The Standard Model of Particle Physics
Lecture II
Radiative corrections, renormalization, and physical observables

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Maria Laach School, Bautzen, September 2011
Outline of Lectures II

• Radiative corrections and Renormalization:
  → renormalization, general structure;
  → SM case: main results.

• Radiative corrections and physical observables:
  → precision tests: testing consistency and constraining unknowns
    ⇐ lepton collider dominated (LEP)
  → obtain best estimate of SM processes for searches of new physics
    ⇐ hadron collider dominated (Tevatron, LHC)

↓

This last point will be expanded in Lectures III and IV
Systematic of renormalization, in a nutshell

Simplest case: scalar \((g\phi^4)\) theory \(\rightarrow \mathcal{L}(\phi_0, \partial_\mu \phi_0, m_0, g_0)\)

\[ \implies \]

Calculating scattering amplitudes \(\langle f|\phi\ldots\phi|i\rangle\) via perturbative approach introduce **divergencies beyond the tree level:**

\[ \rightarrow \textbf{ultraviolet (UV):} \text{ in the } p^2 \rightarrow \infty \text{ region of momentum loop integrals, ex.:} \]

\[ \int \frac{d^4p}{p^2(p^2 - m^2)} \xrightarrow{p \to \infty} \int \frac{dp}{p} \rightarrow \log\text{-}\text{divergence} \]

\[ \rightarrow \textbf{infrared (IR):} \text{ in both loop and real corrections, due to soft } (p^2 \to 0) \text{ or collinear } (p \cdot p_i \to 0) \text{ radiation/loop-momenta;} \]

\[ \int \frac{d^4p}{p^2(p^2 + 2p \cdot q_1)(p^2 + 2p \cdot q_2)} \xrightarrow{p \to 0} \int \frac{dp}{p} \rightarrow \log\text{-}\text{divergence} \]
**Actions taken:** at a given perturbative order,

→ regularize and extract UV and IR singularities, most common:

$$\int d^4 p \rightarrow \mu^{4-d} \int d^d p$$ dimensional regularization

▷ divergencies extracted as poles in \((4 - d)\).

▷ \(\mu\) → (renormalization) scale parameter associated to regularization procedure

→ cancel UV singularities by switching from “bare” to “renormalized” parameters/fields, fixed by suitable renormalization conditions,

\[ \mathcal{L}(\phi_0, \partial_\mu \phi_0, m_0, g_0) \rightarrow \mathcal{L}(\phi, \partial_\mu \phi, m, g) \]

where

\[ m_0^2 = m^2 + \delta m^2 \quad \phi_0 = \sqrt{Z_\phi} \phi \quad g_0 = g + \delta g \]

and \(m, \phi\) and \(g\) (alternatively \(\delta m^2, Z_\phi = 1 + \delta Z_\phi, \delta g\)) are defined by fixing the renormalized proper vertices (or vertex functions) of the theory.

→ cancel IR singularities in the sum of virtual+real corrections (only hard radiation can be resolved).
UV systematics: consider the proper vertices of the theory $\Gamma^{\phi\cdots\phi}$

$\Gamma^{\phi\cdots\phi} \rightarrow$ one-particle irreducible (1PI) diagrams with $n$ external legs.

2-point proper vertex: $\Gamma^{\phi\phi}$

$$\Gamma^{\phi_0\phi_0} = i(p^2 - m_0^2) + i \Sigma(p^2)$$

$$= \quad \quad + \quad \quad$$

$\Sigma=$ self-energy $=$ sum of 1PI (one-particle-irreducible) diagrams (all orders).

