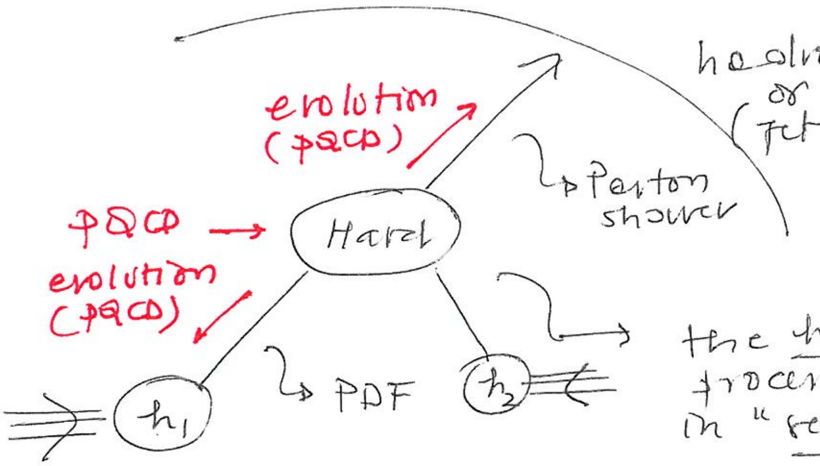
Lecture 3

Pushing the order of pQCD - why -

Given what we know of the role played by pQCD in a generic hadronic event:



the hard scattering process plays a crucial role in "setting the stage" (for further parton-shower evolution)

the further we push the matching between "hard scattering" and parton sh. the better.

push the order of pQCD in $\boxed{\sigma_{hard}}$

push the accuracy in the matching between σ_{hard} and P.S.

we will focus on this part in this lecture → $\boxed{\hat\sigma_{hard}}$ ("parton level")

$$\underbrace{\sigma(h_1 h_2 \to X)}_{\text{inclusive / exclusive}} = \sum_{i,j}\int \underbrace{dx_1 dx_2 f_{i/h_1}(x_1,\mu_F) f_{j/h_2}(x_2,\mu_F)}\, \hat\sigma_{ij}(x_1,x_2,Q,\alpha_s,\mu_R,\mu_F)\;\Theta(\cdots)$$

inclusive

exclusive → functions that select a specific final state

brief discussion during last lecture - PDF accuracy is material for a separate series of lectures!

→ Why are we doing all this? → improve theoretical accuracy

→ How do we improve theoretical accuracy?

- PDF
- $\hat{\sigma}_{hard}$
  - push perturbative order
    - to lower sensitivity to μ_R, μ_F
    - to include all possible initial states (subprocesses) and going enough final-state multiplicity.
  - resum large classes of corrections
    - when process involves very different mass scales
    - when exclusive cuts introduce artificial thresholds
- parton shower

will focus on this part → push perturbative order

we will not have time to cover resummations → resum large classes of corrections

→ in first approximation: $\boxed{\mu_R - \mu_F \text{ variation}}$ should give us a feeling of the theoretical uncertainty involved.

ex.: $\boxed{e^+e^- \to X}$ some QCD final state

$$\alpha_s(\mu_R) \simeq \frac{\alpha_s(Q)}{1+\alpha_s(Q)\,b_0 \ln\frac{\mu_R^2}{Q^2}} + \text{h.o.}$$

$$\alpha_s(Q)\left(1-\alpha_s(Q)\,b_0 \ln\frac{\mu_R^2}{Q^2} + O(\alpha_s^2)\right)$$

$$\sigma^{NLO}(\mu_R) = \sigma_0\left(1 + c_1\alpha_s(\mu_R)\right)$$

$$= \sigma_0\left(1 + c_1\alpha_s(Q) - 2c_1 b_0\,\alpha_s^2(Q)\ln\frac{\mu_R^2}{Q^2} + O(\alpha_s^3)\right)$$

$\boxed{\mu_R \text{ dependence is } O(\alpha_s^2)}$

(3

$$\frac{d\sigma^{ALL}}{d\mu_R} = 0 \longrightarrow \sigma_0 \left(1 + c_1 \alpha_s(\mu_R) + C_2(\mu_R)\, \alpha_s^2(\mu_R) + \text{h.o.}\right)$$

$$C_2(Q) + 2 C_1 b_0 \ln \frac{\mu_R^2}{Q^2}$$

etc.

