* (ϕ_j "rotations")
  conservation of "isospin"-like quantum numbers;
- *local transformations of internal degrees of freedom* ($\phi_j(x)$ "rotations"):
  define the interaction of fermion (s=1/2) and scalar (s=0) particles in terms of exchanged vector (s=1) massless particles $\longrightarrow$ "forces"

Requiring different global and local symmetries defines a theory

AND

Keep in mind that they can be broken

From Global to Local: gauging a symmetry

Abelian case

A theory of free Fermi fields described by the Lagrangian density

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x)$$

is invariant under a global $U(1)$ transformation (α=constant phase)

$$\psi(x) \to e^{i\alpha}\psi(x) \quad \text{such that} \quad \partial_\mu\psi(x) \to e^{i\alpha}\partial_\mu\psi(x)$$

and the corresponding Noether's current is conserved,

$$J^\mu = \bar{\psi}(x)\gamma^\mu\psi(x) \quad \to \quad \partial_\mu J^\mu = 0$$

The same is not true for a local $U(1)$ transformation ($\alpha = \alpha(x)$) since

$$\psi(x) \to e^{i\alpha(x)}\psi(x) \quad \underline{\text{but}} \quad \partial_\mu\psi(x) \to e^{i\alpha(x)}\partial_\mu\psi(x) + ige^{i\alpha(x)}\partial_\mu\alpha(x)\psi(x)$$

Need to introduce a covariant derivative D_μ such that

$$D_\mu\psi(x) \to e^{i\alpha(x)}D_\mu\psi(x)$$

Only possibility: introduce a vector field $A_\mu(x)$ trasforming as

$$A_\mu(x) \to A_\mu(x) - \frac{1}{g}\partial_\mu \alpha(x)$$

and define a covariant derivative D_μ according to

$$D_\mu = \partial_\mu + igA_\mu(x)$$

modifying $\mathcal{L}$ to accomodate D_μ and the gauge field $A_\mu(x)$ as

$$\mathcal{L} = \bar{\psi}(x)(i\not{D} - m)\psi(x) - \frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x)$$

where the last term is the Maxwell Lagrangian for a vector field A^μ, i.e.

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) \ .$$

Requiring invariance under a local $U(1)$ symmetry has:

$\longrightarrow$ promoted a free theory of fermions to an interacting one;

$\longrightarrow$ defined univoquely the form of the interaction in terms of a new vector field $A^\mu(x)$:

$$\mathcal{L}_{int} = -g\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x)$$

$\longrightarrow$ no mass term $A^\mu A_\mu$ allowed by the symmetry $\to$ this is QED.

Non-abelian case: Yang-Mills theories

Consider the same Lagrangian density

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x)$$

where $\psi(x) \to \psi_i(x)$ $(i = 1, \ldots, n)$ is a n-dimensional representation of a non-abelian compact Lie group (e.g. $SU(N)$).

$\mathcal{L}$ is invariant under the global transformation $U(\alpha)$

$$\psi(x) \to \psi'(x) = U(\alpha)\psi(x) \ , \qquad U(\alpha) = e^{i\alpha^a T^a} = 1 + i\alpha^a T^a + O(\alpha^2)$$

where T^a $((a = 1, \ldots, d_{adj}))$ are the generators of the group infinitesimal transformations with algebra,

$$[T^a, T^b] = if^{abc}T^c$$

and the corresponding Noether's current are conserved. However, requiring $\mathcal{L}$ to be invariant under the corresponding local transformation $U(x)$

$$U(x) = 1 + i\alpha^a(x)T^a + O(\alpha^2)$$

brings us to replace ∂_μ by a covariant derivative

$$D_\mu = \partial_\mu - igA^a_\mu(x)T^a$$

in terms of vector fields $A_\mu^a(x)$ that transform as

$$A_\mu^a(x) \to A_\mu^a(x) + \frac{1}{g}\partial_\mu \alpha^a(x) + f^{abc} A_\mu^b(x)\alpha^c(x)$$

such that

$$\begin{aligned} D_\mu &\to U(x) D_\mu U^{-1}(x) \\ D_\mu \psi(x) &\to U(x) D_\mu U^{-1}(x) U(x) \psi = U(x) D_\mu \psi(x) \\ F_{\mu\nu} \equiv \frac{i}{g}[D_\mu, D_\nu] &\to U(x) F_{\mu\nu} U^{-1}(x) \end{aligned}$$

The invariant form of $\mathcal{L}$ or Yang Mills Lagrangian will then be

$$\mathcal{L}_{YM} = \mathcal{L}(\psi, D_\mu \psi) - \frac{1}{4}\mathrm{Tr} F_{\mu\nu} F^{\mu\nu} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

where $F_{\mu\nu} = F_{\mu\nu}^a T^a$ and

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

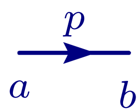
We notice that:

- as in the abelian case:
  $\longrightarrow$ mass terms of the form $A^{a,\mu}A^a_\mu$ are forbidden by symmetry: gauge bosons are massless
  $\longrightarrow$ the form of the interaction between fermions and gauge bosons is fixed by symmetry to be

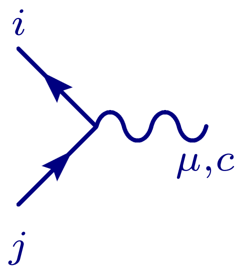
$$\mathcal{L}_{int} = -g\bar{\psi}(x)\gamma_\mu T^a \psi(x) A^{a,\mu}(x)$$

- at difference from the abelian case:
  $\longrightarrow$ gauge bosons carry a group charge and therefore ...
  $\longrightarrow$ gauge bosons have self-interaction.

