Effective Field Theories Across the Universe

The Standard Model Effective Field Theory

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Outline

Introduction to EFT in particle physics

- Main ideas.
- Main strategies.
- Some examples: from Fermi theory to the SM to the SMEFT.

Constructing the SMEFT

- The SM: brief review, strengths and weaknesses.
- Adding SMEFT interactions, how and why.

The SMEFT hands on

- SMEFT effects on SM parameters and SM interactions.
- Calculating observables in the SMEFT.

Constraining SMEFT interactions

- Global fits of collider observables (EW, Higgs, top), flavor observables, low energy observables.
- Matching to UV models.

SMEFT: dim 6

Very similar considerations leads to identify a basis of dim=6 SMEFT operators.

"Warsaw" or GIMR basis: most commonly used

Grzadkowski, Iskrzynski, Misiak, Rosiek, 1008.4884

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC}\widetilde G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK}\widetilde W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar l_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\,\widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar l_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p\sigma^{\mu\nu}T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar e_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\,\widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p\sigma^{\mu\nu}u_r)\tau^I\widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar q_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar q_p\sigma^{\mu\nu}u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\,\widetilde B_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar q_p\sigma^{\mu\nu}T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar u_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar d_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\,\widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p\gamma^\mu d_r)$

With respect to the dim=5 case, the problem arises of identifying a minimal set of independent operators.

(59 operators excluding L- and B-violating ones and suppressing flavor indices).

Considering the flavor structure of the operators: 2499 couplings out of which 1350 are CP-even and 1149 are CP-odd.

hermitian						non-hermitian	
$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$		$(\bar LR)(\bar LR)$ + h.c.	
$Q_{\ell\ell}$	$(\bar\ell_p\gamma_\mu\ell_r)(\bar\ell_s\gamma^\mu\ell_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	$Q_{\ell e}$	$(\bar\ell_p\gamma_\mu\ell_r)(\bar e_s\gamma^\mu e_t)$	$Q_{quqd}^{(1)}$	$(\bar q_p^i u_r)\varepsilon_{ij}(\bar q_s^j d_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	$Q_{\ell u}$	$(\bar\ell_p\gamma_\mu\ell_r)(\bar u_s\gamma^\mu u_t)$	$Q_{quqd}^{(8)}$	$(\bar q_p^i T^A u_r)\varepsilon_{ij}(\bar q_s^j T^A d_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	$Q_{\ell d}$	$(\bar\ell_p\gamma_\mu\ell_r)(\bar d_s\gamma^\mu d_t)$	$Q_{\ell equ}^{(1)}$	$(\bar\ell_p^i e_r)\varepsilon_{ij}(\bar q_s^j u_t)$
$Q_{\ell q}^{(1)}$	$(\bar\ell_p\gamma_\mu\ell_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$	$Q_{\ell equ}^{(3)}$	$(\bar\ell_p^i\sigma_{\mu\nu}e_r)\varepsilon_{ij}(\bar q_s^j\sigma^{\mu\nu}u_t)$
$Q_{\ell q}^{(3)}$	$(\bar\ell_p\gamma_\mu\tau^I\ell_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$		
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$		
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$	$(\bar LR)(\bar RL)$ + h.c.	
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{\ell edq}$	$(\bar\ell_p^i e_r)(\bar d_s q_{ti})$

Effects of SMEFT interactions - recap

- Effective operators at Λ_{EW} induce "direct" and "indirect" contributions of their Wilson coefficients in physical observables.

Modify existent interactions
+
New EFT interactions

Shift fields and parameters from the SM ones

EFT effects in **Lagrangian** $\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \frac{C_i}{\Lambda^2} Q_i$ → EFT effects in **fields and parameters** → EFT effects in **interactions** → EFT effects in **probability amplitudes** → EFT effects in **physical observables**

$$\frac{C_i}{\Lambda^2}$$

$$\frac{C_i}{\Lambda^2}, \frac{C_i^2}{\Lambda^4}, \ldots$$

SMEFT predictions

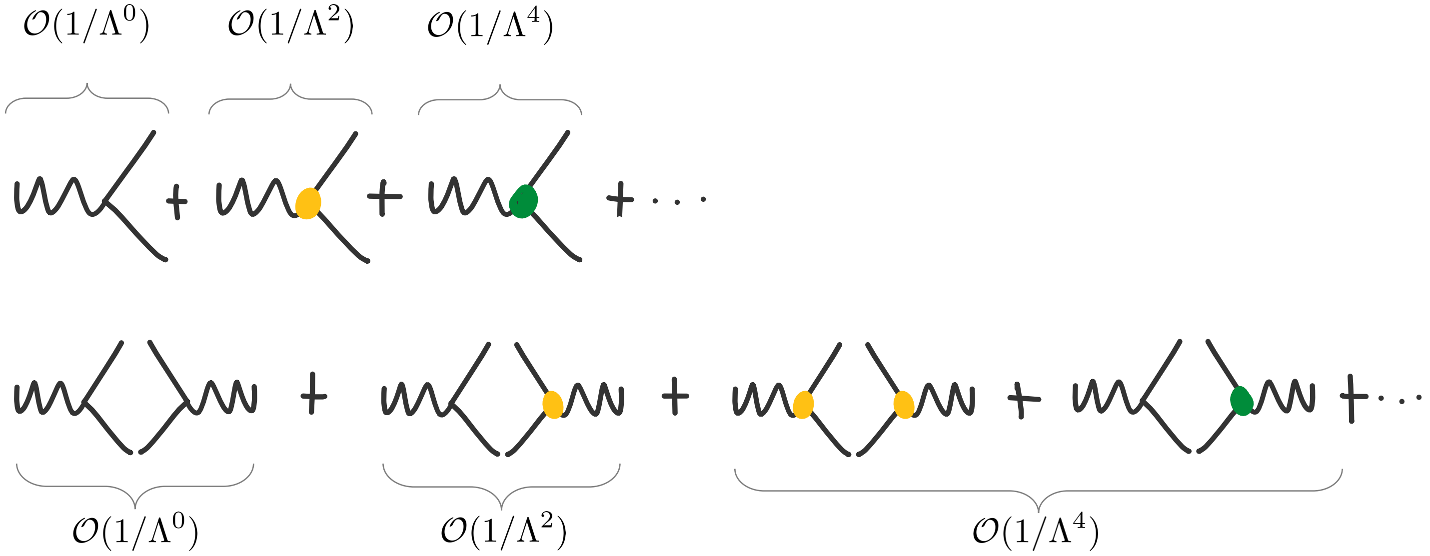
$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \frac{C_i}{\Lambda^2} Q_i$$