Notice relation with all orders propagator:

$$= \quad \quad + \quad \quad$$

$$= \quad \quad + \quad \quad$$

$$= \quad \quad + \quad \quad$$

In terms of bare parameters, $\Gamma^{\phi_0\phi_0}$ is UV divergent: fix $\delta m^2$ and $\delta Z_\phi$ by requiring, e.g., that $\Gamma^{\phi\phi} = Z_\phi^{-1} \Gamma^{\phi_0\phi_0}$ has a pole at the physical mass with unit residue, which gives:

$$\Sigma(m^2) = \delta m^2 \quad \text{and} \quad \Sigma'(m^2) = -\delta Z_\phi$$
4-point proper vertex: $\Gamma^{\phi\phi\phi\phi}$

$$
\Gamma^{\phi_0\phi_0\phi_0\phi_0} = ig_0 + i\Gamma(p_1,p_2,p_3)
$$

In terms of bare parameters, $\Gamma^{\phi_0\phi_0\phi_0\phi_0}$ is UV divergent: fix $\delta g$ by requiring, e.g., that $g$ corresponds to the coupling measured in a specific kinematic realization, which gives:

$$
\delta g = 2g\delta Z_\phi - \Gamma(p_{1exp},p_{2exp},p_{3exp})
$$

where we also use that $\Gamma^{\phi\phi\phi\phi} = Z_\phi^2\Gamma^{\phi_0\phi_0\phi_0\phi_0}$.

All other n-point proper vertices, $\Gamma^{\phi\cdots\phi}$: are obtainable using $\Gamma^{\phi\phi}$ and $\Gamma^{\phi\phi\phi\phi}$ as building blocks, and finite when expressed in terms of $m$ and $g$.

Notice: parameters are now scale-dependent, $m(q^2)$, $g(q^2)$.

Any physical observables calculated in terms of $m$ and $g$ is finite and well defined, although affected by a systematic (perturbative) uncertainty.
Standard Model renormalization: main results

The SM Lagrangian is made of renormalizable field structures,

\[ \mathcal{L}_{SM} = \mathcal{L}_{QCD} + \mathcal{L}_{EW} \]

\[ = \mathcal{L}_{EW}^{\text{ferm}} + \mathcal{L}_{EW}^{\text{gauge}} + \mathcal{L}_{EW}^{SSB} + \mathcal{L}_{EW}^{Yukawa} \]

where,

\[ \mathcal{L}_{QCD} \rightarrow \bar{\psi}(\partial \phi - m)\psi, \bar{\psi}A\psi, \frac{1}{4}G^{a,\mu\nu}G_{\mu\nu} \]

\[ \mathcal{L}_{EW}^{\text{ferm}} \rightarrow \bar{\psi}_L(\partial \phi)\psi_L, \bar{\psi}_L V \psi_L \]

\[ \mathcal{L}_{EW}^{\text{gauge}} \rightarrow \frac{1}{4}F^{a,\mu\nu}F_{\mu\nu}, \frac{1}{4}B^\mu B_{\mu\nu} \]

\[ \mathcal{L}_{EW}^{SSB} \rightarrow \partial^\mu \phi \partial_\mu \phi, \mu^2 \phi^2, \phi^4 \]

\[ \mathcal{L}_{EW}^{Yukawa} \rightarrow \bar{\psi}_L H \psi_R \]

The systematic procedure outlined in these lectures will apply with extra constraints imposed by the presence of a partially spontaneously broken gauge symmetry.
The set of fundamental parameters of the SM Lagrangian is:

\[ g_{s,0}, g_{0}, g'_{0}, \mu_{0}, \lambda_{0}, y_{f,0}, V_{0}^{ij} \]

here taken as bare parameters. Thanks to relations induced by the symmetries of the theory, e.g.

\[ e = g \sin \theta_{W} = g' \cos \theta_{W} \quad \rightarrow \quad e = \frac{gg'}{\sqrt{g^2 + g'^2}} \]

\[ M_{W} = \frac{gv}{2}, \quad M_{Z} = \frac{v\sqrt{g^2 + g'^2}}{2} \quad \rightarrow \quad \frac{M_{W}}{M_{Z}} = \frac{g}{\sqrt{g^2 + g'^2}} = \frac{e}{g'} = \cos \theta_{W} \]

we can trade them for other or “better” sets of input parameters, for example:

\[ g_{s,0}, e_{0}, M_{W,0}, M_{Z,0}, M_{H,0}, m_{f,0}, V_{0}^{ij} \]

and switch to the corresponding set of renormalized or physical parameters upon imposing suitable renormalization conditions.