$\boxed{pp \to X}$ → the same applies to both μ_R AND μ_F dependence.

Q1: how much to vary μ_R/μ_F: no strict rule - Need to study behavior of perturbative expansion.
Conventional: $\frac{1}{2}\mu_0 \leftarrow \mu_0 \rightarrow 2\mu_0$
from some "reasonable" central value determined by

Q2: if difference between LO and NLO larger than μ_R/μ_F dependence
↳ new subprocesses are entering the game ⟹ i.e.: $\boxed{\text{Need higher order to answer these questions!}}$

In the last 20 years: NLO revolution

↳ most SM processes, up to very high multiplicity, (depending on process) can be calculated including pQCD at NLO

→ calculations of $\hat{\sigma}^{\text{hard}}$ have been automatized

→ Fast new techniques have improved available framework and allowed to reach higher multiplicity faster

→ Interface with NLO parton shower have been consolidated (: MC@NLO, POWHEG)

In the last ~ 10 years: <u>NNLO</u> <u>pushed for</u>

$2 \to 1$, $2 \to 2$ processes

$pp \to H$

$pp \to W/Z$

$pp \to t\bar{t}$

$pp \to VV'$ (WW, WZ, ZZ, etc)

$2 \to 3$ is now the "frontier"

<u>Extremely</u> <u>rich</u> <u>field</u>!

→ new analytical / computational / mathematical idea constantly emerging

↓

may lead to breakthroughs in QFT itself!

Most important question: <u>How much</u> <u>precision</u> <u>do we need</u>?

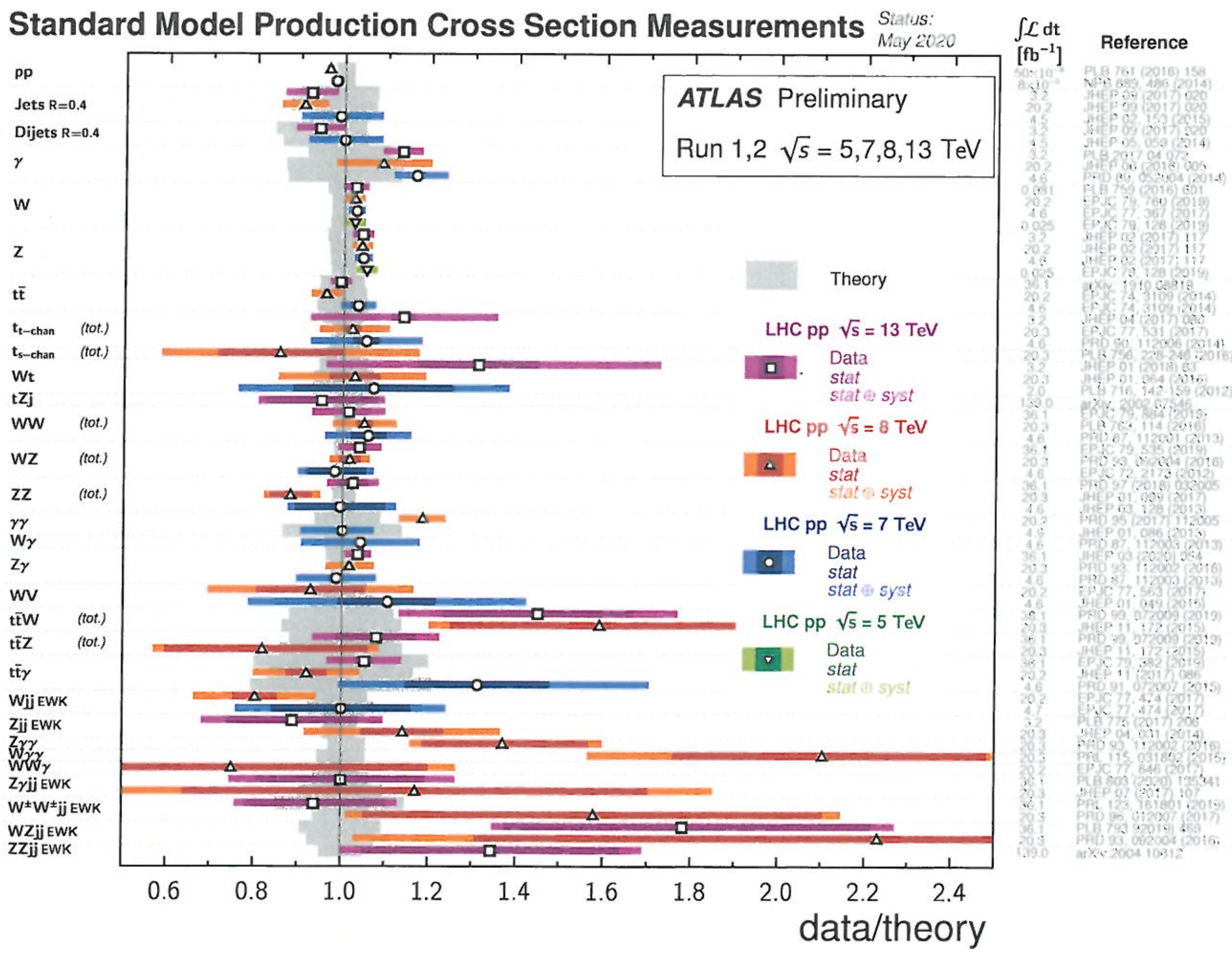
↳ Let's consider the following examples (and comments therein)

