"Perturbative QCD calculations"

Lecture 1
- Review of basic elements of QCD
- Structure of perturbative QCD (pQCD) calculations
  Main focus: collider physics
  (examples from e^+e^-, pp colliders)
- Hard scattering.

Lecture 2
Adding initial & final-state QCD evolution
- → parton distribution functions (PDF)
- → parton-showers
- → jets

Lecture 3
- Pushing the order of pQCD - why.
- tools
- Dissecting specific examples
  (pQCD effects at multiple levels)

References

> Any QFT textbook introduces the basics of QCD
> (aim for books that use "modern" QCD language)

> Dedicated monographies, s.as:
- → Ellis, Stirling, Webber: "QCD and Collider Physics"
- → Campbell, Huston, Krauss: "The Blackbook of QCD"
- → Collins: "Foundations of Perturbative QCD"

⋮

> Lectures at Summer schools:
- → TASI
- → FNAL-CERN summerschool
- → CTEQ "

etc.

① QCD, Asymptotic freedom, running masses
(setting the stage)

Strong interactions described in terms of : Non-abelian gauge theory of Dirac fermions based on SU(3)

"quarks" "color" quantum #

gauge bosons → "gluons"

$$\mathcal{L}_{QCD} = \sum_f \bar{q}_f \,(i\not{D} - m_f)\, q_f - \frac{1}{2}\,\mathrm{Tr}\left(F^{\mu\nu}F_{\mu\nu}\right)$$

flavors of quarks

SU(3) generators $(A = 1,\dots,N^2-1 \underset{N=3}{=} 1,\dots,8)$

$$D_\mu = \partial_\mu + g_s A^A_\mu T^A \qquad [T^A, T^B] = i f^{ABC} T^C$$

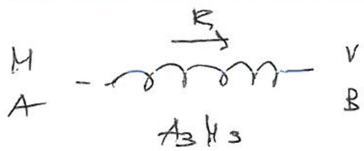
$$F_{\mu\nu} = F^A_{\mu\nu} T^A$$

$$\hookrightarrow \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g_s f^{ABC} T^B T^C$$

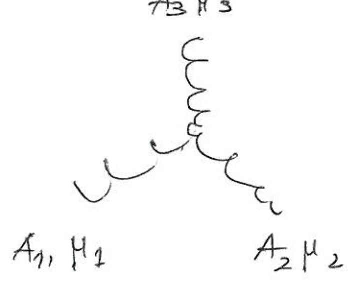
upon quantization $\longrightarrow \mathcal{L}_{QCD} \to \mathcal{L}_{QCD} + \mathcal{L}_{\substack{gauge\\fixing}} + \mathcal{L}_{ghosts}$

Faddeev Popov

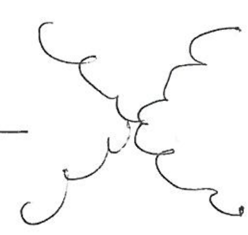
Main novelties :



$$\longrightarrow \frac{i}{k^2+i\epsilon}\,\delta^{AB}\,P^{\mu\nu}(k) \quad \mapsto \text{gauge-dependent spin polarization tensor}$$



→ gauge bosons self-interacts. ⟵



+ ⋯

Note 1 : $\mathcal{L}_{QCD} \subset \mathcal{L}_{SM}$

chiral nature of fermion fields, built into different represents. of $SU(2)_L$ changes "nature" of mass term.