⇒ Relations like \( M_{W}/M_{Z} = \cos \theta_{W} \) will automatically be finite but corrections depend on input parameters (e.g. \( m_{t}, M_{H} \)) \( \rightarrow \) natural relations. Need to specify renormalization scheme and use consistency.
Definitions and renormalization conditions

**QCD** → in the absence of a mass scale, use \( \overline{\text{MS}} \) scheme or minimal subtraction scheme, i.e. subtract just pole parts of each divergent proper vertex.

**EW** → use procedure illustrated in this lecture for a scalar \( g\phi^4 \) toy model → on-shell subtraction scheme.

- mass/coupling renormalization:
  \[
  M_{W,0}^2 = M_W^2 + \delta M_W^2, \quad \ldots \quad m_{f,0} = m_f + \delta m_f, \quad V_{0}^{ij} = V^{ij} + \delta V^{ij}
  \]

- field renormalization:
  \[
  W_{0}^{\pm} = \sqrt{Z_W} W^{\pm}, \quad \begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{ZZ}} & \sqrt{Z_{ZA}} \\ \sqrt{Z_{AZ}} & \sqrt{Z_{AA}} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix} \ldots
  \]

where, the following renormalization conditions are traditionally adopted:

\[
\delta M_W^2 = \text{Re}\left[\Sigma_T^W (M_W^2)\right], \quad \delta Z_W = -\text{Re}\left[\Sigma_T^W (M_W^2)\right], \quad \ldots
\]

and similar ones for other vector+scalar and field renormalization constants ⇒ the bulk of corrections are in the self-energies!
Flavor sector: need to carefully account for the rotation to mass eigenstates (beyond the scope of these lectures).

Finally, the QED electric charge renormalization condition is adopted: $e$ defined as measure in the Thomson limit ($k \to 0$ scattering of photons, on-shell electrons)

$$\alpha(0) = \frac{e^2}{2\pi} \approx \frac{1}{137}$$

Once expressed in terms of the renormalized parameters and fields, any physical observable is finite and can be calculated at the proper perturbative order in QCD+EW and compared with experimental results.

Electroweak precision fits
Electroweak precision fits

An incredible amount of measurements of electroweak observables have been collected over the past many decades:

- **SppS** at CERN (1981-1990) 300 × 300 GeV $p\bar{p}$ collider: discovery of $W$ and $Z$ bosons!

- **LEP I** at CERN (1989-1995) at energies around $\sqrt{s} = M_Z$: $e^+e^- \rightarrow Z \rightarrow f\bar{f}$.

- **LEP II** at CERN (1996-2000) at energies around $\sqrt{s} = 200 - 208$ GeV: $e^+e^- \rightarrow WW \rightarrow 4f$.

- **SLC** at SLAC (1989-1998) at energies up to 100 GeV, polarized beams.

- **Tevatron** at Fermilab, RUN I+II (1987-2011) 0.98 × 0.98 TeV $p\bar{p}$ collider: top-quark discovery! Precision measurement of $M_W$ and $m_t$.

- **LHC** at CERN, now running at $3.5 \times 3.5$ TeV, will go up to $7 \times 7$ TeV: will rediscovery the SM and more!
Measurement of $M_Z$ and $\Gamma_Z$ at LEP I

At the $Z$ pole $e^+e^- \rightarrow f \bar{f}$ ($f \neq e$) dominated by $Z$ exchange:

$$\frac{d\sigma_f^Z}{d\Omega} = \frac{9}{4} \frac{s\Gamma_{ee}\Gamma_{f\bar{f}}/M_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} \left[ (1 + \cos^2 \theta)(1 - P_e A_e) + 2 \cos \theta A_f (-P_e + A_e) \right]$$

$P_e$: polarization of electron beam
$\Gamma_{ee}, \Gamma_{f\bar{f}}$: partial widths for $Z \rightarrow e^+ e^- f \bar{f}$
$A_f$: L-R coupling constant asymmetry

$$A_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2} = \frac{2g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}$$

Scanning at the $Z$ pick and fitting to $\sigma_f^Z$ yields measurements of $M_Z$, $\Gamma_Z$ and $\sigma_{\text{had}} = 12\pi \Gamma_{ee}\Gamma_{\text{had}}/(M_Z^2 \Gamma_Z^2)$. 
Measurement of $M_W$ at LEP II

From $W$ invariant-mass reconstruction: $t$-channel $\nu$-exchange, $s$-channel $\gamma$- and $Z$-exchange $\to$ test of 3$V$-gauge coupling.