Fields and parameters

Interactions

Probability amplitudes

Physical observables



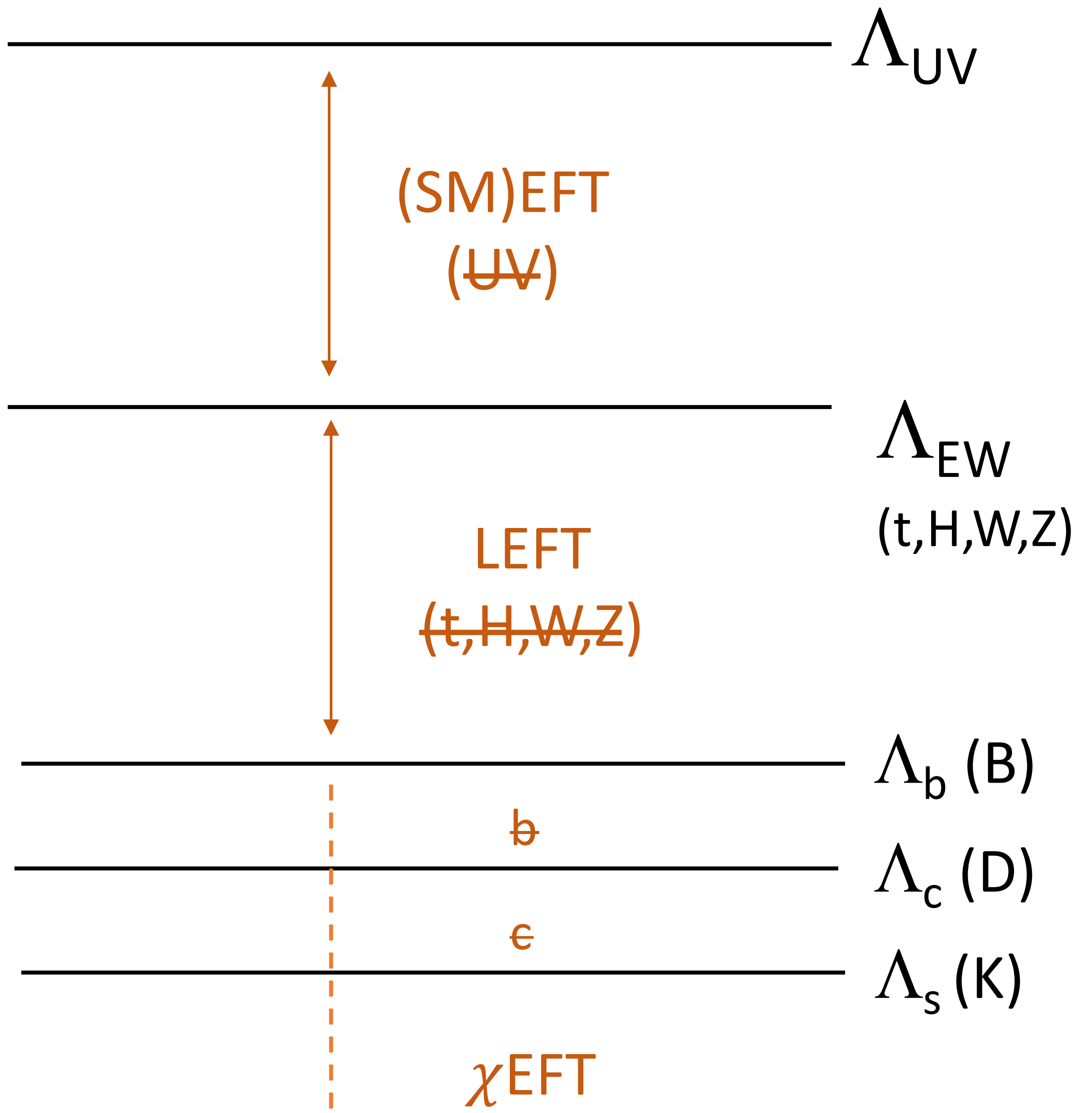
$$O_{SMEFT} = O_{SM} + \Delta O^{(1)} + \Delta O^{(2)} + \dots$$

Constraining the SMEFT

- **Bottom-up**: Global fits of collider observables (EW, Higgs, top), flavor observables, low energy observables.
- **Top-down**: Matching to UV models.

The full picture

Connecting far apart scales (from BSM to flavor) naturally lends itself to the EFT framework



Heavy physics decouples and leaves effective contact interactions of dim > 4

RGE

EFT operators in terms of SM fields

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{i,d} \frac{C_{i,d}^{\rm SMEFT}}{\Lambda^2} \mathcal{O}_{i,d}^{\rm SMEFT}$$

RGE

WC depend on m_t, M_W, M_Z, M_H, …M_X

$$\mathcal{L}_{\rm LEFT} = \mathcal{L}_{\rm QCD+QED} + \sum_{i,d} \frac{C_{i,d}^{\rm LEFT}}{v^2} \mathcal{O}_{i,d}^{\rm LEFT}$$

$$\mu \frac{dC_i}{d\mu} = \gamma_{ij} C_j \longrightarrow C_i(\mu) = U(\mu, \mu_0)_{ij} C_j(\mu_0)$$

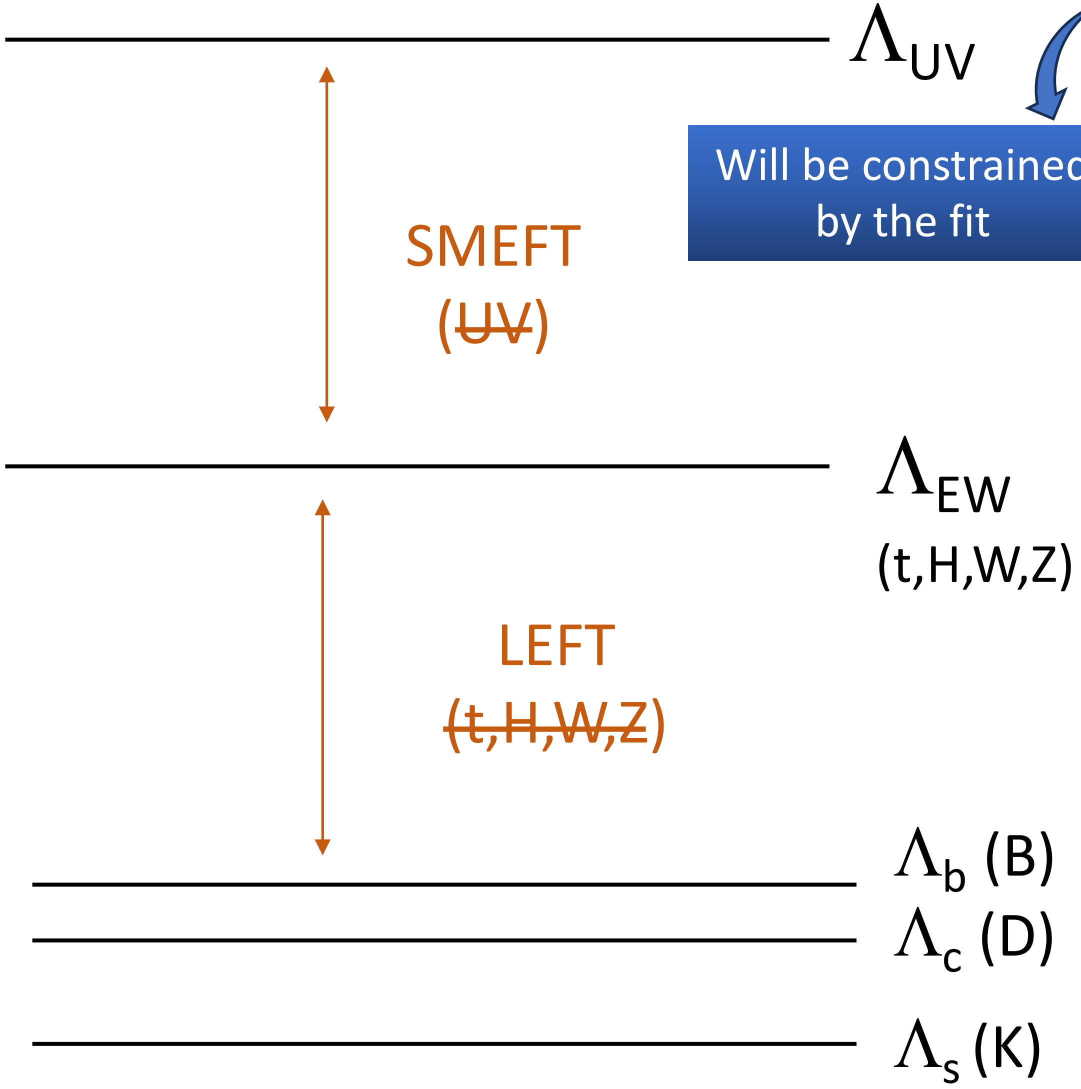
(γ_{ij}) of the corresponding (effective) theory