Note 2 : as for all interacting QFT we do not have an exact solution. Two possible approaches:

→ numerical (ex.: lattice QCD)

→ perturbative → pQCD

expansion in coupling constant g_s ($\alpha_s = \frac{g_s^2}{4\pi}$) better!

$$\sigma_{QCD} = \sum_{k=0}^{\infty} \alpha_s^k C_k(\cdots)$$

generic observable

$$= C_0 + C_1\alpha_s + C_2\alpha_s^2 + C_3\alpha_s^3 + \cdots$$

small ↳ smaller ↳ even smaller

QCD as a renormalizable theory :

$$(g_s, m_f) \longrightarrow (g_s^R, m_f^R)$$

to be defined according to your subtraction scheme of choice (not so clear-cut physical meaning for strong interacts. → see later ↳ choose the technically most efficient one ! MS)

$\boxed{\text{R.G.E.}}$

$$g_s^R \longrightarrow g_s = g_s(\mu)$$

$$m_f^R \longrightarrow m_f = m_f(\mu)$$

$\mu = \mu_R$ renormalization scale

↳ (drop "R" from now on)

Running of strong coupling:

$$\boxed{\mu \frac{\partial g_s(\mu)}{\partial \mu} = \beta(g_s(\mu))}$$

↓ perturbative quantity

or

$$\mu^2 \frac{\partial \alpha_s}{\partial \mu^2} = \beta(\alpha_s) \qquad (\alpha_s = \alpha_s(\mu))$$

$$\beta(\alpha_s) = -\alpha_s^2 \left(b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \dots \right)$$

several orders calculated (up to b_4)

asymptotic freedom

$\alpha_s \downarrow$ with increasing energy/mom.

↕

confinement

(hadronic bound states at low energy/mom.)

renormalized (g_s^R)

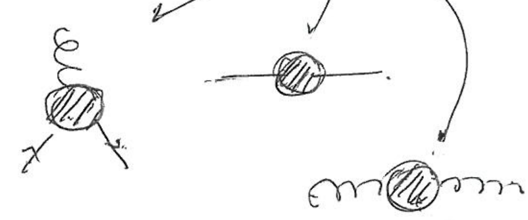
$$g_s = Z_g^{-1} g_{s,0} \quad \text{"bare"}$$

renorm.

$$\underbrace{g_{s,0} Z_q Z_A^{1/2}}_{g_s Z_1} \underbrace{\bar{q}_f A_\mu^A T^A \gamma^\mu q_f}_{\bar{q} A q}$$

$$g_{s,0} = Z_g g_s$$

$$Z_g = Z_1 Z_q^{-1} Z_A^{-1/2}$$



calculated at a given perturbtv. order

$$b_0 = \frac{(11 C_A - 2 N_f T_R)}{12\pi} = \frac{33 - 2N_f}{12\pi} \qquad \text{(1-loop } \beta \text{ function)}$$

$C_A = N \quad (SU(N)) \quad (f^{ABC} f^{DBC} = C_A \delta^{AD})$

$T_R = 1/2 \quad (Tr(T^A T^B))$

N_f = # light flavors

etc.

→ 2004 Nobel prize: Gross, Politzer, Wilczek

↳ QCD can be technically quite complicated, but it is here, with asymptotic freedom, that everything begins → parton models

First order, as an example:

$$\mu^2 \frac{\partial \alpha_s}{\partial \mu^2} = -\alpha_s^2 b_0$$

$$\downarrow$$

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \alpha_s(\mu_0)\, b_0 \ln \frac{\mu^2}{\mu_0^2}} \equiv \frac{1}{b_0 \ln \frac{\mu^2}{\Lambda_{QCD}^2}}$$

notice how terms of the form $\alpha_s(\mu_0) \ln \frac{\mu^2}{\mu_0^2}$ are "resummed" in the definition of $\alpha_s(\mu)$ → "leading logs".

"fundamental scale of QCD"

lower bound of validity for pQCD ← $\Lambda_{QCD} \simeq 0.2$ GeV

↳ extra comments on plots

To complete the picture, masses "run" as well:

$$m(\mu) = m(\mu_0) \exp\left[-\int_{\mu_0}^{\mu} \frac{d\lambda}{\lambda} \left[1 + \gamma_m(\alpha_s(\lambda))\right]\right]$$

mass "anomalous" dimension

$$\gamma_m = \mu \frac{\partial m}{\partial \mu}$$

↳ because of confinement, quark masses do not have a perturbatively clean/obvious physical meaning (not observables, except for top)