From rise of the $WW$ cross section near threshold (statistically limited) but till another test of non-abelian interactions.
Measurement of $m_t$ at the Tevatron

New measurement from LHC should improve precision.
Strategy

Having a variety of measurement for different observables, test the SM by comparing theory and experiment.

- Pick a set of input parameters, typical choice:

\[ \alpha_s, \alpha, G_F, M_Z, M_H, m_t, m_f, \ldots \]

- Compute theoretical predictions, including radiative corrections, in a given renormalization scheme treating the best measured parameters as inputs (\(\alpha, G_F\), fermion masses except \(m_t\) and \(m_c\)), i.e. as fixed parameters.

- Perform a best fit to the electroweak data, defined by a \(\chi^2\) test

\[
\chi^2(\alpha, G_F, \ldots) = \sum_i \frac{(\hat{O}_i^{\text{exp}} - O_i^{\text{th}}(\alpha, G_F, \ldots))^2}{(\Delta \hat{O}_I^{\text{exp}})^2}
\]

This results in a best fit of the non-fixed or floating parameters. Compare best-fit values to measurements if available (ex.: \(M_W, m_t, \alpha_s\), not \(M_H\)!

- For the best-fit values of all input parameters, quote the SM theoretical prediction for each observable and compare with the experimental measurements. “Tensions” may signal new physics …
Fine structure constant, $\alpha$

Measured at low energies,

$$\alpha \equiv \frac{e^2(0)}{4\pi} = \frac{e^2_0}{4\pi(1 - \Delta \alpha(0))} = \frac{1}{137.03599890(50)}$$

then evolved to $M_Z$:

$$\alpha_e(M_Z) = \frac{\alpha}{1 - \Delta \alpha(M_Z)}$$

$\Delta \alpha \rightarrow$ QED and (2-loop) QCD contributions ($\Delta \alpha^{(5)}_{\text{had}}$).

Uncertainties from: h.o. perturbative and nonperturbative corrections, light quark masses (mainly $m_c$), lack of data below 1.8 GeV, slight disagreement in extraction of $\Delta \alpha^{(5)}_{\text{had}}$.

Fermi constant, $G_F$ (or $G_\mu$) From muon lifetime:

$$\tau^{-1}_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} F \left( \frac{m_e^2}{m_\mu^2} \right) \left( 1 + \frac{3}{5} m_\mu^2 M_W^2 \right) \left[ 1 + \left( \frac{25}{8} - \frac{\pi^2}{2} \right) \frac{\alpha(m_\mu)}{\pi} + O(\alpha^2) \right]$$

Measure:

$$G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$$

uncertainty: almost all from residual experimental error.
Example of best fit of floating parameters

W-Boson Mass [GeV]

- TEVATRON: $80.420 \pm 0.031$
- LEP2: $80.376 \pm 0.033$
- Average: $80.399 \pm 0.023$ ($\chi^2$/DoF: 0.9 / 1)
- NuTeV: $80.136 \pm 0.084$
- LEP1/SLD: $80.362 \pm 0.032$
- LEP1/SLD/$m_t$: $80.363 \pm 0.020$

Top-Quark Mass [GeV]

- CDF: $172.5 \pm 1.00$
- DØ: $174.9 \pm 1.4$
- Average: $173.2 \pm 0.90$ ($\chi^2$/DoF: 6.1 / 10)
- LEP1/SLD: $172.6^{+13.5}_{-10.4}$
- LEP1/SLD/$m_W/\Gamma_W$: $179.7^{+11.7}_{-8.7}$

