Lecture 2

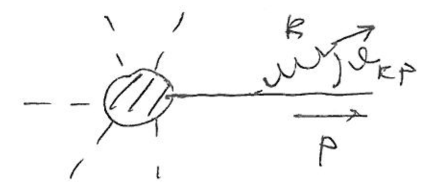
"hadron → parton → hadron"

$\underbrace{\text{hadron} \to \text{parton}}_{\text{initial-state evolution (pQCD)}}$ 2 ; $\underbrace{\text{parton} \to \text{hadron}}_{\text{final-state evolution (pQCD)}}$ 1

1 Parton (level) → hadron (level)

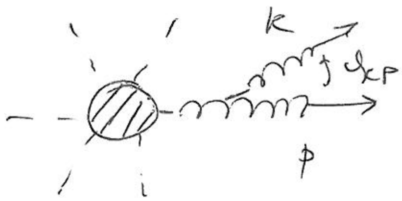
What we learnt for $e^+e^- \to q\bar{q}$ is very general.

Behavior of soft/collinear radiation (gluon emission case):



$$\sim \frac{2\alpha_s C_F}{\pi}\,\frac{d\omega_k}{\omega_k}\,\frac{d\vartheta_{kp}}{\vartheta_{kp}}$$

$$\left\{ \begin{array}{l} C_F = \frac{N^2-1}{2N} \\ C_A = N \end{array} \right.$$



$$\sim \frac{2\alpha_s C_A}{\pi}\,\frac{d\omega_k}{\omega_k}\,\frac{d\vartheta_{kp}}{\vartheta_{kp}}$$

→ "splitting functions"

(include collinear kinematics more precisely)

→ same behavior replicated at all orders

↓

large radiation at small angles and soft energy

↓

important indication for exclusive processes

⇒ energy flow at small angles with respect to initial hard partons or with low energy.

↳ this is the pattern observed in hadron production.

2

$\boxed{\text{Parton shower}}$ ← implement the previous dynamics with a sequential algorithm based on "ordered" emission
→ in angle
→ in energy (k_T)
($k_T \to \omega_k \theta$)

↳ Need to calculate emission probability at each splitting.

Ex: $k_{T,1} > k_{T,2} > \ldots$ (each k_T integration is cutoff by the nested one)

the final cutoff is a scale Q_0 ($\to \Lambda_{QCD}$) at which pQCD reaches is limit of validity → HADRONIZATION MODEL

→ $P(\text{emission} > k_T) = 1 - P(\text{non-emission} > k_T)$
↳ $\Delta(k_T, Q)$

$$\frac{dP(\text{emission} > k_T)}{dk_T} = -\frac{dP(\text{non-emission} > k_T)}{dk_T}$$

"Sudakov form factor" ($q\bar{q} \to q\bar{q}g$)

$$\Delta(k_T, Q) \cong \exp\left\{ -\frac{2\alpha_s C_F}{\pi} \int^{Q} \frac{d\omega_k}{\omega_k} \int^{\pi/2} \frac{d\theta}{\theta}\, \Theta(\omega_k\theta - k_T) \right\}$$

this is the part that can be improved by higher-order pQCD → calculate h.o. corrections to splitting probability

↳ implement this in a Monte Carlo and you have a P.S. event generator!

$$q\bar{q} \xrightarrow[\Delta(k_{T,1}, Q)]{} q\bar{q}g \xrightarrow[\Delta(k_{T,2}, k_{T,1})]{} q\bar{q}gg \longrightarrow \ldots$$

till you reach a scale $Q_0 \simeq \Lambda_{QCD}$

$\boxed{\text{caveat ! this only models the soft/collinear dynamics}}$

$$P_{nodec}(t) = e^{-t/\tau} = e^{-\Gamma t}$$

$$P_{dec}(t) = 1 - P_{nodec}(t) = 1 - e^{-\Gamma t}$$

$$\frac{dP_{dec}(t)}{dt} = \Gamma e^{-\Gamma t} = \Gamma P_{nodec}(t)$$

if $\Gamma = \Gamma(t) \longrightarrow P_{nodec}(t) = e^{-\int_0^t dt' \, \Gamma(t')}$

