Outline of Lecture II

- SM Higgs boson physics depends just on $M_H$: $M_H$ highly constrained!

- **Quantum effects** leading players in constraining $M_H$:
  - $\rightarrow$ branching ratios (see end of last lecture): important corrections;
  - $\rightarrow$ EW precision fits: $M_H$ only unknown.

- SM Higgs so contrained that it can point to scale of new physics.

- Beyond SM Higgs: MSSM (useful example of an extended Higgs sector) and more.

- Need data! and need to understand them ....
SM Higgs boson decay branching ratios and width

Observe difference between light and heavy Higgs

These curves include: **tree level** + **QCD and EW loop corrections**.
Tree level decays: $H \to f \bar{f}$ and $H \to VV$

At lowest order:

\[
\Gamma(H \to f \bar{f}) = \frac{G_F M_H}{4\sqrt{2}\pi} N_{cf} m_f^2 \beta_f^3
\]

\[
\Gamma(H \to VV) = \frac{G_F M_H^3}{16\sqrt{2}\pi} \delta_V \left(1 - \tau_V + \frac{3}{4} \tau_V^2 \right) \beta_V
\]

($\beta_i = \sqrt{1 - \tau_i}$, $\tau_i = 4m_i^2/M_H^2$, $\delta_{W,Z} = 2, 1$, $(N_c)_{l,q} = 1, 3$)

**Ex.1:** Higher order corrections to $H \to q\bar{q}$

QCD corrections dominant:

\[
\Gamma(H \to q\bar{q})_{QCD} = \frac{3G_F M_H}{4\sqrt{2}\pi} \bar{m}_q(M_H) \beta_q^3 \left[\Delta_{QCD} + \Delta_t\right]
\]

\[
\Delta_{QCD} = 1 + 5.67 \frac{\alpha_s(M_H)}{\pi} + (35.94 - 1.36 N_F) \left(\frac{\alpha_s(M_H)}{\pi}\right)^2 + \cdots
\]

\[
\Delta_t = \left(\frac{\alpha_s(M_H)}{\pi}\right)^2 \left[1.57 - \frac{2}{3} \ln \frac{M_H^2}{m_t^2} + \frac{1}{9} \ln^2 \frac{\bar{m}_q(M_H)}{M_H^2}\right] + \cdots
\]
Consist of both virtual and real corrections, e.g.:
• Large Logs absorbed into $\overline{MS}$ quark mass

**Leading Order:** $\bar{m}_Q(\mu) = \bar{m}_Q(m_Q) \left( \frac{\alpha_s(\mu)}{\alpha_s(m_Q)} \right)^{\frac{2b_0}{\gamma_0}}$

**Higher order:** $\bar{m}_Q(\mu) = \bar{m}_Q(m_Q) \frac{f(\alpha_s(\mu)/\pi)}{f(\alpha_s(m_Q)/\pi)}$

where (from renormalization group equation)

$$f(x) = \left( \frac{25}{6} x \right)^{\frac{12}{25}} [1 + 1.014x + \ldots] \quad \text{for } m_c < \mu < m_b$$

$$f(x) = \left( \frac{23}{6} x \right)^{\frac{12}{23}} [1 + 1.175x + \ldots] \quad \text{for } m_b < \mu < m_t$$

$$f(x) = \left( \frac{7}{2} x \right)^{\frac{4}{7}} [1 + 1.398x + \ldots] \quad \text{for } \mu > m_t$$

• Large corrections, when $M_H \gg m_Q$

$$m_b(m_b) \simeq 4.2 \text{ GeV} \quad \longrightarrow \quad \bar{m}_b(M_h \simeq 100 \text{ GeV}) \simeq 3 \text{ GeV}$$

Branching ratio smaller by almost a factor 2.

• Main uncertainties: $\alpha_s(M_Z)$, pole masses: $m_c(m_c)$, $m_b(m_b)$. 
Ex. 2: Higher order corrections $\Gamma(H \rightarrow gg)$

Start from tree level:

\[ \Gamma(H \rightarrow gg) = \frac{G_F \alpha_s^2 M_H^3}{36\sqrt{2}\pi^3} \left| \sum_q A_q^H(\tau_q) \right| \]

where $\tau_q = 4m_q^2/M_H^2$ and

\[ A_q^H(\tau) = \frac{3}{2} \tau \left[ 1 + (1 - \tau)f(\tau) \right] \]

\[ f(\tau) = \begin{cases} 
\arcsin^2 \left( \frac{1}{\sqrt{\tau}} \right) & \tau \geq 1 \\
-\frac{1}{4} \left[ \ln \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi \right]^2 & \tau < 1 
\end{cases} \]

Main contribution from top quark $\rightarrow$ optimal situation to use Low Energy Theorems to add higher order corrections.
QCD corrections dominant:

Difficult task since decay is already a loop effect.