↳ treat them as yet another coupling... ($\overline{MS}$, not on-shell)

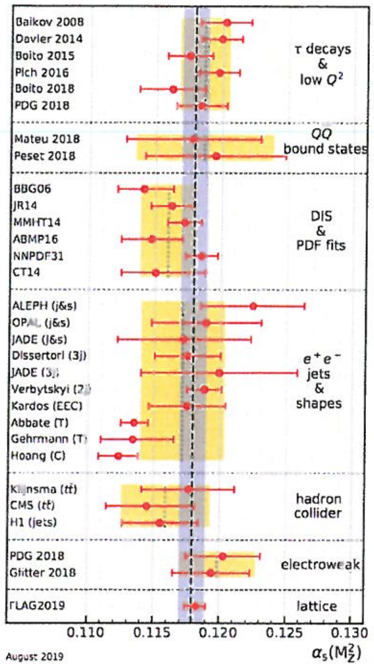
Notice:

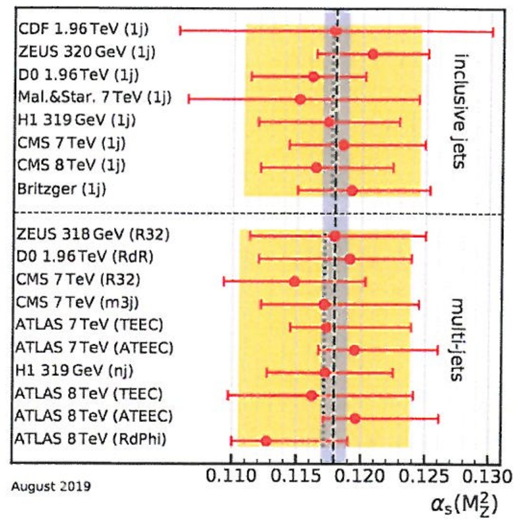
$u, d, s < \Lambda_{QCD}$

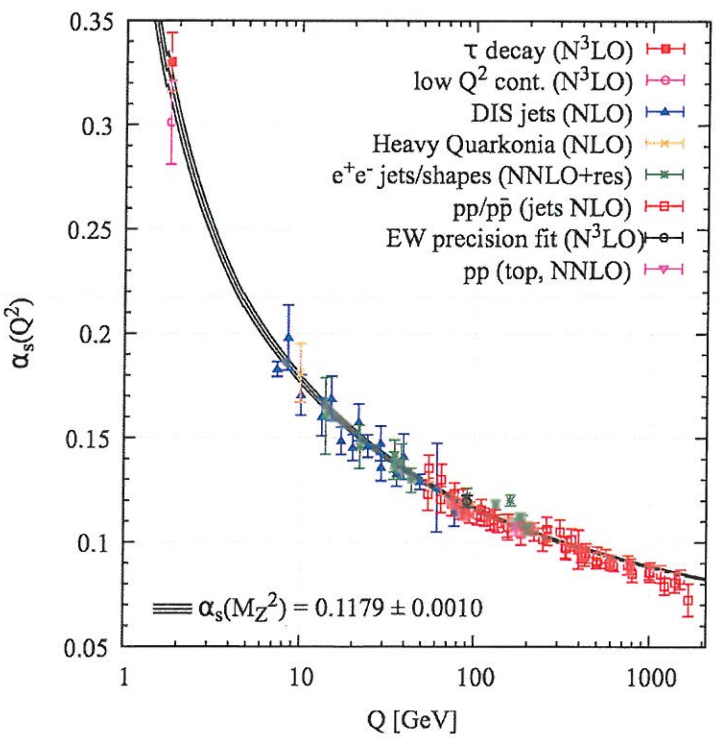
$c, b, t > \Lambda_{QCD}$

→ variety of exp./th opportunities/challenges

lots of physics to be learnt here!







during the evolution:

$$\rightarrow \quad \alpha_s^{(N_f+1)}(\mu) = \alpha_s^{(N_f)}(\mu)\left[1+\sum_{n=1}^{\infty}\sum_{k=0}^{n} c_{nk}\left[\alpha_s^{(N_f)}(\mu)\right]^n \ln^k\frac{\mu^2}{m_h^2}\right]$$