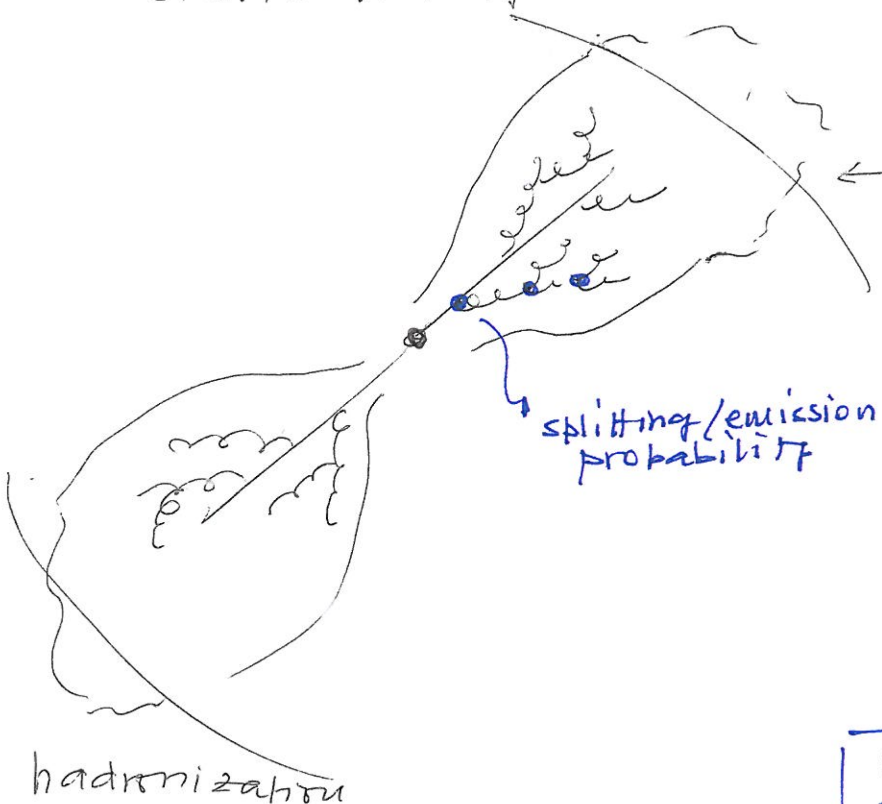
$$\frac{dP_{dec}(t)}{dt} = \Gamma(t) \, P_{nodec}(t)$$

$\uparrow$

reminder to understand form of $\Delta(k_T, Q)$

$e^+e^- \to q\bar{q} \cdots \to$ hadrons

starts looking like this:



hadronic activity (energy) collimated in "narrow" regions around the original hard partons.

↓

pictorially already suggesting the idea of "sprays" of particles or "jets".

↓

$\boxed{\text{Parton Shower event generators translate this property of pQCD into a systematic algorithm.}}$

which can be improved using pQCD itself

LO $\to$ NLO $\to$ NNLO $\to \cdots$

$+O(\alpha_s)$ $\quad +O(\alpha_s)$

$\boxed{\text{Caveat:}}$

Parton Shower e.g. are based on soft/collinear dynamics

↓

fail to correctly reproduce hard radiation

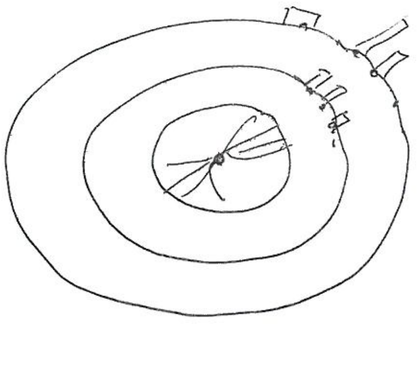
↓

(a) Need h.o. pQCD in σ_{hard} to start with

and

(b) consistent matching procedure

↳ Need to choose meaningful observables!