Global fits of the SMEFT

- **Bottom-up approach**: based on symmetry assumptions used in $\mathcal{L}_{SMEFT}$.
- Effects of new physics can then be constrained using the **broad spectrum of precision measurement available from EW, Higgs, top, flavor physics** and more.
- With **increasing precision** in both theory and experiments, constraints **could start to show intriguing patterns and guide future explorations.**

Bottom-up: global fits of the SMEFT

Connecting far apart scales naturally lends itself to the EFT framework



Matchmakereft, 2112.10787
MATCH2FIT, 2309.04523

$C_{i,d}^{SMEFT}(\Lambda_{UV})$ (from matching to UV theory)

Will be constrained by the fit

Evolved to $C_{i,d}^{SMEFT}(\Lambda_{EW})$ using **RGEsolver++**

Di Noi and Silvestrini, 2210.06838

Based on 1-loop SMEFT anomalous dimension

Jenkins, Manohar, and Trott, 1308.2627, 1310.4838,1312.2014

All fit observables are calculated in terms of $C_{i,d}^{SMEFT}(\Lambda_{EW})$

Match to LEFT operators to calculate flavor observables

Jenkins, Manohar, Stoffer, 1709.04486, 1711.05270

Notice that the LO (1-loop) evolution requires tree-level initial conditions at Λ_{UV} and matrix elements at Λ_{EW} , while 1-loop initial conditions and matrix elements require NLO (2-loop) evolution.

Need a framework

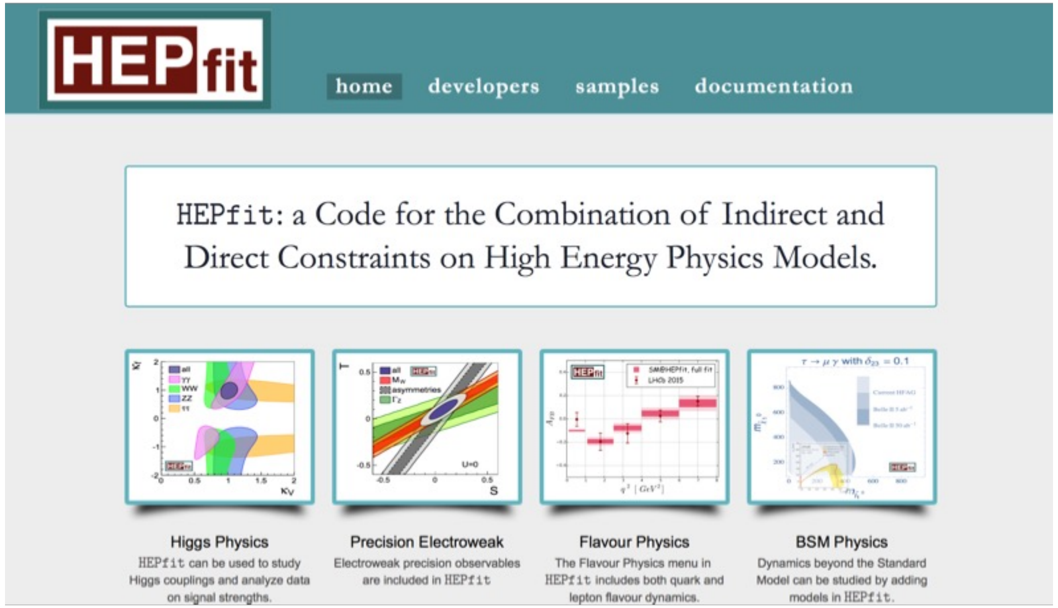
Statistical framework based on a Bayesian MCMC analysis as implemented in
BAT (Bayesian Analysis Toolkit)
Caldwell et al., arXiv:0808.2552

Supports SM (fully implemented) and BSM models, in particular the dim-6 SMEFT

Used for several global fit and future collider projections

New release will include EW, Higgs, top, and flavor observables in the SM and the SMEFT with

- ❑ SM predictions at NLO or higher
- ❑ SMEFT at tree level (dim-6 operators only)
- ❑ RGE running of the SMEFT Wilson coefficients
- ❑ Linear and quadratic effects from dim-6 operators



http://hepfit.roma1.infn.it

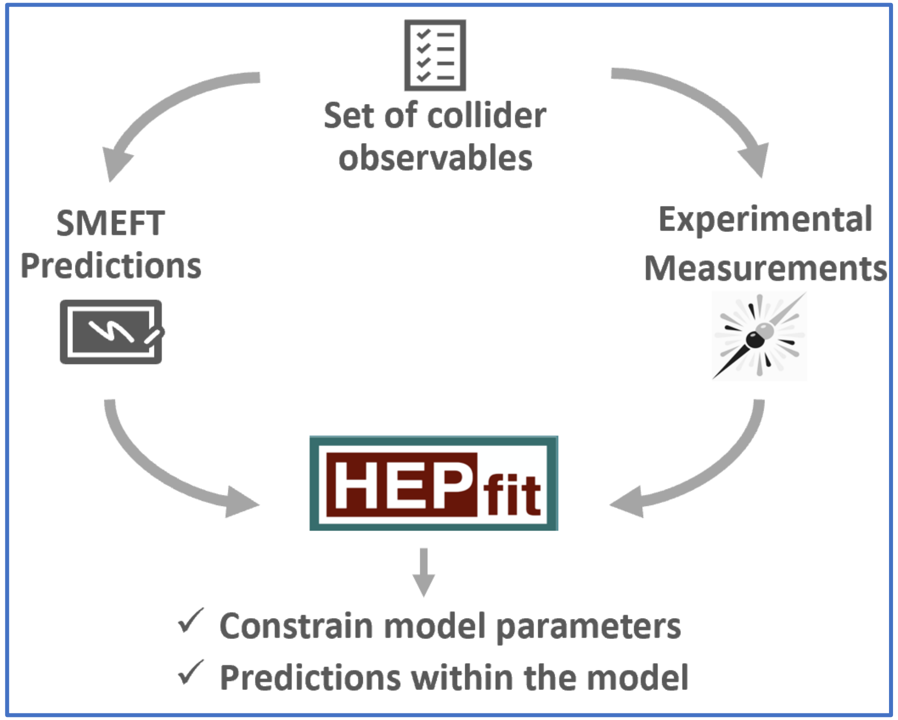
J. De Blas et al., 1910.14012

Other existing frameworks for SMEFT global fits:
SMEFiT, Celada et al. 2105.00006, 2302.06660, 2404.12809
Fitmaker, Ellis et al. 2012.02779
Allwicher et al, 2311.00020
Cirigliano et al. 2311.00021
Bartocci et al. 2311.04963