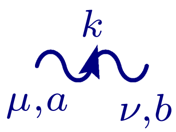
Feynman rules, Yang-Mills theory:



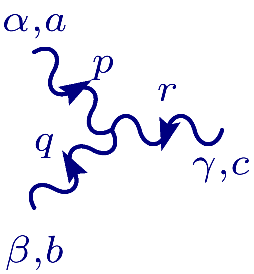
$$= \quad \frac{i\delta^{ab}}{\not{p} - m}$$



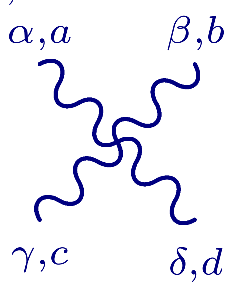
$$= \quad ig\gamma^{\mu}(T^{c})_{ij}$$



$$= \quad \frac{-i}{k^2}\left[g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]\delta^{ab}$$



$$= \quad gf^{abc}\left(g^{\beta\gamma}(q-r)^{\alpha} + g^{\gamma\alpha}(r-p)^{\beta} + g^{\alpha\beta}(p-q)^{\gamma}\right)$$



$$= \quad -ig^2\left[f^{abe}f^{cde}(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}) + f^{ace}f^{bde}(\cdots) + f^{ade}f^{bce}(\cdots)\right]$$

Spontaneous Breaking of a Gauge Symmetry

Abelian Higgs mechanism: one vector field $A^\mu(x)$ and one complex scalar field $\phi(x)$:

$$\boxed{\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi}$$

where

$$\mathcal{L}_A = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

and $(D^\mu = \partial^\mu + igA^\mu)$

$$\mathcal{L}_\phi = (D^\mu\phi)^* D_\mu\phi - V(\phi) = (D^\mu\phi)^* D_\mu\phi - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

$\mathcal{L}$ invariant under local phase transformation, or local $U(1)$ symmetry:

$$\begin{aligned} \phi(x) &\rightarrow e^{i\alpha(x)}\phi(x) \\ A^\mu(x) &\rightarrow A^\mu(x) + \frac{1}{g}\partial^\mu\alpha(x) \end{aligned}$$

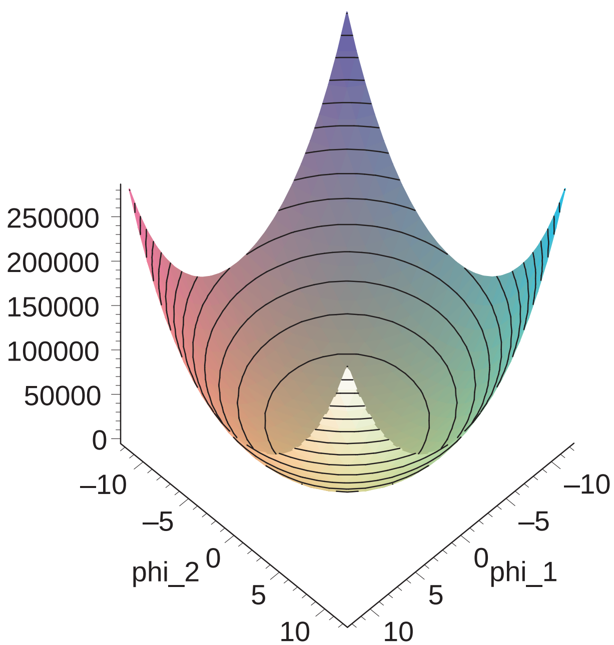
Mass term for A^μ breaks the $U(1)$ gauge invariance.

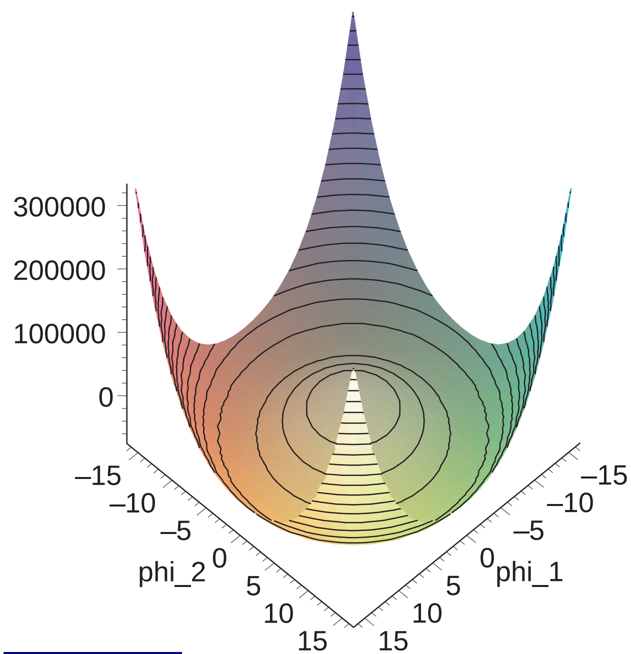
Can we build a gauge invariant massive theory? Yes.