← we see energy depositions with a given pattern. (see before)

↓

need IR safe observable

invariant if $\vec{p}_i = \vec{p}_j + \vec{p}_k$ $\quad \vec{p}_j \parallel \vec{p}_k$ $\quad |\vec{p}_j|, |\vec{p}_k| \to 0$

↓

$\vec{p}_i$ is a momentum from which the observable depends.

Ex.

- gluon multiplicity ×
- charged particle E, p_T ×
- energy flow in a given "cone" ✓.

↓

need "jet" definition.

Long history to find IR-safe definition...

What is a jet for a theorist?

eventually!

jet configurations have to be stable with respect to soft radiation.

→ the more partons the better for a fixed-order calculation.

→ then apply a jet algorithm, based on a "distance" definition, in particular:

$\forall$ "i" parton or "seed"

d_{ij} = distance wrt other partons (seeds)

d_{iB} = " " to the beam (initial partons)

naive: $$d_{ij} = \Delta R_{ij} = \left((y_i - y_j)^2 + (\phi_i - \phi_j)^2 \right)^{1/2}$$

more sofisticated: $$d_{ij} = \min\left(p_{T,i}^{2n}, p_{T,j}^{2n}\right) \frac{\Delta R_{ij}^2}{R_j^2}$$

$$d_{iB} = p_{Ti}^{2n}$$

(k_T, anti-k_T) etc.

$n = 1, 0, -1$ ..

(2) hadron (level) → parton (level)

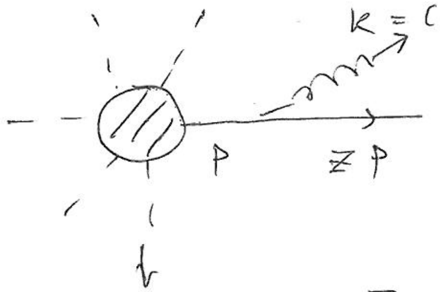
if $(e^+e^-) \to (pp)$ in initial state

↑

how to connect to parton densities in hads.

→ Difference between final-state and initial-state QCD radiation.

Ex.: $q \to qg$



$\sigma_{hard} \to \sigma_h$

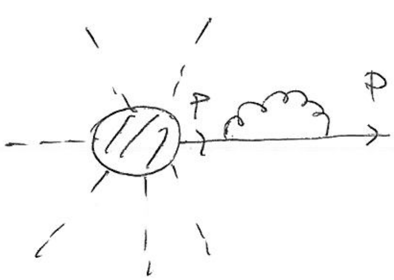
FINAL STATE

$$\sigma_{h+g}(p) \simeq \sigma_h(p) \frac{2\alpha_s C_F}{\pi} \frac{d\omega_k}{\omega_k} \frac{d\theta}{\theta}$$

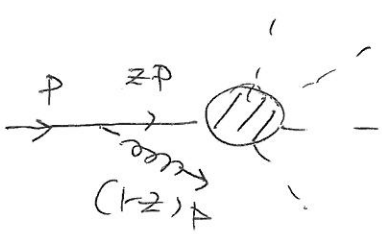
$$\omega_k = (1-z)p$$
$$k_T = \omega_k \sin\theta \simeq \omega_k \theta$$

$$\sigma_h(p) \frac{2\alpha_s C_F}{\pi} \frac{dz}{(1-z)} \frac{dk_T}{k_T}$$

same kinematic. IR div. cancel.

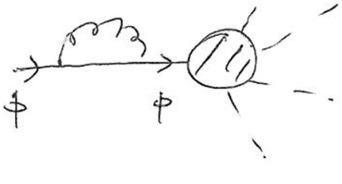


$$\sigma_h^{virt}(p) \simeq -\sigma_h(p) \frac{2\alpha_s C_F}{\pi} \frac{dz}{(1-z)} \frac{dk_T}{k_T}$$



$$\sigma_{h+g}(p) \simeq \sigma_h(zp) \frac{2\alpha_s C_F}{\pi} \frac{dz}{(1-z)} \frac{dk_T}{k_T}$$

mismatch



$$\sigma_h^{virt}(p) \simeq -\sigma_h(p) \frac{2\alpha_s C_F}{\pi} \frac{dz}{(1-z)} \frac{dk_T}{k_T}$$