Fit EW, Higgs, top, DY, di-boson, flavor observables

Constraining new physics through the spectrum of LHC measurements and beyond

- EW precision observables
  - Z-pole observables (LEP I, LEP II, SLD)
  - M_W, Γ_W (Tevatron, LHC)
- Higgs boson observables
  - Signal strengths.
  - Simplified Template Cross Sections (STXS)
- Top quark observables
  - $pp \to t\bar{t}, t\bar{t}Z, t\bar{t}W, t\bar{t}\gamma, tZq, t\gamma q, tW, \ldots$
- Drell-Yan, Di-boson measurements
  - $pp \to W, Z \to f_i\bar{f}_j$
  - $pp \to WZ, WW, ZZ, Z\gamma$
- Flavor observables
  - ΔF=2: $\Delta M_{B_{d,s}}$, $D^0 - \bar{D}^0$, ε_K
  - Leptonic decays: $B_{d,s} \to \mu^+\mu^-$, $B \to \tau\nu$, $D \to \tau\nu$, $K \to \mu\nu$, $\pi \to \mu\nu$
  - Semi-leptonic decays: $B \to D^{(*)}l\nu$, $K \to \pi\nu\bar{\nu}$, $B \to K\nu\bar{\nu}$, $B, K \to \pi l \nu$
  - Radiative B decays ($B \to X_{s,d}\gamma$)



SMEFT predictions

A given observable will be written as

$$O_{\text{SMEFT}} = O_{\text{SM}} + \Delta O^{(1)} + \Delta O^{(2)} + \cdots$$

SM: including SM higher-order corrections

SMEFT: tree level

Observables have been calculated either analytically and via parametrizations obtained using various tools (MG5_aMC@NLO with **SMEFTci2**, a new UFO file developed for this study, Feynart+Feyncalc for loop-induced Higgs decays, …)

Including direct and indirect SMEFT effects from dim-6 operators up to O(1/Λ^4) [by A. Goncalves]

See also, SmeftFR-v3, Dedes et al. 2302.01353

Example 1: EW precision observables

- Z-pole observables, W observables
- Fully analytic expressions

$$\Gamma_{Z,f} = N_f \frac{G_F M_Z^3}{24\sqrt{2}\pi} 4\left[(g_{V,f})^2 + (g_{A,f})^2\right]$$

$$R_e^0 = \frac{\Gamma_{had}}{\Gamma_e} \qquad R_{q,\nu}^0 = \frac{\Gamma_{q,\nu}}{\Gamma_{had}}$$

$$\sigma_{had}^0 = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_{had}}{\Gamma_Z^2}$$

$$A_f = \frac{2\left(\frac{g_{V,f}}{g_{A,f}}\right)}{1+\left(\frac{g_{V,f}}{g_{A,f}}\right)^2} \qquad A_{FB,f} = \frac{3}{4}A_e A_f$$

Z

$$sin^2\theta_{eff,l} = \frac{1}{4}\left(1 - \frac{g_{V,l}}{g_{A,l}}\right)$$

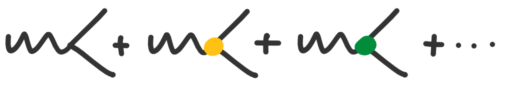
$W^{\pm}$ $\quad M_W \quad \Gamma_{(W\to f_i f_j)} \quad BrW_{f_i f_j}$

$$-\frac{1}{4}G^A_{\mu\nu}G^{A,\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$
$$+ (D_\mu\varphi)^\dagger(D^\mu\varphi) + m^2\varphi^\dagger\varphi - \frac{1}{2}\lambda(\varphi^\dagger\varphi)^2$$
$$+ i\left(\bar{l}'_L \not{D} l'_L + \bar{e}'_R \not{D} e'_R + \bar{q}'_L \not{D} q'_L + \bar{d}'_R D d'_R\right)$$
$$- \left(\bar{l}'_L \Gamma_e e'_R \varphi + \bar{q}'_L \Gamma_u u'_R \tilde{\varphi} + \bar{q}'_L \Gamma_d d'_R \varphi\right) + h.c.$$

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \sum_i \frac{C_i}{\Lambda^2} Q_i + \ldots$$

$$O_{SMEFT} = O_{SM} + \Delta O^{(1)} + \Delta O^{(2)} + \ldots$$

$$g_{V,f} = g_{V,f}^{SM} + \Delta g_{V,f}^{(1)} + \Delta g_{V,f}^{(2)} + \ldots$$
$$g_{A,f} = g_{A,f}^{SM} + \Delta g_{A,f}^{(1)} + \Delta g_{A,f}^{(2)} + \ldots$$



$\psi^2\varphi^2 D$	
$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}'_p \gamma^\mu l'_r)$
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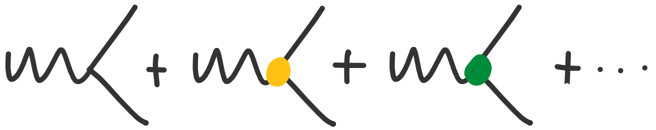
- Z-pole observables: effective couplings



$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \sum_i \frac{C_i}{\Lambda^2} Q_i + \dots$$

$$-\frac{1}{4}G^A_{\mu\nu}G^{A,\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$
$$+ (D_\mu\varphi)^\dagger(D^\mu\varphi) + m^2\varphi^\dagger\varphi - \frac{1}{2}\lambda(\varphi^\dagger\varphi)^2$$
$$+ i\left(\bar{l}'_L \not{D} l'_L + \bar{e}'_R \not{D} e'_R + \bar{q}'_L \not{D} q'_L + \bar{d}'_R \not{D} d'_R\right)$$
$$- \left(\bar{l}'_L \Gamma_e e'_R \varphi + \bar{q}'_L \Gamma_u u'_R \tilde{\varphi} + \bar{q}'_L \Gamma_d d'_R \varphi\right) + h.c.$$

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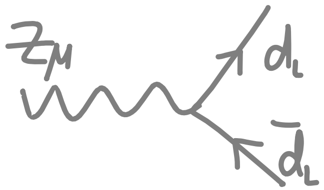
$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}'_p \gamma^\mu d'_r)$

$$\supset \; -\frac{2\widetilde{M}_Z}{\tilde{v}}\left(\sum_f g_{L,f} P_L \gamma^\mu (Z_\mu \bar{f} f) + \sum_f g_{R,f} P_R \gamma^\mu (Z_\mu \bar{f} f)\right)$$