However, full massive calculation of $\Gamma(H \rightarrow gg(q), q\bar{q}g)$ agrees with $m_t \gg M_H$ result at 10%

$$\Gamma(H \rightarrow gg(q), q\bar{q}g) = \Gamma_{LO}(\alpha_s^{(NL)}(M_H)) \left[ 1 + E^{(NL)} \frac{\alpha_s^{(NL)}}{\pi} \right]$$

$$E^{(NL)} \xrightarrow{M_H \ll 4m_q^2} \frac{95}{4} - \frac{7}{6} N_L$$

Dominant soft/collinear radiation do not resolve the Higgs boson coupling to gluons $\rightarrow$ QCD corrections are just a (big) rescaling factor
NLO QCD corrections almost 60 – 70% of LO result in the low mass region:

\[
\delta(H \rightarrow gg) = \Gamma_{\text{LO}} (1 + \delta)
\]

\[
\mu = M_H \quad M_t = 175 \text{ GeV}
\]

solid line \(\rightarrow\) full massive NLO calculation

dashed line \(\rightarrow\) heavy top limit \((M_H^2 \ll 4m_t^2)\)

NNLO corrections calculated in the heavy top limit: add 20%
\(\rightarrow\) perturbative stabilization. Residual theoretical uncertainty \(\simeq 10\%\).
Low-energy theorems, in a nutshell.

• Observing that:

In the $p_H \to 0$ limit: the interactions of a Higgs boson with the SM particles arise by substituting

$$M_i \longrightarrow M_i \left(1 + \frac{H}{v}\right) \quad (i = f, W, Z)$$

In practice: Higgs taken on shell ($p_H^2 = M_H^2$), and limit $p_H \to 0$ is limit of small Higgs masses (e.g.: $M_H^2 \ll 4m_t^2$).

• Then

$$\lim_{p_H \to 0} A(X \to Y + H) = \frac{1}{v} \sum_i M_i \frac{\partial}{\partial M_i} A(X \to Y)$$

very convenient!

• Equivalent to an Effective Theory described by:

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} G^a_{\mu\nu} G^{a\mu\nu}_\nu \frac{H}{v} (1 + O(\alpha_s))$$

including higher order QCD corrections.
For completeness:

\[
\Gamma(H \rightarrow \gamma\gamma) = \frac{G_F \alpha^2 M_H^3}{128\sqrt{2}\pi^3} \left| \sum_f N_{cf} e_f^2 A_f^H(\tau_f) + A_W^H(\tau_W) \right|^2
\]

where \((f(\tau)\) as in \(H \rightarrow gg\)):

\[
A_f^H = 2\tau \left[ 1 + (1 - \tau) f(\tau) \right]
\]

\[
A_W^H(\tau) = -\left[ 2 + 3\tau + 3\tau(2 - \tau) f(\tau) \right]
\]

\[
\Gamma(H \rightarrow Z\gamma) = \frac{G_F^2 M_Z^2 \alpha M_H^3}{64\pi^4} \left( 1 - \frac{M_Z^2}{M_H^2} \right)^3 \left| \sum_f A_f^H(\tau_f, \lambda_f) + A_W^H(\tau_W, \lambda_W) \right|^2
\]

where the form factors \(A_f^H(\tau, \lambda)\) and \(A_W^H(\tau, \lambda)\) can be found in the literature (see, e.g., M. Spira, hep-ph/9705337).

For both decays, both QCD and EW corrections are very small \((\sim 1 - 3\%)\).
**EW precision fits:** perturbatively calculate observables in terms of few parameters:

\[ M_Z, G_F, \alpha(M_Z), M_W, m_f, (\alpha_s(M_Z)) \]

extracted from experiments with high accuracy. Only SM unknown: \( M_H \).