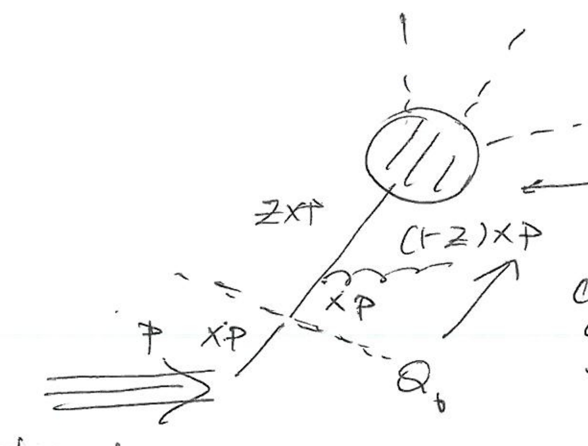
$$\rightarrow \sigma_{h+g} + \sigma_h^{virt} \simeq \frac{2\alpha_s C_F}{\pi} \int_0^{Q} \frac{dk_T}{k_T} \int \frac{dz}{(1-z)} \underbrace{\left[\sigma_h(zp) - \sigma_h(p)\right]}_{\text{finite}}$$

soft limit $(z \to 1)$ soft divergences cancel

collinear limit $(k_T \to 0)$ infinite → collinear div. not cancelled by virtuals

mismatch in longitudinal kinematics.

→ Do not start from $k_T = 0$!
Set a lower bound due to pQCD validity → Q_0 or μ_F
factorization scale



← Q (scale of σ_h)

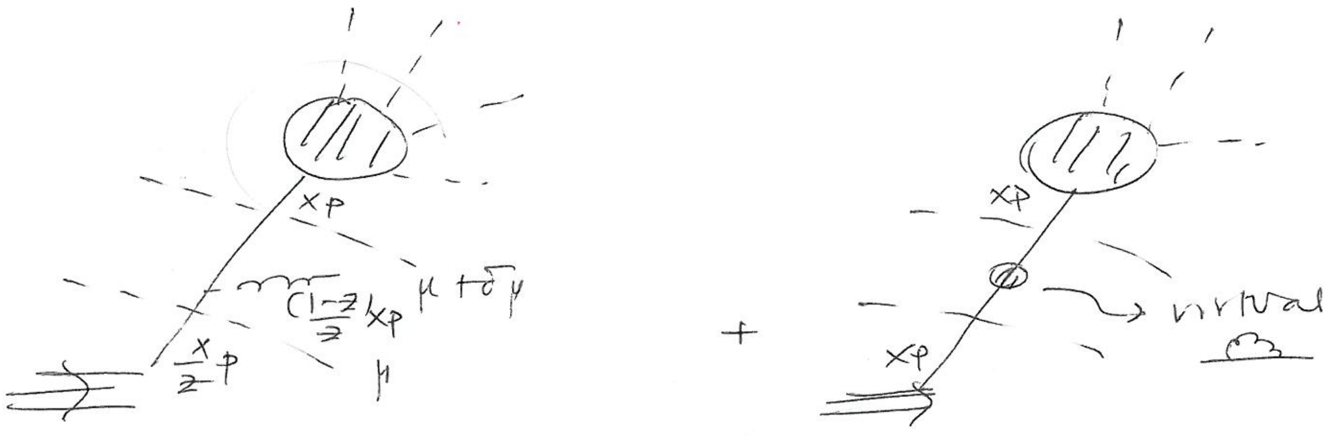
consider only evolution from Q_0 on.

$$\sigma_1 = \frac{2\alpha_s C_F}{\pi} \int_{Q_0}^{\overset{\nearrow \text{large}}{Q}} \frac{dk_T}{k_T} \underbrace{\int \frac{dx\,dz}{1-z} \left[\sigma_h(zxp) - \sigma_h(xp)\right]}_{\text{finite}} \underbrace{q(x, Q_0)}_{\substack{\text{non} \\ \text{perturbative} \\ \text{input}}}$$

↓

you can repeat this process in infinitesimal steps and get the $q(x, Q_0)$ density "evolution"

To get evolution with scale μ, scale the previous picture to look like: (8)



DGLAP evolution → pQCD determines the evolution.