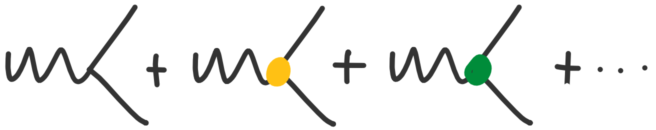
e.g.



$$i\gamma^\mu P_L \Bigg[\frac{3\bar{g}_W^2 + \bar{g}_1^2}{6\sqrt{\bar{g}_1^2 + \bar{g}_W^2}}$$
$$+ \frac{\bar{g}_1 \bar{g}_W (3\bar{g}_1^2 + \bar{g}_W^2)}{6(\bar{g}_1^2 + \bar{g}_W^2)^{3/2}} \hat{C}_{\varphi WB} \bar{v}^2 + \frac{\sqrt{\bar{g}_1^2 + \bar{g}_W^2}}{2}\left(\hat{C}_{\varphi q}^{(1)} + \hat{C}_{\varphi q}^{(3)}\right)\bar{v}^2$$
$$+ \frac{\bar{g}_1 \bar{g}_W (3\bar{g}_1^2 + \bar{g}_W^2)}{6(\bar{g}_1^2 + \bar{g}_W^2)^{3/2}} \hat{C}_{\varphi WB}(\hat{C}_{\varphi W} + \hat{C}_{\varphi B})\bar{v}^4 + \frac{(\bar{g}_1^6 + 8\bar{g}_1^2\bar{g}_W^4 + 3\bar{g}_W^6)}{12(\bar{g}_1^2 + \bar{g}_W^2)^{5/2}} \hat{C}_{\varphi WB}^2 \bar{v}^4 + \frac{\bar{g}_1 \bar{g}_W}{2\sqrt{\bar{g}_1^2 + \bar{g}_W^2}} \hat{C}_{\varphi WB}\left(\hat{C}_{\varphi q}^{(1)} + \hat{C}_{\varphi q}^{(3)}\right)\bar{v}^4 \Bigg]$$

$\{\bar{g}_W, \bar{g}_1, \bar{v}, \lambda\}$ → (tree-level relations) → $\{\bar{\alpha}, \bar{M}_Z, \bar{G}_F, \bar{M}_h\}$ → ($\widetilde{p} = \bar{p}(1 + \delta_p^{(1)} + \delta_p^{(2)} + \dots)$) → $\{\widetilde{\alpha}, \widetilde{M}_Z, \widetilde{G}_F, \widetilde{M}_h\}$

- Z-pole observables: effective couplings



$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \sum_i \frac{C_i}{\Lambda^2} Q_i + \dots$$

$$-\frac{1}{4}G^A_{\mu\nu}G^{A,\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$
$$+ (D_\mu\varphi)^\dagger(D^\mu\varphi) + m^2\varphi^\dagger\varphi - \frac{1}{2}\lambda(\varphi^\dagger\varphi)^2$$
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$$- \left(\bar{l}'_L \Gamma_e e'_R \varphi + \bar{q}'_L \Gamma_u u'_R \tilde{\varphi} + \bar{q}'_L \Gamma_d d'_R \varphi\right) + h.c.$$

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$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}'_p \gamma^\mu l'_r)$
$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{l}'_p \tau^I \gamma^\mu l'_r)$
$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}'_p \gamma^\mu e'_r)$
$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}'_p \gamma^\mu q'_r)$
$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{q}'_p \tau^I \gamma^\mu q'_r)$
$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}'_p \gamma^\mu u'_r)$
$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}'_p \gamma^\mu d'_r)$
$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}'_p \gamma^\mu d'_r)$

$$\supset -\frac{2\widetilde{M}_Z}{\tilde{v}}\left(\sum_f g_{L,f} P_L \gamma^\mu (Z_\mu \bar{f} f) + \sum_f g_{R,f} P_R \gamma^\mu (Z_\mu \bar{f} f)\right)$$

$$g_{R,f} = g^{SM}_{R,f} + \Delta g^{(1)}_{R,f} + \Delta g^{(2)}_{R,f} + \dots$$
$$g_{L,f} = g^{SM}_{L,f} + \Delta g^{(1)}_{L,f} + \Delta g^{(2)}_{L,f} + \dots$$

Left & Right couplings

$$g_V = g_L + g_R$$
$$g_A = g_L - g_R$$

$$g_{V,f} = g^{SM}_{V,f} + \Delta g^{(1)}_{V,f} + \Delta g^{(2)}_{V,f} + \dots$$
$$g_{A,f} = g^{SM}_{A,f} + \Delta g^{(1)}_{A,f} + \Delta g^{(2)}_{A,f} + \dots$$

Vector & Vector-Axial couplings

- Z-pole observables: EFT expansion

For example,

$$sin^2\theta_{eff} = \frac{1}{4}\left(1 - \frac{g_V}{g_A}\right) \xrightarrow[g_A = g_A^{SM} + \Delta g_A^{(1)} + \Delta g_A^{(2)} + \dots]{g_V = g_V^{SM} + \Delta g_V^{(1)} + \Delta g_V^{(2)} + \dots} sin^2\theta_{eff} = sin^2\theta_{eff}^{SM} + \Delta sin^2\theta_{eff}^{(1)} + \Delta sin^2\theta_{eff}^{(2)} + \dots$$

$$\Delta sin^2\theta_{eff}^{(1)}: \quad -\frac{1}{4}\frac{g_A^{SM}\ \Delta g_V^{(1)} - g_V^{SM}\ \Delta g_A^{(1)}}{(g_A^{SM})^2}$$

$$\Delta sin^2\theta_{eff}^{(2)}: \quad \frac{1}{4}\frac{\Delta g_V^{(1)}\ \Delta g_A^{(1)}}{(g_A^{SM})^2} - \frac{1}{4}\frac{g_V^{SM}\ (\Delta g_A^{(1)})^2}{(g_A^{SM})^3} - \frac{1}{4}\frac{g_A^{SM}\ \Delta g_V^{(2)} - g_V^{SM}\ \Delta g_A^{(2)}}{(g_A^{SM})^2}$$

And most generally,

$$O_{SMEFT} = O_{SM} + \Delta O^{(1)} + \Delta O^{(2)} + \dots$$

- W-pole observables:

$$\Gamma_{(W\to f_i f_j)} \quad \left\{ \quad BrW_{f_i f_j} = \frac{\Gamma_{(W\to f_i f_j)}}{\Gamma_W} \right.$$

$$M_W$$

→ similar description as for the Z-observables

EW observables: adding quadratic terms

Typical effect: lifting degeneracies among contributing coefficients

Observable	$C_{\varphi D}$	$C_{\varphi WB}$	$C_{\varphi L}^{(3)}$	C_{LL}	$C_{\varphi L}^{(1)}$	$C_{\varphi e}$	$C_{\varphi Q}^{(1)}$	$C_{\varphi Q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$	$C_{\varphi B}$	$C_{\varphi W}$	$C_{\varphi ud}$
A_l, A_{FB}^l, P_τ^{pol}, $\sin\theta_{eff,l}^2$	✓	✓	✓	✓	✓	✓					✓	✓	
A_c, R_c^0	✓	✓	✓	✓			✓	✓	✓		✓	✓	
A_b, A_s, R_b^0	✓	✓	✓	✓			✓	✓		✓	✓	✓	
A_{FB}^c	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	
A_{FB}^b	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	
R_l^0, Γ_Z, σ_{had}^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
M_W	✓	✓	✓	✓									
Γ_W, BrW	✓	✓	✓	✓	✓		✓			✓	✓	✓	✓

