The Standard Model of Particle Physics
Lecture III
Probing the Higgs mechanism of electroweak symmetry breaking

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Outline of Lecture III

- SM Higgs so contrained that it can point to scale of new physics.
  - constrain from precision fits (see Lecture II);
  - validity of the SM up to a scale $\Lambda$ translate into bounds on $M_H$
    (and vice versa $M_H \rightarrow \Lambda$).

- Looking for a SM Higgs boson at hadron colliders:
  - parton level production processes;
  - branching ratios;
  - Tevatron Higgs physics program;
  - LHC Higgs physics program; (see S Caron’s lecture this morning)
  - preparing the ground for tomorrow’s lecture: overview of inclusive
    theoretical predictions;
  - what we haven’t discussed . . .

- SM Higgs physics at linear colliders: brief glance
Theoretical constraints on $M_H$ in the Standard Model

SM as an effective theory valid up to a scale $\Lambda$. The Higgs sector of the SM actually contains two unknowns: $M_H$ and $\Lambda$.

Bounds given by:

- $\rightarrow$ unitarity
- $\rightarrow$ triviality
- $\rightarrow$ vacuum stability
- $\rightarrow$ fine tuning

$M_H^2 = 2\lambda v^2$ $\rightarrow$ $M_H$ determines the weak/strong coupling behavior of the theory, i.e. the limit of validity of the perturbative approach.
Unitarity: longitudinal gauge boson scattering cross section at high energy grows with $M_H$.

Electroweak Equivalence Theorem:

in the high energy limit ($s \gg M_V^2$)

$$\mathcal{A}(V_L^i \ldots V_L^n \rightarrow V_L^i \ldots V_L^m) = (i)^n (-i)^m \mathcal{A}(\omega^1 \ldots \omega^n \rightarrow \omega^1 \ldots \omega^m) + O \left( \frac{M_V^2}{s} \right)$$

($V_L^i =$longitudinal weak gauge boson; $\omega^i =$associated Goldstone boson).

Example: $\boxed{W_L^+ W_L^- \rightarrow W_L^+ W_L^-}$

$$\mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) \sim -\frac{1}{v^2} \left( -s - t + \frac{s^2}{s - M_H^2} + \frac{t^2}{t - M_H^2} \right)$$

$$\mathcal{A}(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) = -\frac{M_H^2}{v^2} \left( \frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right)$$

$$\Downarrow$$

$$\mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \mathcal{A}(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) + O \left( \frac{M_W^2}{s} \right)$$
Using partial wave decomposition:

\[ \mathcal{A} = 16\pi \sum_{l=0}^{\infty} (2l + 1) P_l(\cos \theta) a_l \]

\[ \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{A}|^2 \quad \rightarrow \quad \sigma = \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l + 1) |a_l|^2 = \frac{1}{s} \text{Im} [A(\theta = 0)] \]

\[ \downarrow \]

\[ |a_l|^2 = \text{Im}(a_l) \quad \rightarrow \quad |\text{Re}(a_l)| \leq \frac{1}{2} \]

Most constraining condition for \( W^+_L W^-_L \rightarrow W^+_L W^-_L \) from

\[ a_0 (\omega^+ \omega^- \rightarrow \omega^+ \omega^-) = -\frac{M_H^2}{16\pi v^2} \left[ 2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log \left( 1 + \frac{s}{M_H^2} \right) \right] \quad s \gg M_H^2 \rightarrow \frac{M_H^2}{8\pi v^2} \]

\[ |\text{Re}(a_0)| < \frac{1}{2} \quad \rightarrow \quad M_H < 870 \text{ GeV} \]

Best constraint from coupled channels \( (2W^+_L W^-_L + Z_L Z_L) \):

\[ a_0 \quad s \gg M_H^2 \rightarrow \quad \frac{5M_H^2}{32\pi v^2} \quad \rightarrow \quad M_H < 780 \text{ GeV} \]
Observe that: if there is no Higgs boson, i.e. $M_H \gg s$:

$$a_0(\omega^+\omega^- \rightarrow \omega^+\omega^-) \xrightarrow{M_H^2 \gg s} \frac{s}{32\pi v^2}$$

Imposing the unitarity constraint $\rightarrow \sqrt{s_c} < 1.8 \text{ TeV}$

Most restrictive constraint $\rightarrow \sqrt{s_c} < 1.2 \text{ TeV}$

$\downarrow$

New physics expected at the TeV scale
Triviality: a $\lambda\phi^4$ theory cannot be perturbative at all scales unless $\lambda=0$.