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha}{2\pi}\int_x^1 \frac{dz}{z} P_{qq}(z)\, q\left(\frac{x}{z},\mu^2\right)$$

splitting function $\quad P_{qq}(z) = \left(\frac{1+z^2}{1-z}\right)_+$

$$\int_0^1 dz\,[h(z)]_+ f(z) = \int_0^1 dz\, h(z)\,(f(z)-f(1))$$

→ to be extended to all possible emissions

$q \to qg$

$g \to gg$

$g \to q\bar{q}$

$$\longrightarrow \frac{d}{d\ln\mu^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}\begin{pmatrix} q \\ g \end{pmatrix}$$

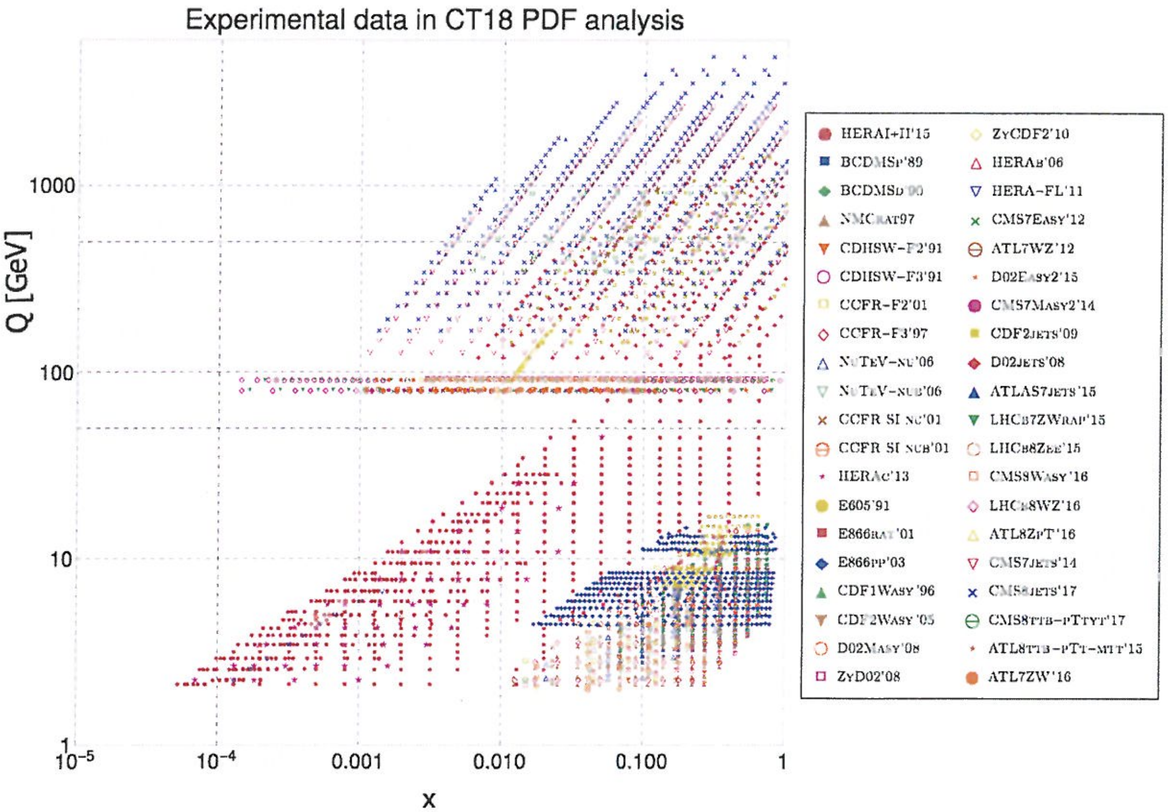
→ Many available sets: NNPDF, CTEQ, MRST/MSTW/ MMHT, .. PDF4LHC ... { NLO, NNLO