- SM needs Higgs boson to cancel infinities, e.g.

  \[ M_W, M_Z \rightarrow \]

  \[ \begin{array}{c}
  \text{H} \\
  \text{w} \text{Z} \\
  \text{w} \text{Z}
  \end{array} \]

- Finite **logarithmic contributions** survive, e.g. radiative corrections to \( \rho = M_W^2 / (M_Z^2 \cos^2 \theta_W) \):

  \[
  \rho = 1 - \frac{11g^2}{96\pi^2 \tan^2 \theta_W} \ln \left( \frac{M_H}{M_W} \right)
  \]

  Main effects in **oblique radiative corrections** \((S,T\text{-parameters})\)

- Same constraints apply to any model of new physics.
SM Higgs-boson mass range: constrained by EW precision fits

Increasing precision will continue to provide an invaluable tool to test the consistency of the SM and its extensions.

\[ m_W = 80.399 \pm 0.023 \text{ GeV} \]
\[ m_t = 173.3 \pm 1.1 \text{ GeV} \]

\[ M_H = 89^{+35}_{-26} \text{ GeV} \]
\[ M_H < 158 \text{ (185) GeV} \]

plus exclusion limits (95\% c.l.):

\[ M_H > 114.4 \text{ GeV (LEP)} \]
\[ M_H \neq 158 - 175 \text{ GeV (Tevatron)} \]

focus is now on exclusion limits and discovery!
Other theoretical constraints on $M_H$ in the Standard Model

SM as an effective theory valid up to a scale $\Lambda$. The Higgs sector of the SM actually contains two unknowns: $M_H$ and $\Lambda$.

Bounds given by:

- → unitarity
- → triviality
- → vacuum stability
- → fine tuning

$M_H^2 = 2\lambda v^2$ → $M_H$ determines the weak/strong coupling behavior of the theory, i.e. the limit of validity of the perturbative approach.
Unitarity: longitudinal gauge boson scattering cross section at high energy grows with $M_H$.

**Electroweak Equivalence Theorem:**
in the high energy limit ($s \gg M^2_V$)

$$\mathcal{A}(V^1_L \ldots V^n_L \to V^1_L \ldots V^m_L) = (i)^n(-i)^m \mathcal{A}(\omega^1 \ldots \omega^n \to \omega^1 \ldots \omega^m) + O\left(\frac{M^2_V}{s}\right)$$

($V^i_L =$longitudinal weak gauge boson; $\omega^i =$associated Goldstone boson).

**Example:** $\boxed{W^+_L W^-_L \to W^+_L W^-_L}$

$$\mathcal{A}(W^+_L W^-_L \to W^+_L W^-_L) \sim -\frac{1}{v^2} \left(-s - t + \frac{s^2}{s - M^2_H} + \frac{t^2}{t - M^2_H}\right)$$

$$\mathcal{A}(\omega^+ \omega^- \to \omega^+ \omega^-) = -\frac{M^2_H}{v^2} \left(\frac{s}{s - M^2_H} + \frac{t}{t - M^2_H}\right)$$

$$\Downarrow$$

$$\mathcal{A}(W^+_L W^-_L \to W^+_L W^-_L) = \mathcal{A}(\omega^+ \omega^- \to \omega^+ \omega^-) + O\left(\frac{M^2_W}{s}\right)$$
Using partial wave decomposition:

\[ \mathcal{A} = 16\pi \sum_{l=0}^{\infty} (2l + 1) P_l(\cos \theta) a_l \]

\[ \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{A}|^2 \quad \rightarrow \quad \sigma = \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l + 1) |a_l|^2 = \frac{1}{s} \text{Im}[\mathcal{A}(\theta = 0)] \]

\[ \downarrow \]

\[ |a_l|^2 = \text{Im}(a_l) \quad \rightarrow \quad |\text{Re}(a_l)| \leq \frac{1}{2} \]

Most constraining condition for \( W^+_L W^-_L \rightarrow W^+_L W^-_L \) from

\[ a_0(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) = -\frac{M_H^2}{16\pi v^2} \left[ 2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log \left( 1 + \frac{s}{M_H^2} \right) \right] \xrightarrow{s \gg M_H^2} \frac{5M_H^2}{32\pi v^2} \]

\[ |\text{Re}(a_0)| < \frac{1}{2} \quad \rightarrow \quad M_H < 870 \text{ GeV} \]

Best constraint from coupled channels \( 2W^+_L W^-_L + Z_L Z_L \):

\[ a_0 \xrightarrow{s \gg M_H^2} - \frac{5M_H^2}{32\pi v^2} \quad \rightarrow \quad M_H < 780 \text{ GeV} \]
Observe that: if there is no Higgs boson, i.e. $M_H \gg s$:

$$a_0(\omega^+\omega^- \to \omega^+\omega^-) \xrightarrow{M_H^2 \gg s} \frac{s}{32\pi v^2}$$

Imposing the unitarity constraint $\sqrt{s_c} < 1.8$ TeV

Most restrictive constraint $\sqrt{s_c} < 1.2$ TeV

↓

New physics expected at the TeV scale

Exciting !!
this is the range of energies of both Tevatron and LHC
Triviality: a $\lambda \phi^4$ theory cannot be perturbative at all scales unless $\lambda = 0$.