Unfortunately, each single example could take a lesson to really explain it!

I hope to have conveyed the general approach with which you can understand the results you see.

Ask more if you like!

the LHC at a glance:



Experiments are reaching % level precision for several $(2\to1)$ and $(2\to2)$ processes.

Next decade → 20-fold increase in datasets @ LHC.

"attobarn Era"

↓

reaching % accuracy for $2\to3$ (or more) processes.

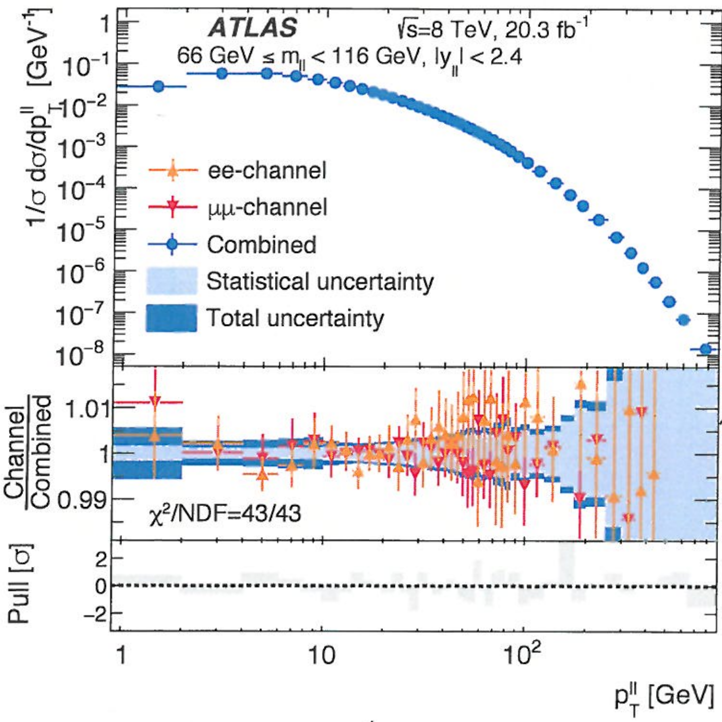