In the SM the scale evolution of $\lambda$ is more complicated:

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2g^2 + \frac{9}{8}g^4 - 24y_t^4 + \cdots$$

($t=\ln(Q^2/Q_0^2)$, $y_t = m_t/v \rightarrow$ top quark Yukawa coupling).

Still, for large $\lambda$ ($\leftrightarrow$ large $M_H$) the first term dominates and (at 1-loop):

$$\lambda(Q) = \frac{\lambda(Q_0)}{1 - \frac{3}{4\pi^2}\lambda(Q_0) \ln \left(\frac{Q^2}{Q_0^2}\right)}$$

when $Q$ grows $\rightarrow$ $\lambda(Q)$ hits a pole $\rightarrow$ triviality

Imposing that $\lambda(Q)$ is finite, gives a scale dependent bound on $M_H$:

$$\frac{1}{\lambda(\Lambda)} > 0 \rightarrow M_H^2 < \frac{8\pi^2v^2}{3\log\left(\frac{\Lambda^2}{v^2}\right)}$$

where we have set $Q \rightarrow \Lambda$ and $Q_0 \rightarrow v$. 
Vacuum stability: \[ \lambda(Q) > 0 \]

For small \( \lambda \) (\( \leftrightarrow \) small \( M_H \)) the last term in \( d\lambda/dt = \ldots \) dominates and:

\[
\lambda(\Lambda) = \lambda(v) - \frac{3}{4\pi^2} y_t^4 \log \left( \frac{\Lambda^2}{v^2} \right)
\]

from where a first rough lower bound is derived:

\[
\lambda(\Lambda) > 0 \implies M_H^2 > \frac{3v^2}{2\pi^2} y_t^4 \log \left( \frac{\Lambda^2}{v^2} \right)
\]

More accurate analyses use 2-loop renormalization group improved \( V_{eff} \).
**Fine-tuning:** $M_H$ is unstable to ultraviolet corrections

$$M_H^2 = (M_H^0)^2 + \frac{g^2}{16\pi^2} \Lambda^2 \cdot \text{constant} + \text{higher orders}$$

$M_H^0 \rightarrow$ fundamental parameter of the SM  
$\Lambda \rightarrow$ UV-cutoff scale

Unless $\Lambda \simeq$ EW-scale, **fine-tuning is required to get** $M_H \simeq$ EW-scale.

More generally, the all order calculation of $V_{eff}$ would give:

$$\bar{\mu}^2 = \mu^2 + \Lambda^2 \sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/Q)$$

**Veltman condition:** the absence of large quadratic corrections is guaranteed by:

$$\sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/M_H) = 0 \quad \text{or better} \quad \sum_{n=0}^{n_{max}} c_n(\lambda_i) \log^n(\Lambda/M_H) < \frac{v^2}{\Lambda^2}$$

where: $n_{max} = 0, 1, 2 \rightarrow \Lambda \simeq 2, 15, 50$ TeV.
Beyond SM: new physics at the TeV scale can be a better fit

Example: MSSM

▷ a light scalar Higgs boson, along with a heavier scalar, a pseudoscalar and a charged scalar;
▷ similar although less constrained pattern in any 2HDM;
▷ MSSM main uncertainty: unknown masses of SUSY particles.
▷ precise measurement of mass spectrum and couplings will be crucial.
... mass spectrum at a glance: $\Lambda \sim \text{TeV}$

(MasterCode by Buchmüller et al., '09)

- CMSSM/NUHM1 (different choice of soft SUSY breaking mass terms);
- all available data (exp.) and all known corrections (th.) included in fit;
- most masses accessible to early LHC.
The Higgs bosons of the MSSM: example of 2HDM (some basics)