Consider the potential of the scalar field:

$$\boxed{V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2}$$

where $\lambda > 0$ (to be bounded from below), and observe that:





$\boxed{\mu^2 > 0}$ → unique minimum:

$$\phi^* \phi = 0$$

$\boxed{\mu^2 < 0}$ → degeneracy of minima:

$$\phi^* \phi = \frac{-\mu^2}{2\lambda}$$

- $\mu^2 > 0 \longrightarrow$ electrodynamics of a massless photon and a massive scalar field of mass μ ($g = -e$).

- $\mu^2 < 0 \longrightarrow$ when we choose a minimum, the original $U(1)$ symmetry is spontaneously broken or hidden.

$$\phi_0 = \left(-\frac{\mu^2}{2\lambda}\right)^{1/2} = \frac{v}{\sqrt{2}} \quad \longrightarrow \quad \phi(x) = \phi_0 + \frac{1}{\sqrt{2}}\left(\phi_1(x) + i\phi_2(x)\right)$$

$$\Downarrow$$

$$\mathcal{L} = \underbrace{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}g^2v^2A^\mu A_\mu}_{\text{massive vector field}} + \underbrace{\frac{1}{2}(\partial^\mu\phi_1)^2 + \mu^2\phi_1^2}_{\text{massive scalar field}} + \underbrace{\frac{1}{2}(\partial^\mu\phi_2)^2 + gvA_\mu\partial^\mu\phi_2}_{\text{Goldstone boson}} + \ldots$$

Side remark: The ϕ_2 field actually generates the correct transverse structure for the mass term of the (now massive) A^μ field propagator:

$$\langle A^\mu(k)A^\nu(-k)\rangle = \frac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) + \cdots$$

More convenient parameterization (unitary gauge):

$$\phi(x) = \frac{e^{i\frac{\chi(x)}{v}}}{\sqrt{2}}(v + H(x)) \quad \xrightarrow{U(1)} \quad \frac{1}{\sqrt{2}}(v + H(x))$$

The $\chi(x)$ degree of freedom (Goldstone boson) is rotated away using gauge invariance, while the original Lagrangian becomes:

$$\mathcal{L} = \mathcal{L}_A + \frac{g^2 v^2}{2} A^\mu A_\mu + \frac{1}{2}\left(\partial^\mu H \partial_\mu H + 2\mu^2 H^2\right) + \ldots$$

which describes now the dynamics of a system made of:

- a massive vector field A^μ with $m_A^2 = g^2 v^2$;
- a real scalar field H of mass $m_H^2 = -2\mu^2 = 2\lambda v^2$: the Higgs field.

$$\Downarrow$$

Total number of degrees of freedom is balanced

Non-Abelian Higgs mechanism: several vector fields $A^a_\mu(x)$ and several (real) scalar field $\phi_i(x)$:

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi \quad , \quad \mathcal{L}_\phi = \frac{1}{2}(D^\mu\phi)^2 - V(\phi) \quad , \quad V(\phi) = \mu^2\phi^2 + \frac{\lambda}{2}\phi^4$$

($\mu^2 < 0$, $\lambda > 0$) invariant under a non-Abelian symmetry group G:

$$\phi_i \longrightarrow (1 + i\alpha^a t^a)_{ij}\phi_j \overset{t^a = iT^a}{\longrightarrow} (1 - \alpha^a T^a)_{ij}\phi_j$$

(s.t. $D_\mu = \partial_\mu + g A^a_\mu T^a$). In analogy to the Abelian case:

$$\frac{1}{2}(D_\mu\phi)^2 \quad \longrightarrow \quad \ldots + \frac{1}{2}g^2(T^a\phi)_i(T^b\phi)_i A^a_\mu A^{b\mu} + \ldots$$

$$\overset{\phi_{min}=\phi_0}{\longrightarrow} \quad \ldots + \frac{1}{2}\underbrace{g^2(T^a\phi_0)_i(T^b\phi_0)_i}_{m^2_{ab}} A^a_\mu A^{b\mu} + \ldots =$$

$\boxed{T^a\phi_0 \neq 0}$ $\longrightarrow$ massive vector boson + (Goldstone boson)

$\boxed{T^a\phi_0 = 0}$ $\longrightarrow$ massless vector boson + massive scalar field

Classical $\longrightarrow$ Quantum : $\boxed{V(\phi) \longrightarrow V_{eff}(\varphi_{cl})}$