↓

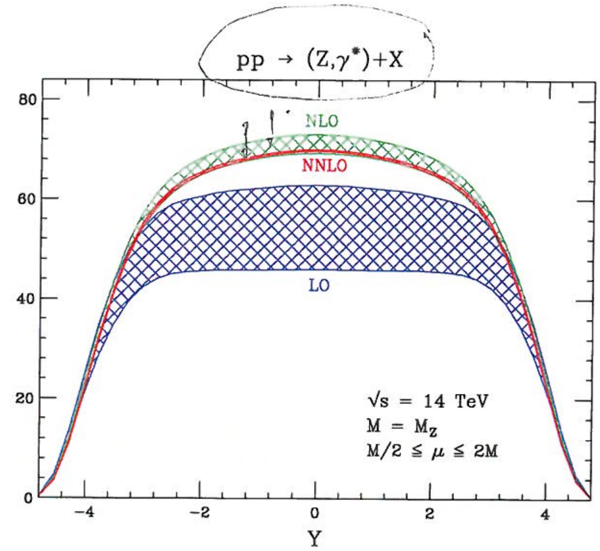
theory need to reach same level of precision/accuracy

Ex. 1

Drell-Yan production → measured exp. at % level !

a "standard candle" for collider physics





exper. precision at % level

need for NNLO precision to stabilize perturbative predictions

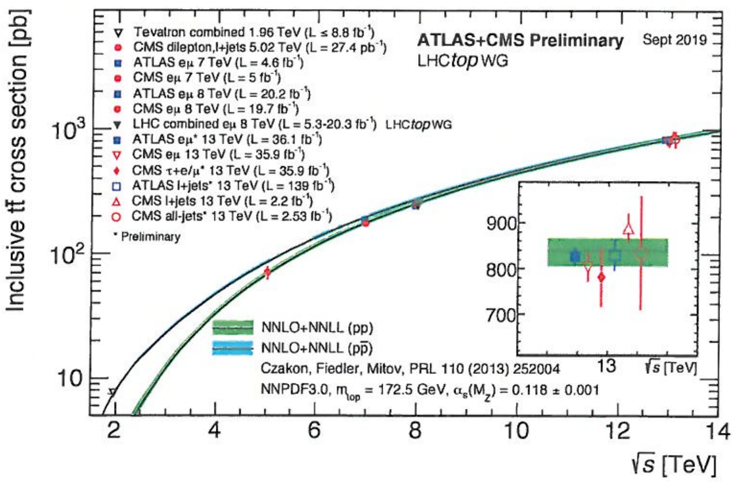
↓

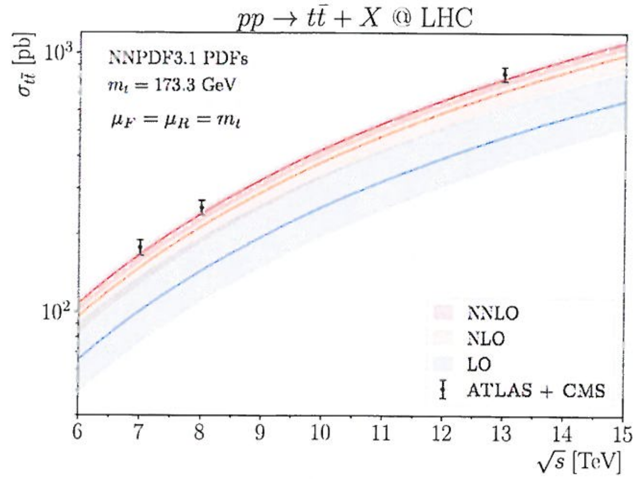
can disentangle parton luminosities.

Ex. 2:

$t\bar{t}$ production at hadron colliders.