Two complex $SU(2)_L$ doublets, with hypercharge $Y = \pm 1$:

$$\Phi_u = \left( \begin{array}{c} \phi_u^+ \\ \phi_u^0 \end{array} \right), \quad \Phi_d = \left( \begin{array}{c} \phi_d^0 \\ \phi_d^- \end{array} \right)$$

and (super)potential (Higgs part only):

$$V_H = (|\mu|^2 + m_u^2)|\Phi_u|^2 + (|\mu|^2 + m_d^2)|\Phi_d|^2 - \mu B \epsilon_{ij} (\Phi_u^i \Phi_d^j + h.c.)$$

$$+ \frac{g^2 + g'^2}{8} (|\Phi_u|^2 - |\Phi_d|^2)^2 + \frac{g^2}{2} |\Phi_u^\dagger \Phi_d|^2$$

The EW symmetry is spontaneously broken by choosing:

$$\langle \Phi_u \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_u \end{array} \right), \quad \langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} v_d \\ 0 \end{array} \right)$$

normalized to preserve the SM relation:

$$M_W^2 = \frac{g^2 (v_u^2 + v_d^2)}{4} = \frac{g^2 v^2}{4}.$$
Five physical scalar/pseudoscalar degrees of freedom:

\[ h^0 = - (\sqrt{2} \text{Re}\Phi^0_d - v_d) \sin \alpha + (\sqrt{2} \text{Re}\Phi^0_u - v_u) \cos \alpha \]
\[ H^0 = (\sqrt{2} \text{Re}\Phi^0_d - v_d) \cos \alpha + (\sqrt{2} \text{Re}\Phi^0_u - v_u) \sin \alpha \]
\[ A^0 = \sqrt{2} (\text{Im}\Phi^0_d \sin \beta + \text{Im}\Phi^0_u \cos \beta) \]
\[ H^\pm = \Phi^\pm_d \sin \beta + \Phi^\pm_u \cos \beta \]

where \[ \tan \beta = v_u/v_d \].

All masses can be expressed (at tree level) in terms of \[ \tan \beta \] and \[ M_A \] :

\[ M^2_{H^\pm} = M^2_A + M^2_W \]
\[ M^2_{H,h} = \frac{1}{2} \left( M^2_A + M^2_Z \pm ((M^2_A + M^2_Z)^2 - 4M^2_Z M^2_A \cos^2 2\beta)^{1/2} \right) \]

Notice: tree level upper bound on \[ M_h \]: \[ M^2_h \leq M^2_Z \cos 2\beta \leq M^2_Z \]!
Higgs masses greatly modified by radiative corrections.

In particular, the upper bound on $M_h$ becomes:

$$M_h^2 \leq M_Z^2 + \frac{3g^2m_t^2}{8\pi^2M_W^2} \left[ \log \left( \frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right]$$

where $M_S \equiv (M_{t_1}^2 + M_{t_2}^2)/2$ while $X_t$ is the top squark mixing parameter:

$$X_t \equiv A_t - \mu \cot \beta.$$

$$D_L = (1/2 - 2/3 \sin \theta_W)M_Z^2 \cos 2\beta$$

$$D_R = 2/3 \sin^2 \theta_W M_Z^2 \cos 2\beta$$
Higgs boson couplings to SM gauge bosons:

Some phenomelogically important ones:

\[ g_{hVV} = g_V M_V \sin(\beta - \alpha)g^{\mu\nu}, \quad g_{HVV} = g_V M_V \cos(\beta - \alpha)g^{\mu\nu} \]

where \( g_V = 2M_V/v \) for \( V = W, Z \), and

\[ g_{hAZ} = \frac{g \cos(\beta - \alpha)}{2 \cos \theta_W} (p_h - p_A)^\mu, \quad g_{HAZ} = -\frac{g \sin(\beta - \alpha)}{2 \cos \theta_W} (p_H - p_A)^\mu \]

Notice: \( g_{AZZ} = g_{AWW} = 0 \), \( g_{H^\pm ZZ} = g_{H^\pm WW} = 0 \)

Decoupling limit: \( M_A \gg M_Z \) \( \rightarrow \) \[ \begin{cases} M_h \simeq M_h^{\max} \\ M_H \simeq M_{H^\pm} \simeq M_A \end{cases} \]

\[ \cos^2(\beta - \alpha) \simeq \frac{M_Z^4 \sin^2 4\beta}{M_A^4} \] \( \rightarrow \) \[ \begin{cases} \cos(\beta - \alpha) \rightarrow 0 \\ \sin(\beta - \alpha) \rightarrow 1 \end{cases} \]

The only low energy Higgs is \( h \simeq H_{SM} \).
Higgs boson couplings to quarks and leptons:

Yukawa type couplings, \( \Phi_u \) to up-component and \( \Phi_d \) to down-component of \( SU(2)_L \) fermion doublets. Ex. (3\(^{rd}\) generation quarks):

\[
\mathcal{L}_{Yukawa} = h_t \left[ \bar{t} P_L t \Phi_u^0 - \bar{t} P_L b \Phi_u^+ \right] + h_b \left[ \bar{b} P_L b \Phi_d^0 - \bar{b} P_L t \Phi_d^- \right] + \text{h.c.}
\]

and similarly for leptons. The corresponding couplings can be expressed as \((y_t, y_b \rightarrow \text{SM})\):

\[
\begin{align*}
g_{ht\bar{t}} &= \frac{\cos \alpha}{\sin \beta} y_t = [\sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha)] y_t \\
g_{hb\bar{b}} &= -\frac{\sin \alpha}{\cos \beta} y_b = [\sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha)] y_b \\
g_{Ht\bar{t}} &= \frac{\sin \alpha}{\sin \beta} y_t = [\cos(\beta - \alpha) - \cot \beta \sin(\beta - \alpha)] y_t \\
g_{Hb\bar{b}} &= \frac{\cos \alpha}{\cos \beta} y_b = [\cos(\beta - \alpha) + \tan \beta \sin(\beta - \alpha)] y_b \\
g_{At\bar{t}} &= \cot \beta y_t, \quad g_{Ab\bar{b}} = \tan \beta y_b \\
g_{H^\pm t\bar{b}} &= \frac{g}{2\sqrt{2}M_W} \left[ m_t \cot \beta(1 + \gamma_5) + m_b \tan \beta(1 - \gamma_5) \right]
\end{align*}
\]

Notice: consistent decoupling limit behavior.
SM Higgs boson decay branching ratios and width

Observe difference between light and heavy Higgs

These curves include:  

- tree level
- QCD and EW loop corrections
MSSM Higgs boson branching ratios, possible scenarios:
\textbf{pp, \bar{p}p colliders: SM Higgs production modes}

\begin{align*}
\text{gg} & \rightarrow H \\
\text{qq} & \rightarrow qqH \\
\text{qq} & \rightarrow WH, ZH
\end{align*}

\begin{align*}
\text{q\bar{q}, gg} & \rightarrow t\bar{t}H, b\bar{b}H
\end{align*}
Schematically …

The hard cross section is calculated perturbatively

$$\hat{\sigma}(ij \rightarrow X) = \alpha_s^k \sum_{m=0}^{n} \hat{\sigma}^{(m)}_{ij} \alpha_s^m$$

n=0 : Leading Order (LO), or tree level or Born level
n=1 : Next to Leading Order (NLO), include $O(\alpha_s)$ corrections
……

and convoluted with initial state parton densities at the same order.

Renormalization and factorization scale dependence left at any fixed order.

Setting $\mu_R = \mu_F = \mu$:

$$\sigma(pp, p\bar{p} \rightarrow X) = \sum_{ij} \int dx_1 dx_2 f_i^p (x_1, \mu) f_j^{p,\bar{p}} (x_2, \mu) \sum_{m=0}^{n} \hat{\sigma}^{(m)}_{ij} (\mu, Q^2) \alpha_s^{m+k} (\mu)$$

Systematic theoretical error from:

- PDF and $\alpha_s(\mu)$;
- left over scale dependence;
- input parameters.
Tevatron: great potential for a light SM-like Higgs boson

Lower mass region:

\[ q\bar{q}' \rightarrow WH, H \rightarrow b\bar{b} \]

Higher mass region:

\[ gg \rightarrow H, H \rightarrow W^+W^- \]

(smaller impact:

\[ q\bar{q} \rightarrow q'\bar{q}'H, q\bar{q}, gg \rightarrow t\bar{t}H \]

(M. Spira, Fortsch.Phys. 46 (1998) 203)

\[ \rightarrow \text{Exclusion region very important for LHC search strategies.} \]
LHC: entire SM Higgs-boson mass range accessible

Many channels have been studied:

Below 130-140 GeV:

\( gg \rightarrow H \), \( H \rightarrow \gamma\gamma, WW, ZZ \)