The stable vacuum configurations of the theory are now determined by the extrema of the Effective Potential:

$$V_{eff}(\varphi_{cl}) = -\frac{1}{VT}\Gamma_{eff}[\phi_{cl}] \;\;,\;\; \phi_{cl} = \text{constant} = \varphi_{cl}$$

where

$$\Gamma_{eff}[\phi_{cl}] = W[J] - \int d^4y J(y)\phi_{cl}(y) \quad , \quad \phi_{cl}(x) = \frac{\delta W[J]}{\delta J(x)} = \langle 0|\phi(x)|0\rangle_J$$

$W[J] \longrightarrow$ generating functional of connected correlation functions

$\Gamma_{eff}[\phi_{cl}] \longrightarrow$ generating functional of 1PI connected correlation functions

$V_{eff}(\varphi_{cl})$ can be organized as a loop expansion (expansion in $\hbar$), s.t.:

$$V_{eff}(\varphi_{cl}) = V(\varphi_{cl}) + \text{loop effects}$$

SSB $\longrightarrow$ non trivial vacuum configurations

Gauge fixing : the R_ξ gauges. Consider the abelian case:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^* D_\mu\phi - V(\phi)$$

upon SSB:

$$\phi(x) = \frac{1}{\sqrt{2}}((v + \phi_1(x)) + i\phi_2(x))$$

$$\Downarrow$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial^\mu\phi_1 + gA^\mu\phi_2)^2 + \frac{1}{2}(\partial^\mu\phi_2 - gA^\mu(v + \phi_1))^2 - V(\phi)$$

Quantizing using the gauge fixing condition:

$$G = \frac{1}{\sqrt{\xi}}(\partial_\mu A^\mu + \xi g v \phi_2)$$

in the generating functional

$$Z = C\int \mathcal{D}A\mathcal{D}\phi_1\mathcal{D}\phi_2 \exp\left[\int d^4x\left(\mathcal{L} - \frac{1}{2}G^2\right)\right]\det\left(\frac{\delta G}{\delta\alpha}\right)$$

($\alpha \longrightarrow$ gauge transformation parameter)

$$\begin{aligned}
\mathcal{L} - \frac{1}{2}G^2 &= -\frac{1}{2}A_\mu \left(-g^{\mu\nu}\partial^2 + \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu - (gv)^2 g^{\mu\nu}\right) A_\nu \\
& \quad \frac{1}{2}(\partial_\mu\phi_1)^2 - \frac{1}{2}m^2_{\phi_1}\phi_1^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 - \frac{\xi}{2}(gv)^2\phi_2^2 + \cdots \\
&+ \\
\mathcal{L}_{ghost} &= \bar{c}\left[-\partial^2 - \xi(gv)^2\left(1 + \frac{\phi_1}{v}\right)\right]c
\end{aligned}$$

such that:

$$\begin{aligned}
\langle A^\mu(k)A^\nu(-k)\rangle &= \frac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) + \frac{-i\xi}{k^2 - \xi m_A^2}\left(\frac{k^\mu k^\nu}{k^2}\right) \\
\langle \phi_1(k)\phi_1(-k)\rangle &= \frac{-i}{k^2 - m^2_{\phi_1}} \\
\langle \phi_2(k)\phi_2(-k)\rangle &= \langle c(k)\bar{c}(-k)\rangle = \frac{-i}{k^2 - \xi m_A^2}
\end{aligned}$$

Goldtone boson ϕ_2, $\Longleftrightarrow$ longitudinal gauge bosons

Towards the Standard Model of particle physics

Translating experimental evidence into the right gauge symmetry group.

- Electromagnetic interactions $\rightarrow$ QED
  - ▷ well established example of abelian gauge theory
  - ▷ extremely succesful quantum implementation of field theories
  - ▷ useful but very simple template
- Strong interactions $\rightarrow$ QCD
  - ▷ evidence for strong force in hadronic interactions
  - ▷ Gell-Mann-Nishijima quark model interprets hadron spectroscopy
  - ▷ need for extra three-fold quantum number (color) (ex.: hadronic spectroscopy, $e^+e^- \rightarrow$hadrons, ...)
  - ▷ natural to introduce the gauge group $\rightarrow SU(3)_C$
  - ▷ DIS experiments: confirm parton model based on $SU(3)_C$
  - ▷ ... and much more!
- Weak interactions $\rightarrow$ most puzzling ...
  - ▷ discovered in neutron β-decay: $n \rightarrow p + e^- + \bar{\nu}_e$
  - ▷ new force: small rates/long lifetimes
  - ▷ universal: same strength for both hadronic and leptonic processes ($n \rightarrow pe^-\nu_e$, $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$, $\mu^- \rightarrow e^-\bar{\nu}_e + \nu_\mu$, ...)