(2) Structure of pQCD calculations (in a nutshell)

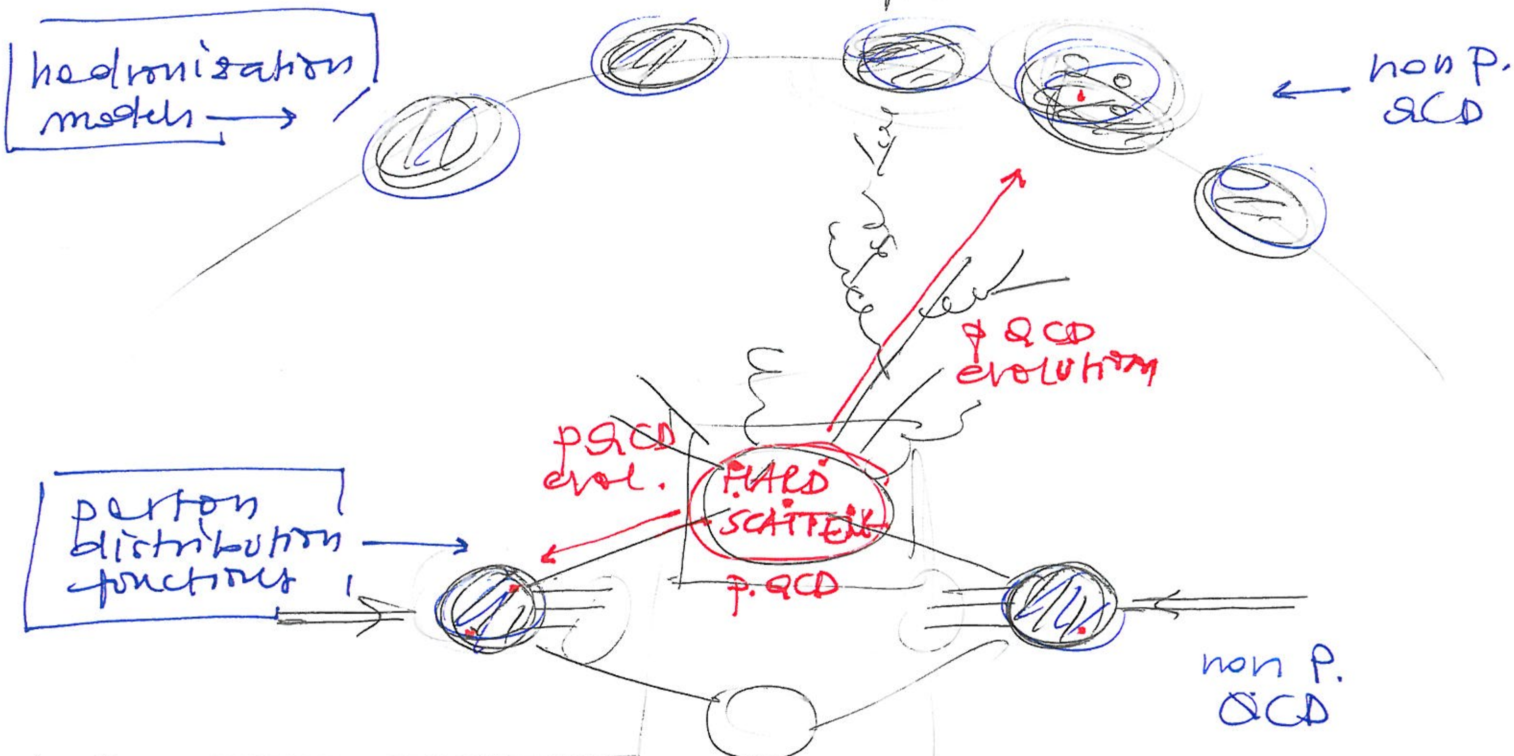
Where pQCD most matters : high-energy scattering processes