O(1/Λ⁴) : degeneracy is (analytically) lifted

O(1/Λ²) : Constrain 8 independent relations

$$\hat{\mathbf{C}}_{\varphi\mathbf{L}}^{(\mathbf{3})} = \hat{C}_{\varphi L}^{(3)} + \frac{1}{4}\frac{\widetilde{c_W}^2}{\widetilde{s_W}^2}\hat{C}_{\varphi D} + \frac{\widetilde{c_W}}{\widetilde{s_W}}\hat{C}_{\varphi WB}$$

$$\hat{\mathbf{C}}_{\varphi\mathbf{Q}}^{(\mathbf{3})} = \hat{C}_{\varphi Q}^{(3)} + \frac{1}{4}\frac{\widetilde{c_W}^2}{\widetilde{s_W}^2}\hat{C}_{\varphi D} + \frac{\widetilde{c_W}}{\widetilde{s_W}}\hat{C}_{\varphi WB}$$

$$\hat{\mathbf{C}}_{\varphi\mathbf{L}}^{(\mathbf{1})} = \hat{C}_{\varphi L}^{(1)} + \frac{1}{4}\hat{C}_{\varphi D}$$

$$\hat{\mathbf{C}}_{\varphi\mathbf{Q}}^{(\mathbf{1})} = \hat{C}_{\varphi Q}^{(1)} - \frac{1}{12}\hat{C}_{\varphi D}$$

$$\hat{\mathbf{C}}_{\varphi\mathbf{e}} = \hat{C}_{\varphi e} + \frac{1}{2}\hat{C}_{\varphi D}$$

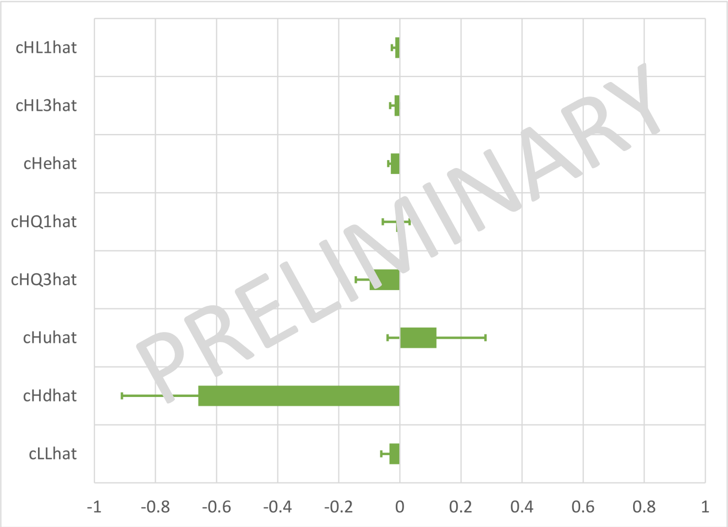
$$\hat{\mathbf{C}}_{\varphi\mathbf{u}} = \hat{C}_{\varphi e} - \frac{1}{3}\hat{C}_{\varphi D}$$

$$\hat{\mathbf{C}}_{\mathbf{LL}} = \hat{C}_{LL}$$

EW observables: adding quadratic terms

- **Preliminary Global Fit of EW observables at quadratic order in the d=6 SMEFT:**