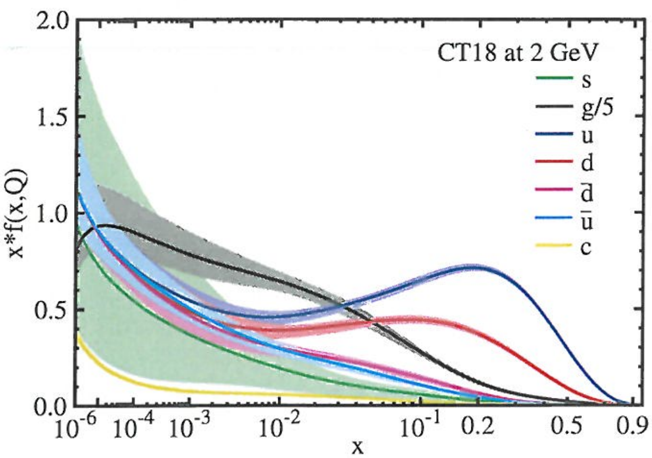
↓

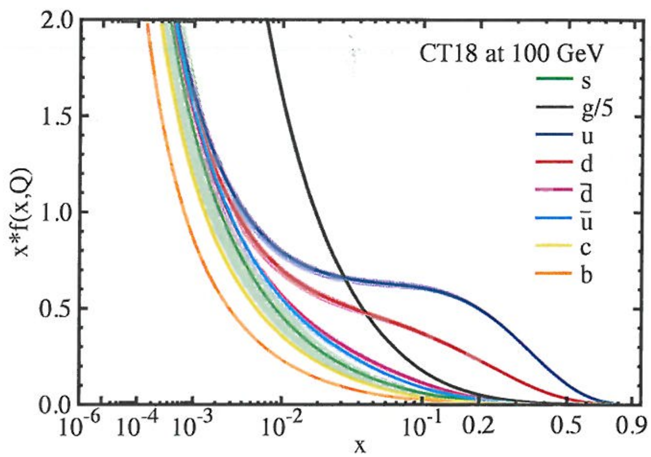
uncertainty depend on (Q,x)

↳ see plots

overall : 5–10% range → systematically improvable


Experimental data in CT18 PDF analysis
Q [GeV]
1000
100
10
1
x
10^{-5}
10^{-4}
0.001
0.010
0.100
1
HERAI+II'15
BCDMSp'89
BCDMSd'90
NMCrat97
CDHSW-F2'91
CDHSW-F3'91
CCFR-F2'01
CCFR-F3'97
NuTeV-nu'06
NuTeV-nub'06
CCFR SI nu'01
CCFR SI nub'01
HERAc'13
E605'91
E866rat'01
E866pp'03
CDF1Wasy'96
CDF2Wasy'05
D02Masy'08
ZyD02'08
ZyCDF2'10
HERAb'06
HERA-FL'11
CMS7Easy'12
ATL7WZ'12
D02Easy2'15
CMS7Masy2'14
CDF2jets'09
D02jets'08
ATLAS7jets'15
LHCb7ZWrap'15
LHCb8Zee'15
CMS8Wasy'16
LHCb8WZ'16
ATL8ZpT'16
CMS7jets'14
CMS8jets'17
CMS8ttb-pTtyt'17
ATL8ttb-pTt-mtt'15
ATL7ZW'16


CT18 at 2 GeV
s
g/5
u
d
$\bar{d}$
$\bar{u}$
c
x*f(x,Q)
2.0
1.5
1.0
0.5
0.0
10^{-6} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 0.2 0.5 0.9
x


CT18 at 100 GeV
s
g/5
u
d
$\bar{d}$
$\bar{u}$
c
b
x*f(x,Q)
2.0
1.5
1.0
0.5
0.0
10^{-6} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 0.2 0.5 0.9
x

From G. Salam.

UNCERTAINTIES ON PARTONIC LUMINOSITIES — V. RAPIDITY(Y) AND MASS