$\updownarrow$

HE colliders

Let's build the stages of a hadron-hadron collision at high energies (say at the LHC, $pp(\sqrt{s}=13\,\text{TeV})$)

↳ a h.e. e^+e^- collision will be just a simplified version of it



To realize the hard scattering part no need to use $pp \longrightarrow e^+e^-$

$e^+e^- \longrightarrow q\bar{q}$

(3)

$$R = \frac{\sigma(e^+e^- \longrightarrow \text{hadrons})}{\sigma(e^+e^- \longrightarrow \mu^+\mu^-)} \simeq N \sum_q e_q^2 \ldots$$

$\boxed{e^+e^- \longrightarrow \text{hadrons}}$ inclusive!

③ Hard scattering cross section — $\boxed{\text{parton level}}$

↳ our observable

Consider: $\sigma(e^+e^- \to \text{hadrons})$ ↓ $\boxed{\text{inclusive!}}$

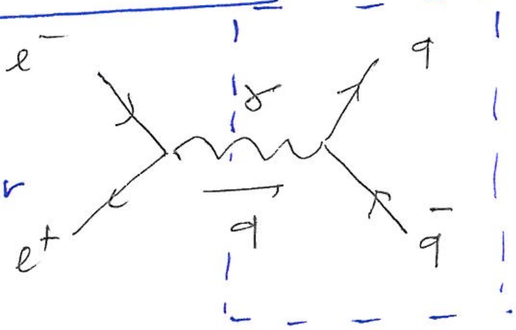
(crucial observable for pQCD $R = \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$)

↓ evidence of SU(3) "colour" symmetry

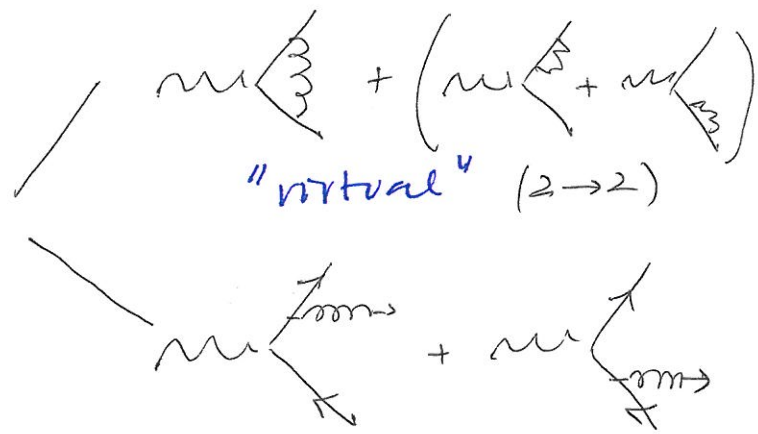
$\boxed{\text{parton level}}$ → $e^+e^- \to q\bar{q}$ (tree-level) = first order at which the process appear

$\boxed{\text{Adding QCD corrections}}$ → QCD-wise

1st order



$+\, O(\alpha_s)$



"virtual" (2→2)

"real" (2→3)

same order in α_s at the xsec level

$$\sigma(e^+e^- \to q\bar{q}) \overset{O(\alpha_s)}{\simeq} \underbrace{\int dPS_{2\to2}\, \overbrace{|M_{2\to2}|^2}^{\sigma_{born}+\sigma_{virtual}} + \int dPS_{2\to3}\, \overbrace{|M_{2\to3}|^2}^{\sigma_{real}}}_{\text{at the same order in } \alpha_s}$$

$$|M^0_{2\to2} + M^1_{2\to2}|^2 = |M^0_{2\to2}|^2 + \underbrace{(M^{0*}M^1 + M^{1*}M^0)}_{O(\alpha_s)} + \cancel{|M^1_{2\to2}|^2}_{\,\hookrightarrow O(\alpha_s^2)}$$

$M_{q\bar{q}}$ ←

$|M_{2\to3}|^2 \longrightarrow O(\alpha_s)$ → need to consider both when inclusive on final-state