In the SM the scale evolution of $\lambda$ is more complicated:

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2g^2 + \frac{9}{8}g^4 - 24y_t^4 + \cdots$$

($t = \ln(Q^2/Q_0^2)$, $y_t = m_t/v \to$ top quark Yukawa coupling).

Still, for large $\lambda$ ($\leftrightarrow$ large $M_H$) the first term dominates and (at 1-loop):

$$\lambda(Q) = \frac{\lambda(Q_0)}{1 - \frac{3}{4\pi^2}\lambda(Q_0) \ln \left(\frac{Q^2}{Q_0^2}\right)}$$

when $Q$ grows $\rightarrow$ $\lambda(Q)$ hits a pole $\rightarrow$ triviality

Imposing that $\lambda(Q)$ is finite, gives a scale dependent bound on $M_H$:

$$\frac{1}{\lambda(\Lambda)} > 0 \quad \rightarrow \quad M_H^2 < \frac{8\pi^2 v^2}{3 \log \left(\frac{\Lambda^2}{v^2}\right)}$$

where we have set $Q \to \Lambda$ and $Q_0 \to v$. 
Vacuum stability: $\lambda(Q) > 0$

For small $\lambda$ (↔ small $M_H$) the last term in $d\lambda/dt = \ldots$ dominates and:

$$\lambda(\Lambda) = \lambda(v) - \frac{3}{4\pi^2} y_t^4 \log \left( \frac{\Lambda^2}{v^2} \right)$$

from where a first rough lower bound is derived:

$$\lambda(\Lambda) > 0 \quad \rightarrow \quad M_H^2 > \frac{3v^2}{2\pi^2} y_t^4 \log \left( \frac{\Lambda^2}{v^2} \right)$$

More accurate analyses use 2-loop renormalization group improved $V_{eff}$. 
Fine-tuning: $M_H$ is unstable to ultraviolet corrections

$$M_H^2 = (M_H^0)^2 + \frac{g^2}{16\pi^2}\Lambda^2 \cdot \text{constant} + \text{higher orders}$$

$M_H^0 \rightarrow$ fundamental parameter of the SM
$\Lambda \rightarrow$ UV-cutoff scale

Unless $\Lambda \simeq$ EW-scale, fine-tuning is required to get $M_H \simeq$ EW-scale.

More generally, the all order calculation of $V_{eff}$ would give:

$$\bar{\mu}^2 = \mu^2 + \Lambda^2 \sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/Q)$$

Veltman condition: the absence of large quadratic corrections is guaranteed by:

$$\sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/M_H) = 0 \quad \text{or better} \quad \sum_{n=0}^{n_{max}} c_n(\lambda_i) \log^n(\Lambda/M_H) < \frac{v^2}{\Lambda^2}$$

where: $n_{max} = 0, 1, 2 \quad \rightarrow \Lambda \simeq 2, 15, 50$ TeV.
Beyond SM: new physics at the TeV scale can be a better fit

Ex. 1: MSSM

- a light scalar Higgs boson, along with a heavier scalar, a pseudoscalar and a charged scalar;
- similar although less constrained pattern in any 2HDM;
- MSSM main uncertainty: unknown masses of SUSY particles.
- precise measurement of mass spectrum and couplings will be crucial.
... mass spectrum at a glance ...  