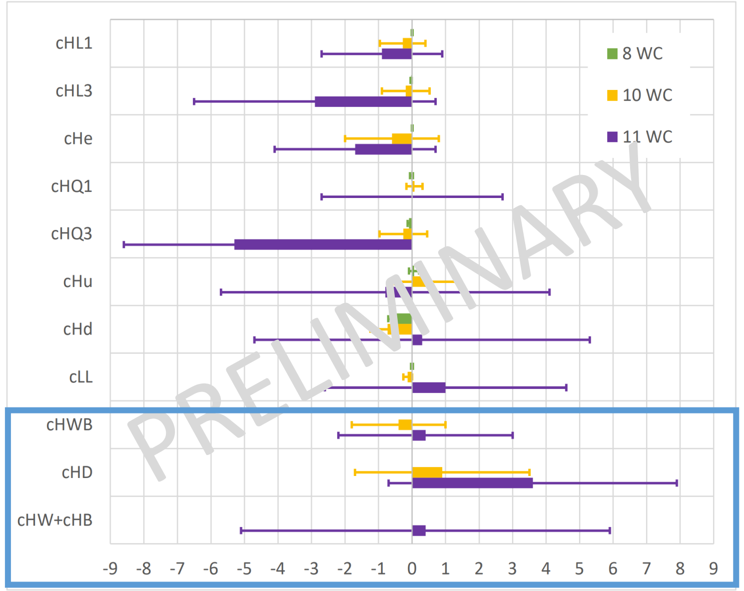
$O(1/\Lambda^2)$



Fit parameters	Analytically	Numerically
≤8	✓	✓
>8	✗	✗

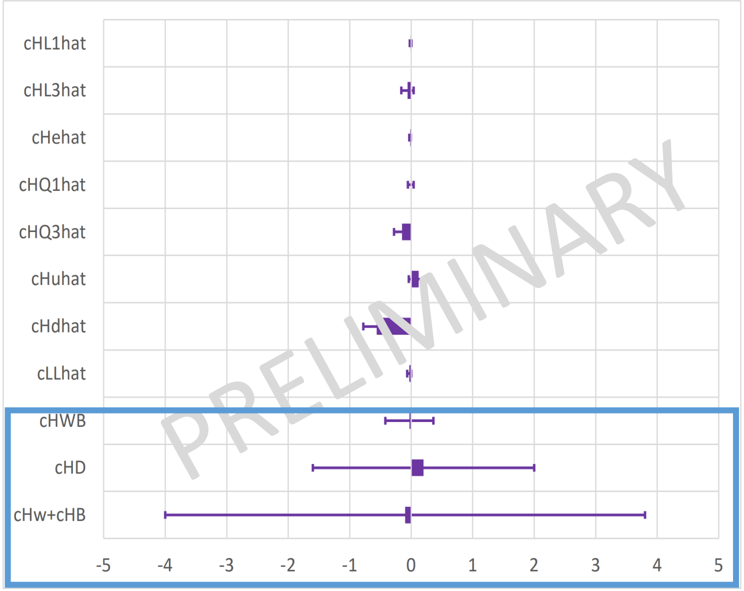
Flat distributions
full correlations

$O(1/\Lambda^4)$ original-representation



Fit parameters	Analytically	Numerically
≤8	✓	✓
>8	✓	✗

$O(1/\Lambda^4)$ hat-representation

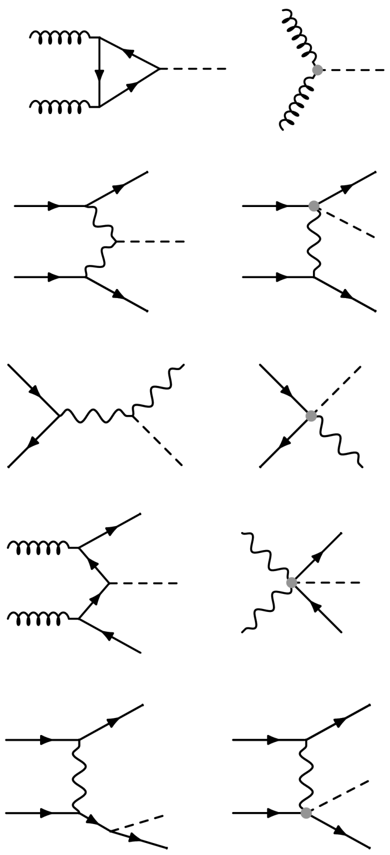


Fit parameters	Analytically	Numerically
≤8	✓	✓
>8	✓	✓

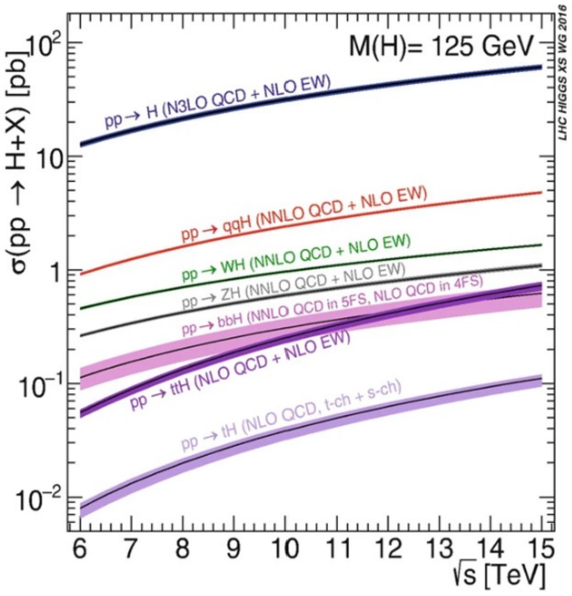
improve sensitivity
for {CHWB, CHD, CHW+CHB}

Example 2: Higgs observables

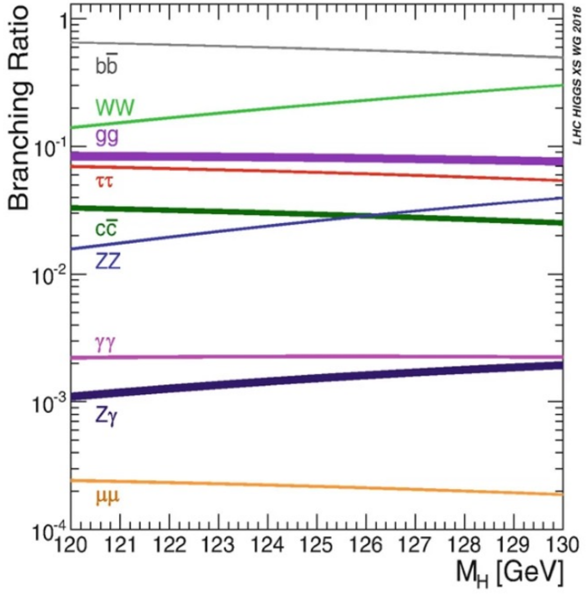
- Higgs-boson production cross-sections and branching ratios :

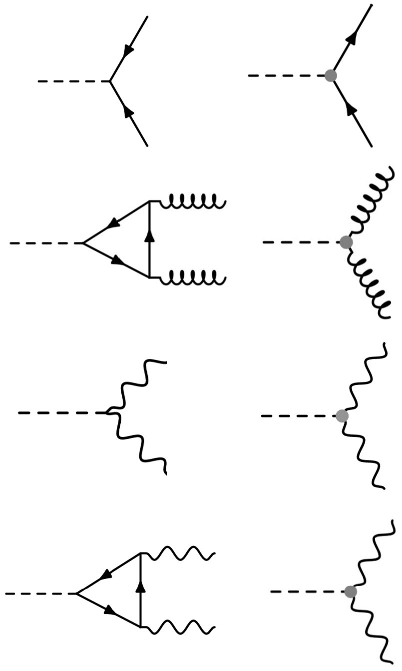


$$ggH,\ VBF,\ WH,\ ZH, t\bar{t}H, tH$$



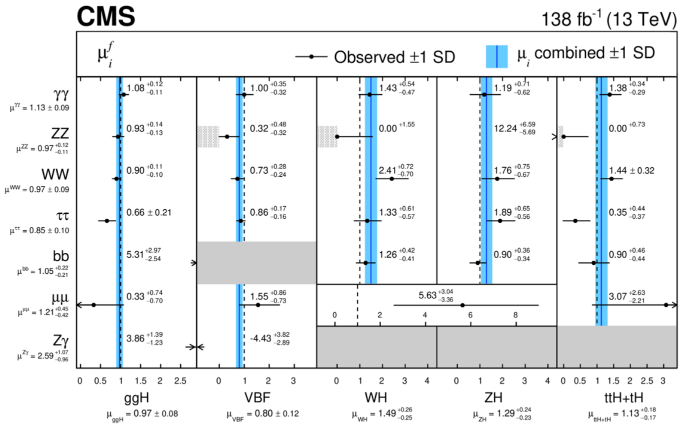
$$H \to \{\bar{f}f,\ gg,\ WW^*,\ ZZ^*,\ \gamma\gamma,\ Z\gamma,\}$$