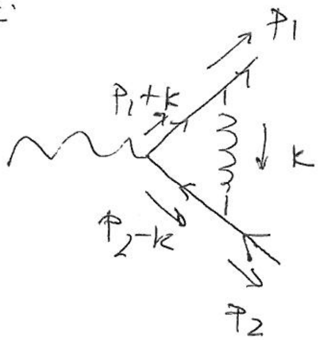
$M_{q\bar{q}+g}$ ←

$\sigma_{virtual}$ — $\rightarrow$ $O(\alpha_s)$ corrections to $\sim\!\!<^{q}_{\bar{q}}$ $(\gamma^* \rightarrow q\bar{q})$

UV divergences $\longrightarrow$ cancelled by renormalization procedure $(g_s^{(0)}, m_f^{(0)} \rightarrow g^{(R)}, m_f^{(R)})$

IR divergences $\rightarrow$ still present in loop integrations

Ex.



$$\longrightarrow \int \frac{d^4k}{(2\pi)^4} \frac{N(k,p_1,p_2)}{k^2(k+p_1)^2(k-p_2)^2}$$

$$\downarrow$$

$$k^2(2k\cdot p_1)(2k\cdot p_2)$$

$$\simeq \omega_k^4(1-\cos\theta_{kp_1})(1-\cos\theta_{k\cdot p_2})$$

$\omega_k \rightarrow 0$ "soft" IR divergence

$\left\{ \begin{matrix} \theta_{k\cdot p_1} \\ \theta_{k\cdot p_2} \end{matrix} \right. \longrightarrow 0$ "collinear" IR divergence

dimensional regularization $\longrightarrow$ $d = 4-2\epsilon_{IR}$ $\longrightarrow$ extract them as poles in ϵ_{IR}.

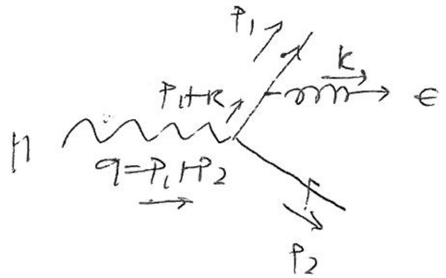
$$\sigma_{virtual} = \sigma^{IR}_{virtual} + \sigma^{finite}_{virtual}$$

$$\downarrow$$

$$\frac{c_2}{\epsilon_{IR}^2} + \frac{c_1}{\epsilon_{IR}}$$

$\longrightarrow$ same divergence (but opposite sign!) appears in σ_{real}.

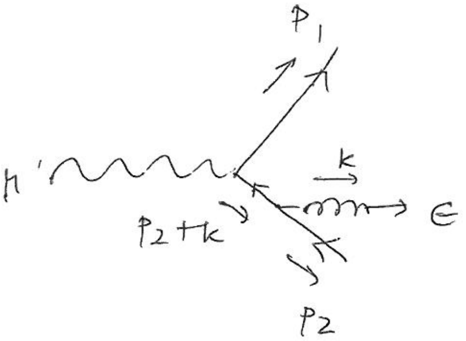
σ_{real} $(\gamma^* \longrightarrow q\bar q + g)$



$$\bar u(p_1)\, i g_s \not\epsilon\, T^A \frac{i}{(\not p_1+\not k)}\, i e_q \gamma^\mu v(p_2)$$

$\downarrow$ $k\to 0$ (soft limit)

$$\ldots\ \bar u(p_1)\not\epsilon\,\frac{(\not p_1+\not k)}{2p_1\cdot k}\gamma^\mu v(p_2) \longrightarrow \ldots \frac{\cancel{2}p_1\cdot\epsilon}{\cancel{2}p_1\cdot k}\,\bar u(p_1)\gamma^\mu v(p_2)$$



$$-\bar u(p_1)\, i e_q \gamma^\mu \frac{i}{(\not p_2+\not k)}\, i g_s \not\epsilon\, T^A v(p_2)$$