NNLO QCD





NNLO Calculations by:

Czakon, Fiedler, Mitov (2013)

Czakon, Fiedler, Heymes, Mitov (2015-16)

+EW

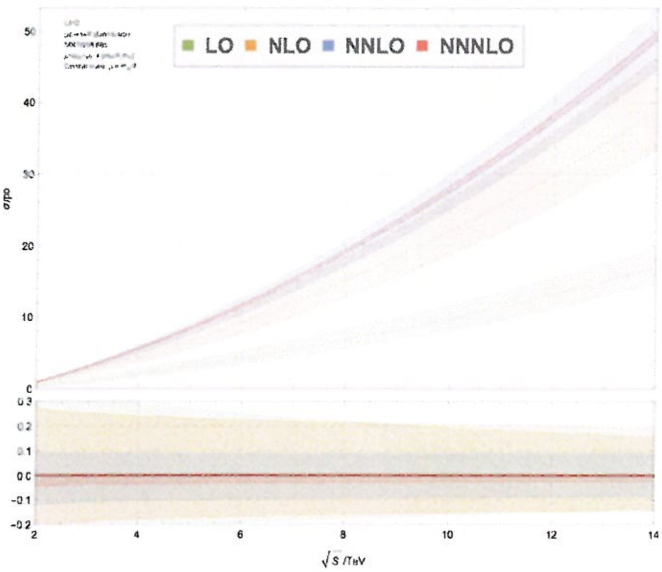
+Resummation

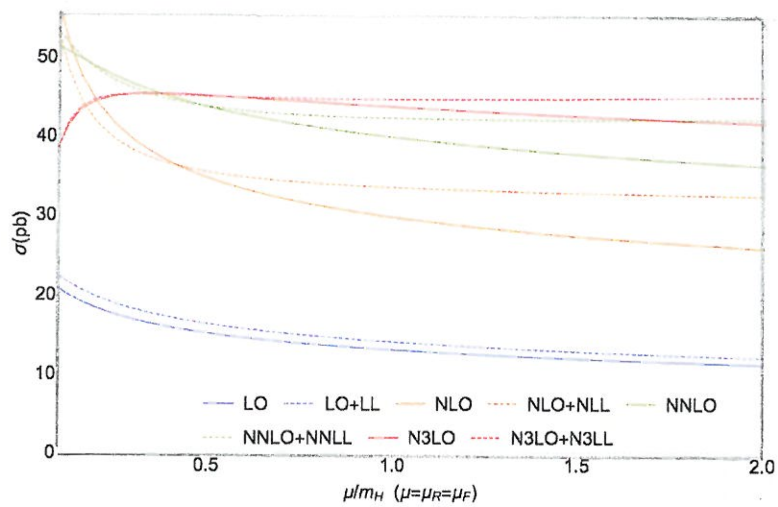
Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan (2019)

Ex. 3

Higgs production via $gg \to H$.

N^3LO pQCD /  rEFT





N^3LO prediction: accuracy of 5-6%

$$\sigma(13\ \text{TeV}) = 48.58^{+2.22}_{-3.27}\ (th) + 1.56\ (\alpha_s + PDF)\ \text{pb}$$

$$\begin{aligned}
48.58\ \text{pb} = \ & 16.00\ \text{pb} \quad (+32.9\%) \quad (\text{LO, rEFT}) \\
& + 20.84\ \text{pb} \quad (+42.9\%) \quad (\text{NLO, rEFT}) \\
& - 2.05\ \text{pb} \quad (-4.2\%) \quad (\{t,b,c\}, \text{exact NLO}) \\
& + 9.56\ \text{pb} \quad (+19.7\%) \quad (\text{NNLO, rEFT}) \\
& + 0.34\ \text{pb} \quad (+0.2\%) \quad (\text{NNLO}, 1/m_t) \\
& + 2.40\ \text{pb} \quad (+4.9\%) \quad (\text{EW, QCD-EW}) \\
& + 1.49\ \text{pb} \quad (+3.1\%) \quad (N^3LO, \text{rEFT})
\end{aligned}$$

Ref.: Anastasiou et al. arXiv:1503.06056, 1602.00695