- violates parity (P)
- charged currents only affect left-handed particles (right-handed antiparticles)
- neutral currents not of electromagnetic nature
- First description: Fermi Theory (1934)

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} (\bar{p}\gamma_\mu(1-\gamma_5)n)(\bar{e}\gamma^\mu(1-\gamma_5)\nu_e)$$

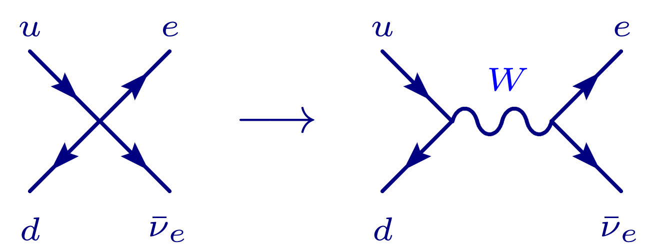
$G_F \to$ Fermi constant, $[G_F] = m^{-2}$ (in units of $c = \hbar = 1$).

- Easely accomodates a <u>massive</u> intermediate vector boson

$$\mathcal{L}_{IVB} = \frac{g}{\sqrt{2}} W_\mu^+ J_\mu^- + \text{h.c.}$$

with (in a proper quark-based notation)

$$J_\mu^- = \bar{u}\gamma_\mu \frac{1-\gamma_5}{2} d + \bar{\nu_e}\gamma^\mu \frac{1-\gamma_5}{2} e$$



provided that,

$$q^2 \ll M_W^2 \quad \longrightarrow \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

- Promote it to a gauge theory: natural candidate $SU(2)_L$, but if $T^{1,2}$ can generate the charged currents ($T^{\pm} = (T^1 \pm iT^2)$), T^3 cannot be the electromagnetic charge (Q) ($T^3 = \sigma^3/2$'s eigenvalues do not match charges in $SU(2)$ doublets)
- Need extra $U(1)_Y$, such that $Y = T^3 - Q$!
- Need massive gauge bosons → SSB

$$\Downarrow$$

$$SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_Q$$

$$\mathcal{L}_{SM} = \mathcal{L}_{QCD} + \mathcal{L}_{EW}$$

where

$$\mathcal{L}_{EW} = \mathcal{L}_{EW}^{\text{ferm}} + \mathcal{L}_{EW}^{\text{gauge}} + \mathcal{L}_{EW}^{SSB} + \mathcal{L}_{EW}^{Yukawa}$$

Strong interactions: Quantum Chromodynamics

Exact Yang-Mills theory based on $SU(3)_C$ (quark fields only):

$$\mathcal{L}_{QCD} = \sum_i \bar{Q}_i (i\not{D} - m_i)\, Q_i - \frac{1}{4} F^{a,\mu\nu} F^a_{\mu\nu}$$

with

$$D_\mu = \partial_\mu - i g A^a_\mu T^a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

- $Q_i \rightarrow (i = 1, \ldots, 6 \rightarrow u, d, s, c, b, t)$ fundamental representation of $SU(3) \rightarrow$ triplets:

$$Q_i = \begin{pmatrix} Q_i \\ Q_i \\ Q_i \end{pmatrix}$$

- $A^a_\mu \rightarrow$ adjoint representation of $SU(3) \rightarrow N^2 - 1 = 8$ massless gluons
  $T^a \rightarrow SU(3)$ generators (Gell-Mann's matrices)

Electromagnetic and weak interactions: unified into Glashow-Weinberg-Salam theory

Spontaneously broken Yang-Mills theory based on $SU(2)_L \times U(1)_Y$.

- $SU(2)_L \rightarrow$ weak isospin group, gauge coupling g:
  - ▷ three generators: $T^i = \sigma^i/2$ (σ^i = Pauli matrices, $i = 1, 2, 3$)
  - ▷ three gauge bosons: W_1^μ, W_2^μ, and W_3^μ
  - ▷ $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$ fields are doublets of $SU(2)$
  - ▷ $\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$ fields are singlets of $SU(2)$
  - ▷ mass terms not allowed by gauge symmetry
- $U(1)_Y \rightarrow$ weak hypercharge group ($Q = T_3 + Y$), gauge coupling g':
  - ▷ one generator $\rightarrow$ each field has a Y charge
  - ▷ one gauge boson: B^μ

Example: first generation

$$L_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}_{Y=-1/2} \qquad (\nu_{eR})_{Y=0} \qquad (e_R)_{Y=-1}$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}_{Y=1/6} \qquad (u_R)_{Y=2/3} \qquad (d_R)_{Y=-1/3}$$

Three fermionic generations, summary of gauge quantum numbers:

				$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_Q$
$Q^i_L =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$	$\frac{2}{3}$ $-\frac{1}{3}$
$u^i_R =$	u_R	c_R	t_R	3	1	$\frac{2}{3}$	$\frac{2}{3}$
$d^i_R =$	d_R	s_R	b_R	3	1	$-\frac{1}{3}$	$-\frac{1}{3}$
$L^i_L =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$	0 -1
$e^i_R =$	e_R	μ_R	τ_R	1	1	-1	-1
$\nu^i_R =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0	0

where a minimal extension to include ν^i_R has been allowed (notice however that it has zero charge under the entire SM gauge group!)