$\downarrow$ $k\to 0$

$$\ldots\ \frac{\cancel{2}p_2\cdot\epsilon}{\cancel{2}p_2\cdot k}\,\bar u(p_1)\gamma^\mu v(p_2)$$

$\downarrow$

$$\underset{(2\to 3)}{M_{q\bar q+g}} \simeq \bar u(p_1)\, i e_q \gamma^\mu T^A v(p_2)\left(\frac{p_1\cdot\epsilon}{p_1\cdot k}-\frac{p_2\cdot\epsilon}{p_2\cdot k}\right)$$

$\downarrow$ soft limit

$$T^A T^A = C_F \overset{SU(N)}{=} \frac{N^2-1}{2N} \overset{N=3}{=} \frac{4}{3}$$

$\downarrow$

$$\sum_{\substack{\text{color}\\\text{pol.}}} |M_{q\bar q+g}|^2 = |M_{q\bar q}|^2\, C_F\cdot g_s^2\cdot \underbrace{\frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}}_{\text{gluon emission characteristics}}$$

$\swarrow$ independent PS integration

$$\sigma_{real} \xrightarrow[\text{soft limit}]{} \int dPS_{2\to3} |M_{q\bar{q}+g}|^2 \underset{\text{soft limit}}{\simeq} \underbrace{\int dPS_{2\to2} |M_{q\bar{q}}|^2}_{\text{Hard } e^+e^-\to q\bar{q} \text{ part (tree level)}} \underbrace{\frac{d^3k}{2\omega_k(2\pi)^3} C_F g_s^2 \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}}_{dS \to \text{ soft gluon emission (eik)}}$$

(also contain collinear piece)

$$dS = \omega_k d\omega_k \, d\cos\theta \frac{d\phi}{2\pi} \frac{2\alpha_s C_F}{\pi} \underbrace{\frac{2p_1\cdot p_2}{(2p_1\cdot k)(2p_2\cdot k)}}_{\frac{1}{\omega_k^2(1-\cos^2\theta)}}$$

$$dS = \frac{2\alpha_s C_F}{\pi} \frac{d\omega_k}{\omega_k} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

$$\begin{cases} \omega_k \to 0 & \text{IR "soft" divergence} \\ \theta \to 0, \pi & \text{IR "collinear" divergence} \end{cases}$$

↳ integral can be performed in $d = 4 - 2\epsilon_{IR}$

↳ same pole structure as $\sigma_{virtual}$

↳ $-\frac{c_2}{\epsilon_{IR}^2} - \frac{c_1}{\epsilon_{IR}}$

$$\sigma(e^+e^- \to q\bar{q}) = \sigma_{q\bar{q}} \left\{ 1 + \frac{2\alpha_s C_F}{\pi} \int \frac{d\omega_k}{\omega_k} \int \frac{d\theta}{\sin\theta} \left[R\left(\frac{\omega_k}{Q}, \theta\right) - V\left(\frac{\omega_k}{Q}, \theta\right) \right] \right\}$$

thinkites

$$\lim_{\omega_k \to 0} (R - V) = 0, \quad \lim_{\theta \to 0, \pi} (R - V) = 0$$

$\to \mu \sim Q$ right scale! (not affected by soft dynamics)

In general : Kinoshita - Lee - Naunberg theorem

for sufficiently inclusive quantities
IR divergences cancel $\hookrightarrow \sigma = \int_{n+1} d\sigma^R + \int_n d\sigma^V$

Note 1 : PS is flat, but matrix elements are enhanced in soft & collinear regions
$\hookrightarrow$ hadronic final state formed by jets of collinear particles, plus soft particles (!)
(we can see why!)

Note 2 : finite residual contains soft/collinear logs.
$\hookrightarrow$ if large need to know them.

Note 3 : when considering hadronic i.s. !
only 1 parton per hadron in initial state, not inclusive enough!

Note 4 : what about "exclusive" quantities?
How do we handle residual IR divergences?