(MasterCode by Buchmüller et al., ’09)

- CMSSM/NUHM1 (different choice of soft SUSY breaking mass terms);
- all available data (exp.) and all known corrections (th.) included in fit;
- most masses accessible to early LHC, several within reach of ILC.
The Higgs bosons of the MSSM: example of 2HDM

Two complex $SU(2)_L$ doublets, with hypercharge $Y = \pm 1$:

$$
\Phi_u = \begin{pmatrix}
\phi^+_u \\
\phi^0_u
\end{pmatrix}, \quad \Phi_d = \begin{pmatrix}
\phi^0_d \\
\phi^-_d
\end{pmatrix}
$$

and (super)potential (Higgs part only):

$$
V_H = (|\mu|^2 + m^2_u)|\Phi_u|^2 + (|\mu|^2 + m^2_d)|\Phi_d|^2 - \mu B \varepsilon_{ij} (\Phi^i_u \Phi^j_d + h.c.) + \frac{g^2 + g'^2}{8} (|\Phi_u|^2 - |\Phi_d|^2)^2 + \frac{g^2}{2} |\Phi^\dagger_u \Phi_d|^2
$$

The EW symmetry is spontaneously broken by choosing:

$$
\langle \Phi_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 \\
v_u
\end{pmatrix}, \quad \langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix}
v_d \\
0
\end{pmatrix}
$$

normalized to preserve the SM relation:

$$
M^2_W = g^2 (v_u^2 + v_d^2)/4 = g^2 v^2 / 4.
$$
Five physical scalar/pseudoscalar degrees of freedom:

\[
\begin{align*}
    h^0 &= -\left(\sqrt{2} \text{Re}\Phi^0_d - v_d\right) \sin \alpha + \left(\sqrt{2} \text{Re}\Phi^0_u - v_u\right) \cos \alpha \\
    H^0 &= \left(\sqrt{2} \text{Re}\Phi^0_d - v_d\right) \cos \alpha + \left(\sqrt{2} \text{Re}\Phi^0_u - v_u\right) \sin \alpha \\
    A^0 &= \sqrt{2} \left(\text{Im}\Phi^0_d \sin \beta + \text{Im}\Phi^0_u \cos \beta\right) \\
    H^\pm &= \Phi^\pm_d \sin \beta + \Phi^\pm_u \cos \beta
\end{align*}
\]

where \[\tan \beta = v_u / v_d\].

All masses can be expressed (at tree level) in terms of \[\tan \beta\] and \[M_A\]:

\[
M^2_{H^\pm} = M^2_A + M^2_W
\]

\[
M^2_{H,h} = \frac{1}{2} \left(M^2_A + M^2_Z \pm ((M^2_A + M^2_Z)^2 - 4M^2_Z M^2_A \cos^2 2\beta)^{1/2}\right)
\]

Notice: tree level upper bound on \[M_h\]: \[M^2_h \leq M^2_Z \cos 2\beta \leq M^2_Z\] !
Higgs masses greatly modified by radiative corrections.

In particular, the upper bound on $M_h$ becomes:

$$M_h^2 \leq M_Z^2 + \frac{3g^2m_t^2}{8\pi^2M_W^2} \left[ \log \left( \frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right]$$

where $M_S \equiv (M_{t_1}^2 + M_{t_2}^2)/2$ while $X_t$ is the top squark mixing parameter:

\[
\begin{pmatrix}
M_{Q_t}^2 + m_t^2 + D_L^t \\
M_{R_t}^2 + m_t^2 + D_R^t
\end{pmatrix}
\begin{pmatrix}
m_t X_t \\
M_{R_t}^2 + m_t^2 + D_R^t
\end{pmatrix}
\]

with $X_t \equiv A_t - \mu \cot \beta$.

$$D_L^t = \left(1/2 - 2/3 \sin \theta_W\right)M_Z^2 \cos 2\beta$$

$$D_R^t = 2/3\sin^2 \theta_W M_Z^2 \cos 2\beta$$
Higgs boson couplings to SM gauge bosons:

Some phenomelogically important ones:

\[ g_{hVV} = g_V M_V \sin(\beta - \alpha) g^{\mu\nu}, \quad g_{HVV} = g_V M_V \cos(\beta - \alpha) g^{\mu\nu} \]

where \( g_V = 2M_V/v \) for \( V = W, Z \), and

\[ g_{hAZ} = \frac{g \cos(\beta - \alpha)}{2 \cos \theta_W} (p_h - p_A)^\mu, \quad g_{HAZ} = -\frac{g \sin(\beta - \alpha)}{2 \cos \theta_W} (p_H - p_A)^\mu \]

Notice: \( g_{AZZ} = g_{AWW} = 0 \), \( g_{H\pm ZZ} = g_{H\pm WW} = 0 \)

Decoupling limit: \( M_A \gg M_Z \) → \( \begin{cases} M_h \sim M_h^{max} \\ M_H \sim M_{H\pm} \sim M_A \end{cases} \)