Lagrangian of fermion fields

For each generation (here specialized to the first generation):

$$\mathcal{L}_{EW}^{\text{ferm}} = \bar{L}_L(i\not{D})L_L + \bar{e}_R(i\not{D})e_R + \bar{\nu}_{eR}(i\not{D})\nu_{eR} + \bar{Q}_L(i\not{D})Q_L + \bar{u}_R(i\not{D})u_R + \bar{d}_R(i\not{D})d_R$$

where in each term the covariant derivative is given by

$$D_\mu = \partial_\mu - igW_\mu^i T^i - ig'\frac{1}{2}YB_\mu$$

and $T^i = \sigma^i/2$ for L-fields, while $T^i = 0$ for R-fields ($i = 1, 2, 3$), i.e.

$$\begin{aligned} D_{\mu,L} &= \partial_\mu - \frac{ig}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} gW_\mu^3 - g'YB_\mu & 0 \\ 0 & -gW_\mu^3 - g'YB_\mu \end{pmatrix} \\ D_{\mu,R} &= \partial_\mu + ig'\frac{1}{2}YB_\mu \end{aligned}$$

with

$$W^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

$\mathcal{L}_{EW}^{\text{ferm}}$ can then be written as

$$\mathcal{L}_{EW}^{\text{ferm}} = \mathcal{L}_{kin}^{\text{ferm}} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

where

$$\begin{aligned}
\mathcal{L}_{kin}^{\text{ferm}} &= \bar{L}_L(i\partial\!\!\!/)L_L + \bar{e}_R(i\partial\!\!\!/)e_R + \ldots \\
\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}}W_\mu^+\bar{\nu}_{eL}\gamma^\mu e_L + W_\mu^-\bar{e}_L\gamma^\mu\nu_{eL} + \ldots \\
\mathcal{L}_{NC} &= \frac{g}{2}W_\mu^3\left[\bar{\nu}_{eL}\gamma^\mu\nu_{eL} - \bar{e}_L\gamma^\mu e_L\right] + \frac{g'}{2}B_\mu\left[Y(L)(\bar{\nu}_{eL}\gamma^\mu\nu_{eL} + \bar{e}_L\gamma^\mu e_L)\right. \\
&+ \left. Y(e_R)\bar{\nu}_{eR}\gamma^\mu\nu_{eR} + Y(e_R)\bar{e}_R\gamma^\mu e_R\right] + \ldots
\end{aligned}$$

where

$W^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right) \rightarrow$ mediators of Charged Currents
W_μ^3 and $B_\mu \rightarrow$ mediators of Neutral Currents.

$$\Downarrow$$

<u>However</u> neither W_μ^3 nor B_μ can be identified with the photon field A_μ, because they couple to neutral fields.

Rotate W_μ^3 and B_μ introducing a weak mixing angle (θ_W)

$$\begin{aligned} W_\mu^3 &= \sin\theta_W A_\mu + \cos\theta_W Z_\mu \\ B_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z_\mu \end{aligned}$$

such that the kinetic terms are still diagonal and the neutral current lagrangian becomes

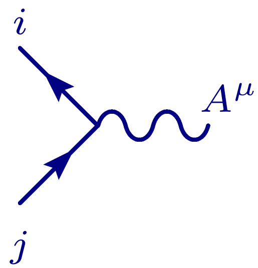
$$\mathcal{L}_{NC} = \bar\psi\gamma^\mu\left(g\sin\theta_W T^3 + g'\cos\theta_W \frac{Y}{2}\right)\psi A_\mu + \bar\psi\gamma^\mu\left(g\cos\theta_W T^3 - g'\sin\theta_W \frac{Y}{2}\right)\psi Z_\mu$$

for $\psi^T = (\nu_{eL}, e_L, \nu_{eR}, e_R, \dots)$. One can then identify ($Q \to$ e.m. charge)

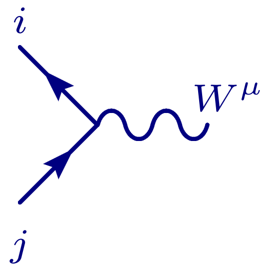
$$eQ = g\sin\theta_W T^3 + g'\cos\theta_W \frac{Y}{2}$$

and, e.g., from the leptonic doublet L_L derive that

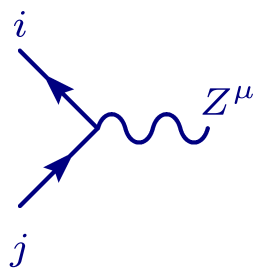
$$\left\{ \begin{aligned} \tfrac{g}{2}\sin\theta_W - \tfrac{g'}{2}\cos\theta_W &= 0 \\ -\tfrac{g}{2}\sin\theta_W - \tfrac{g'}{2}\cos\theta_W &= -e \end{aligned} \right. \longrightarrow g\sin\theta_W = g'\cos\theta_W = e$$



$$= \quad -ieQ_f\gamma^\mu$$



$$= \quad \frac{ie}{2\sqrt{2}s_w}\gamma^\mu(1-\gamma_5)$$



$$= \quad ie\gamma^\mu(v_f - a_f\gamma_5)$$

where

$$v_f = -\frac{s_w}{c_w}Q_f + \frac{T_f^3}{2s_W c_W}$$

$$a_f = \frac{T_f^3}{2s_W c_W}$$

Lagrangian of gauge fields

$$\mathcal{L}_{EW}^{\text{gauge}} = -\frac{1}{4} W_{\mu\nu}^{a} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

where

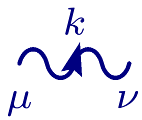
$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W_{\mu\nu}^{a} &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc} W_\mu^b W_\nu^c \end{aligned}$$

in terms of physical fields:

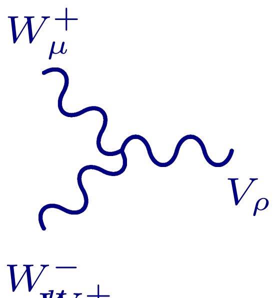
$$\mathcal{L}_{EW}^{\text{gauge}} = \mathcal{L}_{kin}^{\text{gauge}} + \mathcal{L}_{EW}^{3V} + \mathcal{L}_{EW}^{4V}$$

where

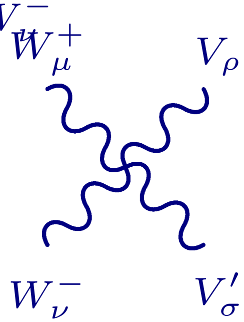
$$\begin{aligned} \mathcal{L}_{kin}^{\text{gauge}} &= -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) \\ &- \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ \mathcal{L}_{EW}^{3V} &= (\text{3-gauge-boson vertices involving } ZW^+W^- \text{ and } AW^+W^-) \\ \mathcal{L}_{EW}^{4V} &= (\text{4-gauge-boson vertices involving } ZZW^+W^-,\ AAW^+W^-, \\ &\quad AZW^+W^-, \text{ and } W^+W^-W^+W^-) \end{aligned}$$



$$= \quad \frac{-i}{k^2 - M_V^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_V^2}\right)$$



$$= \quad ieC_V\left[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\right]$$



$$= \quad ie^2 C_{VV'}\left(2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right)$$

where

$$C_\gamma = 1 \ , \quad C_Z = -\frac{c_W}{s_W}$$

and

$$C_{\gamma\gamma} = -1 \ , \quad C_{ZZ} = -\frac{c_W^2}{s_W^2} \ , \quad C_{\gamma Z} = \frac{c_W}{s_W} \ , \quad C_{WW} = \frac{1}{s_W^2}$$

The Higgs sector of the Standard Model: $SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_Q$

Introduce one complex scalar doublet of $SU(2)_L$ with $Y = 1/2$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \longleftrightarrow \quad \mathcal{L}_{EW}^{SSB} = (D^\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

where $D_\mu \phi = (\partial_\mu - i g W_\mu^a T^a - i g' Y_\phi B_\mu)$, ($T^a = \sigma^a/2$, $a = 1,2,3$).

The SM symmetry is spontaneously broken when $\langle \phi \rangle$ is chosen to be (e.g.):

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{with} \quad v = \left(\frac{-\mu^2}{\lambda} \right)^{1/2} \quad (\mu^2 < 0,\ \lambda > 0)$$

The gauge boson mass terms arise from:

$$\begin{aligned} (D^\mu \phi)^\dagger D_\mu \phi \quad &\longrightarrow \quad \cdots + \frac{1}{8} (0 \ \ v) \left(g W_\mu^a \sigma^a + g' B_\mu \right) \left(g W^{b\mu} \sigma^b + g' B^\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} + \cdots \\ &\longrightarrow \quad \cdots + \frac{1}{2} \frac{v^2}{4} \left[g^2 (W_\mu^1)^2 + g^2 (W_\mu^2)^2 + (-g W_\mu^3 + g' B_\mu)^2 \right] + \cdots \end{aligned}$$

And correspond to the weak gauge bosons:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2) \longrightarrow \boxed{M_W = g\frac{v}{2}}$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g W_\mu^3 - g' B_\mu) \longrightarrow \boxed{M_Z = \sqrt{g^2+g'^2}\frac{v}{2}}$$

while the linear combination orthogonal to Z_μ remains massless and corresponds to the photon field:

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g' W_\mu^3 + g B_\mu) \longrightarrow \boxed{M_A = 0}$$

Notice: using the definition of the weak mixing angle, θ_w:

$$\cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}} \quad , \quad \sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}$$

the W and Z masses are related by: $\boxed{M_W = M_Z \cos\theta_w}$

The scalar sector becomes more transparent in the unitary gauge:

$$\phi(x) = \frac{e^{\frac{i}{v}\vec{\chi}(x)\cdot\vec{\tau}}}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \xrightarrow{SU(2)} \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

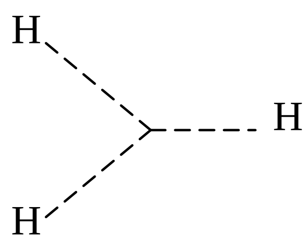
after which the Lagrangian becomes

$$\mathcal{L} = \mu^2 H^2 - \lambda v H^3 - \frac{1}{4} H^4 = -\frac{1}{2} M_H^2 H^2 - \sqrt{\frac{\lambda}{2}} M_H H^3 - \frac{1}{4} \lambda H^4$$