All following plots from:
The LHC Electroweak Working Group (http://lepewwg.web.cern.ch/LEPEWWG/)
Summary of various “pulls”: theory vs experiment

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Fit</th>
<th>$\sigma_{\text{meas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \alpha_{\text{had}}^{(s)}(m_Z)$</td>
<td>0.02750 ± 0.00033</td>
<td>0.02759</td>
</tr>
<tr>
<td>$m_Z$ [GeV]</td>
<td>91.1875 ± 0.0021</td>
<td>91.1874</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>2.4952 ± 0.0023</td>
<td>2.4959</td>
</tr>
<tr>
<td>$\sigma_{\text{had}}^0$ [nb]</td>
<td>41.540 ± 0.037</td>
<td>41.478</td>
</tr>
<tr>
<td>$R_l$</td>
<td>20.767 ± 0.025</td>
<td>20.742</td>
</tr>
<tr>
<td>$A_{l,b}^{0,l}$</td>
<td>0.01714 ± 0.00095</td>
<td>0.01646</td>
</tr>
<tr>
<td>$A_l(P_T)$</td>
<td>0.1465 ± 0.0032</td>
<td>0.1482</td>
</tr>
<tr>
<td>$R_b$</td>
<td>0.21629 ± 0.00066</td>
<td>0.21579</td>
</tr>
<tr>
<td>$R_c$</td>
<td>0.1721 ± 0.0030</td>
<td>0.1722</td>
</tr>
<tr>
<td>$A_{l,b}^{0,l}$</td>
<td>0.0992 ± 0.0016</td>
<td>0.1039</td>
</tr>
<tr>
<td>$A_{l,b}^{0,c}$</td>
<td>0.0707 ± 0.0035</td>
<td>0.0743</td>
</tr>
<tr>
<td>$A_l$</td>
<td>0.923 ± 0.020</td>
<td>0.935</td>
</tr>
<tr>
<td>$A_c$</td>
<td>0.670 ± 0.027</td>
<td>0.668</td>
</tr>
<tr>
<td>$A_l$(SLD)</td>
<td>0.1513 ± 0.0021</td>
<td>0.1482</td>
</tr>
<tr>
<td>$\sin^2 \theta_{\text{eff}}(Q_{fb})$</td>
<td>0.2324 ± 0.0012</td>
<td>0.2314</td>
</tr>
<tr>
<td>$m_W$ [GeV]</td>
<td>80.399 ± 0.023</td>
<td>80.378</td>
</tr>
<tr>
<td>$\Gamma_W$ [GeV]</td>
<td>2.085 ± 0.042</td>
<td>2.092</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>173.20 ± 0.90</td>
<td>173.27</td>
</tr>
</tbody>
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July 2011
SM Higgs-boson mass range: constrained by EW precision fits

Increasing precision will continue to provide an invaluable tool to test the consistency of the SM and its extensions.

\[ m_W = 80.399 \pm 0.023 \text{ GeV} \]
\[ m_t = 173.2 \pm 0.90 \text{ GeV} \]
\[ M_H < 161 \ (185) \text{ GeV} \]

plus exclusion limits (95% c.l.):
\[ M_H > 114.4 \text{ GeV (LEP)} \]
\[ M_H \neq 156 \ - \ 177 \text{ GeV (Tevatron)} \]

focus is now on exclusion limits and discovery!
Disentangling $m_t - M_H$ and $M_W - M_H$ correlations

\[
M_W/(\text{GeV}) = 80.409 - 0.507 \left( \frac{\Delta \alpha_h^{(5)}}{0.02767} - 1 \right) + 0.542 \left[ \left( \frac{m_t}{178 \text{ GeV}} \right)^2 - 1 \right] \\
- 0.05719 \ln \left( \frac{M_H}{100 \text{ GeV}} \right) - 0.00898 \ln^2 \left( \frac{M_H}{100 \text{ GeV}} \right)
\]

A. Ferroglia, G. Ossola, M. Passera, A. Sirlin, PRD 65 (2002) 113002

W. Marciano, hep-ph/0411179