\( \cos^2(\beta - \alpha) \simeq \frac{M^4_Z \sin^2 4\beta}{M^4_A} \) → \( \begin{cases} \cos(\beta - \alpha) \rightarrow 0 \\ \sin(\beta - \alpha) \rightarrow 1 \end{cases} \)

The only low energy Higgs is \( h \simeq H_{SM} \).
Higgs boson couplings to quarks and leptons:

Yukawa type couplings, $\Phi_u$ to up-component and $\Phi_d$ to down-component of $SU(2)_L$ fermion doublets. Ex. (3$^\text{rd}$ generation quarks):

$$\mathcal{L}_{Yukawa} = h_t \left[ \bar{t} P_L t \Phi_u^0 - \bar{t} P_L b \Phi_u^+ \right] + h_b \left[ \bar{b} P_L b \Phi_d^0 - \bar{b} P_L t \Phi_d^- \right] + \text{h.c.}$$

and similarly for leptons. The corresponding couplings can be expressed as $(y_t, y_b \rightarrow \text{SM})$:

$$g_{ht\bar{t}} = \frac{\cos \alpha}{\sin \beta} y_t = \left[ \sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha) \right] y_t$$

$$g_{hb\bar{b}} = -\frac{\sin \alpha}{\cos \beta} y_b = \left[ \sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha) \right] y_b$$

$$g_{Ht\bar{t}} = \frac{\sin \alpha}{\sin \beta} y_t = \left[ \cos(\beta - \alpha) - \cot \beta \sin(\beta - \alpha) \right] y_t$$

$$g_{Hb\bar{b}} = \frac{\cos \alpha}{\cos \beta} y_b = \left[ \cos(\beta - \alpha) + \tan \beta \sin(\beta - \alpha) \right] y_b$$

$$g_{At\bar{t}} = \cot \beta y_t \quad , \quad g_{Ab\bar{b}} = \tan \beta y_b$$

$$g_{H^{\pm}t\bar{b}} = \frac{g}{2\sqrt{2}M_W} \left[ m_t \cot \beta (1 + \gamma_5) + m_b \tan \beta (1 - \gamma_5) \right]$$

Notice: consistent decoupling limit behavior.
Higgs couplings modified by radiative corrections

Most important effects:

- **Corrections to** $\cos(\beta - \alpha)$: crucial in decoupling behavior.

\[
\cos(\beta - \alpha) = K \left[ \frac{M_Z^2 \sin 4\beta}{2M_A^2} + \mathcal{O}\left(\frac{M_Z^4}{M_A^4}\right) \right]
\]

where

\[
K \equiv 1 + \frac{\delta M_{11}^2 - \delta M_{22}^2}{2M_Z^2 \cos 2\beta} - \frac{\delta M_{12}^2}{M_Z^2 \sin 2\beta}
\]

$\delta M_{ij} \to$ corrections to the CP-even scalar mass matrix.

- **Corrections to 3rd generation Higgs-fermion Yukawa couplings.**

\[-\mathcal{L}_{eff} = \epsilon_{ij} \left[(h_b + \delta h_b) \bar{b}_R H_d^i Q_L^j + (h_t + \delta h_t) \bar{t}_R H_u^j Q_L^i \right] + \Delta h_t \bar{t}_R Q_L^k H_d^{k*} + \Delta h_b \bar{b}_R Q_L^k H_u^{k*} + \text{h.c.}\]

\[
m_b = \frac{h_b v}{\sqrt{2}} \cos \beta \left(1 + \frac{\delta h_b}{h_b} + \Delta h_b \tan \beta\right) \equiv \frac{h_b v}{\sqrt{2}} \cos \beta(1 + \Delta_b)
\]

\[
m_t = \frac{h_t v}{\sqrt{2}} \sin \beta \left(1 + \frac{\delta h_t}{h_t} + \Delta h_t \tan \beta\right) \equiv \frac{h_t v}{\sqrt{2}} \sin \beta(1 + \Delta_t)
\]
MSSM Higgs boson branching ratios, possible scenarios:
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Beyond SM: new physics at the TeV scale can be a better fit

Ex. 2: “Fat Higgs” models

(Harnik, Kribs, Larson, and Murayama, PRD 70 (2004) 015002)

- supersymmetric theory of a composite Higgs boson;
- moderately heavy lighter scalar Higgs boson, along with a heavier scalar, a pseudoscalar and a charged scalar;
- consistent with EW precision measurements without fine tuning.