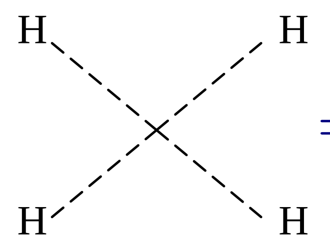
Three degrees of freedom, the $\chi^a(x)$ Goldstone bosons, have been reabsorbed into the longitudinal components of the $W_\mu^\pm$ and Z_μ weak gauge bosons. One real scalar field remains:

the Higgs boson, H, with mass $\boxed{M_H^2 = -2\mu^2 = 2\lambda v^2}$

and self-couplings:

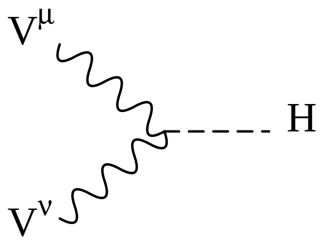


$= -3i\frac{M_H^2}{v}$

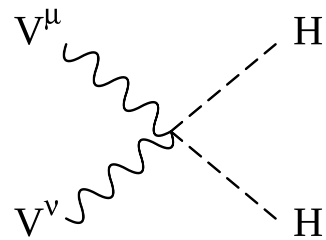


$= -3i\frac{M_H^2}{v^2}$

From $(D^\mu \phi)^\dagger D_\mu \phi \longrightarrow$ Higgs-Gauge boson couplings:



$$= 2i \frac{M_V^2}{v} g^{\mu\nu}$$



$$= 2i \frac{M_V^2}{v^2} g^{\mu\nu}$$

Notice: The entire Higgs sector depends on only two parameters, e.g.

M_H and v

v measured in μ-decay:

$v = (\sqrt{2} G_F)^{-1/2} = 246$ GeV $\longrightarrow$ **SM Higgs Physics depends on M_H**

Higgs boson couplings to quarks and leptons

The gauge symmetry of the SM also forbids fermion mass terms $(m_{Q_i} Q_L^i u_R^i, \ldots)$, but all fermions are massive.

$$\Downarrow$$

Fermion masses are generated via gauge invariant Yukawa couplings:

$$\mathcal{L}_{EW}^{Yukawa} = -\Gamma_u^{ij} \bar{Q}_L^i \phi^c u_R^j - \Gamma_d^{ij} \bar{Q}_L^i \phi d_R^j - \Gamma_e^{ij} \bar{L}_L^i \phi l_R^j + \text{h.c.}$$

such that, upon spontaneous symmetry breaking:

$$\begin{aligned} \mathcal{L}_{EW}^{Yukawa} &= -\Gamma_u^{ij} \bar{u}_L^i \frac{v+H}{\sqrt{2}} u_R^j - \Gamma_d^{ij} \bar{d}_L^i \frac{v+H}{\sqrt{2}} d_R^j - \Gamma_e^{ij} \bar{l}_L^i \frac{v+H}{\sqrt{2}} l_R^j + \text{h.c.} \\ &= -\sum_{f,i,j} \bar{f}_L^i M_f^{ij} f_R^j \left(1 + \frac{H}{v}\right) + \text{h.c.} \end{aligned}$$

where

$$M_f^{ij} = \Gamma_f^{ij} \frac{v}{\sqrt{2}}$$

is a non-diagonal mass matrix.

Upon diagonalization (by unitary transformation U_L and U_R)

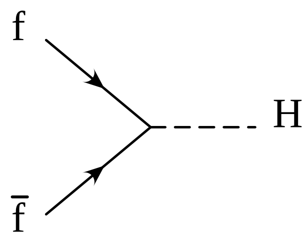
$$M_D = (U_L^f)^\dagger M_f U_R^f$$

and defining mass eigenstates:

$$f_L'^{\,i} = (U_L^f)_{ij} f_L^j \quad \text{and} \quad f_R'^{\,i} = (U_R^f)_{ij} f_R^j$$

the fermion masses are extracted as

$$\begin{aligned}
\mathcal{L}_{EW}^{Yukawa} &= \sum_{f,i,j} \bar{f}_L'^{\,i} [(U_L^f)^\dagger M_f U_R^f] f_R'^{\,j} \left(1 + \frac{H}{v}\right) + \text{h.c.} \\
&= \sum_{f,i,j} m_f \left(\bar{f}_L' f_R' + \bar{f}_R' f_L'\right) \left(1 + \frac{H}{v}\right)
\end{aligned}$$



$$= -i \frac{m_f}{v} = -i y_t$$

In terms of the new mass eigenstates the quark part of $\mathcal{L}_{CC}$ now reads

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \bar{u}'^{\,i}_L [(U^u_L)^\dagger U^d_R] \gamma^\mu d^j_L + \text{h.c.}$$

where

$$V_{CKM} = (U^u_L)^\dagger U^d_R$$

is the Cabibbo-Kobayashi-Maskawa matrix, origin of flavor mixing in the SM.

$$\Downarrow$$

see G. Buchalla's lectures at this school