\( qq \rightarrow qqH \), \( H \rightarrow \gamma\gamma, WW, ZZ, \tau\tau \)

\( q\bar{q}, gg \rightarrow t\bar{t}H \), \( H \rightarrow \gamma\gamma, b\bar{b}, \tau\tau \)

\( q\bar{q}' \rightarrow WH \), \( H \rightarrow \gamma\gamma, b\bar{b} \)

Above 130-140 GeV:

\( gg \rightarrow H \), \( H \rightarrow WW, ZZ \)

\( qq \rightarrow qqH \), \( H \rightarrow \gamma\gamma, WW, ZZ \)

\( q\bar{q}, gg \rightarrow t\bar{t}H \), \( H \rightarrow \gamma\gamma, WW \)

\( q\bar{q}' \rightarrow WH \), \( H \rightarrow WW \)

(M. Spira, Fortsch.Phys. 46 (1998) 203)
With $\sqrt{s} = 7$ TeV and a few fb$^{-1}$ …

Combining only $H \rightarrow W^+W^-$, $H \rightarrow ZZ$, $H \rightarrow \gamma\gamma$, ATLAS and CMS indicate that,

- if no signal, the SM Higgs can be excluded up to 500 GeV;
- a 5$\sigma$ significance for a SM Higgs in the 140 – 170 GeV mass range;
- in the low mass region (new strategies, new ideas).

where also $WH$, $H \rightarrow b\bar{b}$ (highly boosted) and VBF with $H \rightarrow \tau\tau$ were used.
We have not discussed: study of Higgs properties

At the LHC:

- **Color** and **charge** will be given by the measurement of a given (production+decay) channel.

- The Higgs boson **mass** will be measured with 0.1% accuracy in $H \to ZZ^* \to 4l^\pm$, complemented by $H \to \gamma\gamma$ in the low mass region. Above $M_H \simeq 400$ GeV precision deteriorates to $\simeq 1\%$ (lower rates).

- The Higgs boson **width** can be measured in $H \to ZZ^* \to 4l^\pm$ above $M_H \simeq 200$ GeV. The best accuracy of $\simeq 5\%$ is reached for $M_H \simeq 400$ GeV.

- The Higgs boson **spin** could be measured through angular correlations between fermions in $H \to VV \to 4f$: need for really high statistics.

- The Higgs boson **couplings** will be measured combining multiple channels:
  \[
  (\sigma_p(H)\text{Br}(H \to dd))^{exp} = \frac{\sigma_{p}^{th}(H)}{\Gamma_p^{th}} \frac{\Gamma_d \Gamma_p}{\Gamma_H}
  \]
  Higgs self-couplings will be very hard!
$e^+e^-$ colliders: SM Higgs production modes
Program:

One or more Higgs bosons will be observed over the entire mass spectrum. A high energy $e^+e^-$ collider will then have the unique possibility of:

• Measure $\sigma(e^+e^- \rightarrow ZH)$ at the 2% level: extract $\text{Br}(H \rightarrow xx)$ in model independent way!

• Measure $M_H$ from the recoiling $f\bar{f}$ mass in $ZH \rightarrow Hf\bar{f}$. Accuracies of the order of 50-80 MeV can be obtained.

spin measured by:

▷ onset slope of the $e^+e^- \rightarrow ZH$
▷ correlations in $e^+e^- \rightarrow ZH \rightarrow 4f, \ldots$
▷ phase space distributions in $e^+e^- \rightarrow t\bar{t}H$
• Measure $\Gamma_H$ below $M_H \simeq 200$ Gev combining $\text{Br}(H \rightarrow W^+W^-)$ (from $e^+e^- \rightarrow ZH$) and $g_{HWW}$ (from $e^+e^- \rightarrow H\nu\bar{\nu}$), with a $\simeq 6\%$ accuracy.

**Ex.**: SM Higgs boson, $\sqrt{s}=500$ GeV, 500 fb$^{-1}$ (Except $HHH$, 1 ab$^{-1}$)

<table>
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<th>Coupling:</th>
<th>$Hb\bar{b}$</th>
<th>$H\tau^+\tau^-$</th>
<th>$Hc\bar{c}$</th>
<th>$HWW$</th>
<th>$HZZ$</th>
<th>$Htt$</th>
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<td>1.2%</td>
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(Djouadi, ’05, using HFITTER/HDECAY)