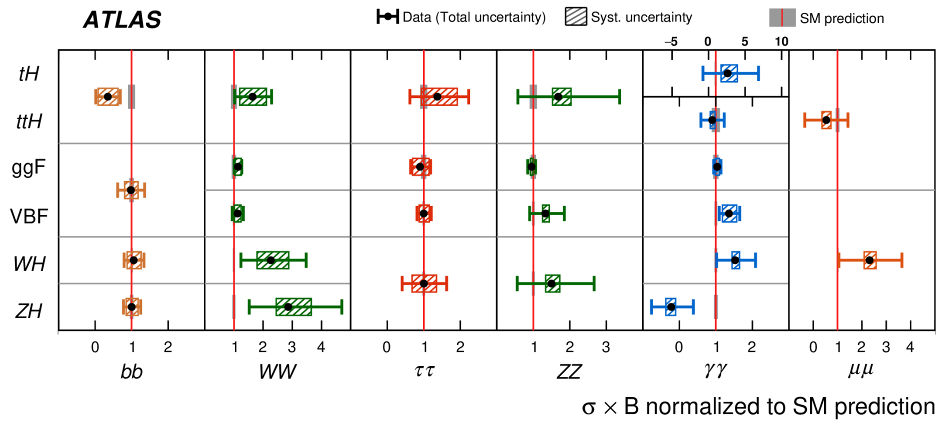


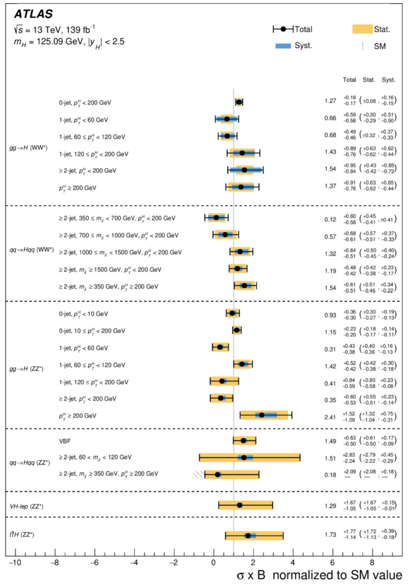


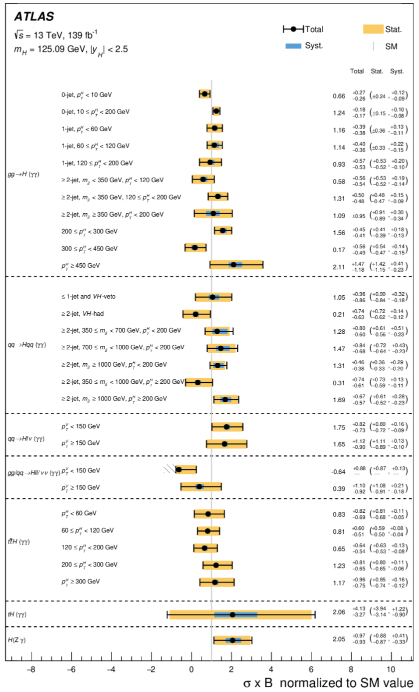
Higgs-boson Observables: exp. measurements

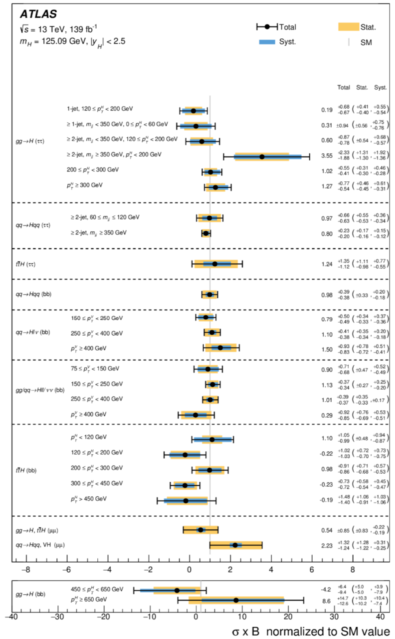
- **Higgs-boson inclusive and fiducial $\mu' s$ measured by ATLAS and CMS:**







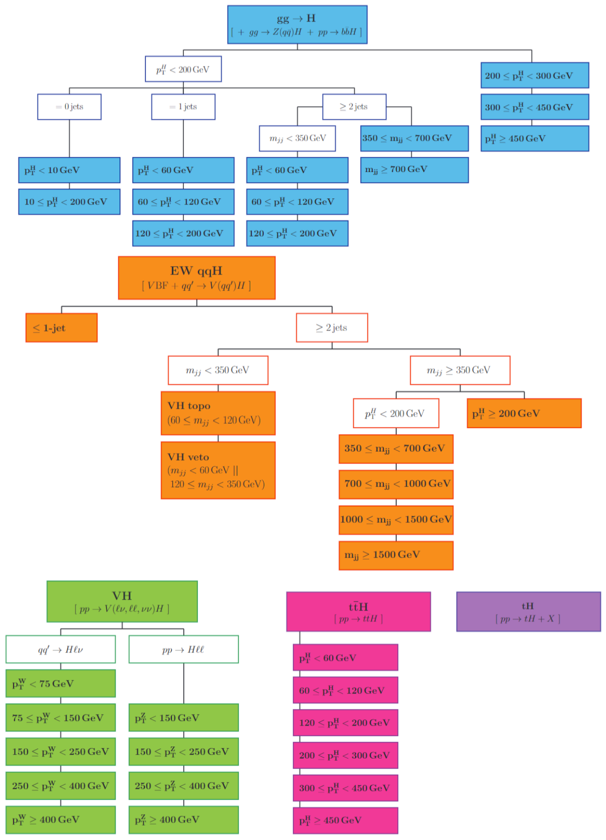




SMEFT Predictions: Higgs-boson Observables

- Higgs-boson production cross-sections and branching ratios
- **Signal strength modifiers:**
  - **Production cross-sections as inclusive or fiducial observables through *Simplified Template Cross-Sections* (STXS)**
  - **SMEFT predictions obtained differently depending on their complexity: Analytic vs. Numeric computations with *Madgraph***

[J. Alwall, et al, arXiv:1405.0301]



$$\mu_{ij} = \frac{\sigma_i \times Br_j}{\sigma_i^{SM} \times Br_j^{SM}}$$

With SMEFT expansion:

$$\mu_{ij} = 1 + \left(\frac{\Delta\sigma_i^{(1)}}{\sigma_i^{SM}} + \frac{\Delta Br_j^{(1)}}{Br_j^{SM}}\right) + \left(\frac{\Delta\sigma_i^{(2)}}{\sigma^{SM}} + \frac{\Delta Br_j^{(2)}}{Br^{SM}} + \frac{\Delta\sigma_i^{(1)}\Delta Br_j^{(1)}}{\sigma_i^{SM} Br_j^{SM}}\right) + \ldots$$

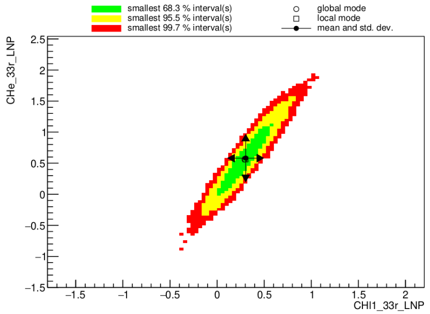
$$Br_j^{SM} = \frac{\Gamma_j^{SM}}{\Gamma_H^{SM}} \qquad \Delta Br_j^{(1)} = \frac{\Delta\Gamma_j^{(1)}}{\Gamma_j^{SM}} - \frac{\Delta\Gamma_H^{(1)}}{\Gamma_H^{SM}} \qquad \Delta Br_j^{(2)} = \frac{\Delta\Gamma_j^{(2)}}{\Gamma_j^{SM}} - \frac{\Delta\Gamma_H^{(2)}}{\Gamma_H^{SM}} - \frac{\Delta\Gamma_j^{(1)}}{\Gamma_j^{SM}}\frac{\Delta\Gamma_H^{(1)}}{\Gamma_H^{SM}} + \left(\frac{\Delta\Gamma_H^{(1)}}{\Gamma_H^{SM}}\right)^2$$

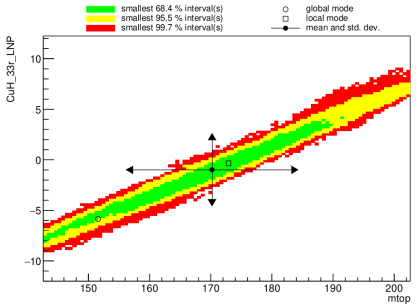
"Building blocks"

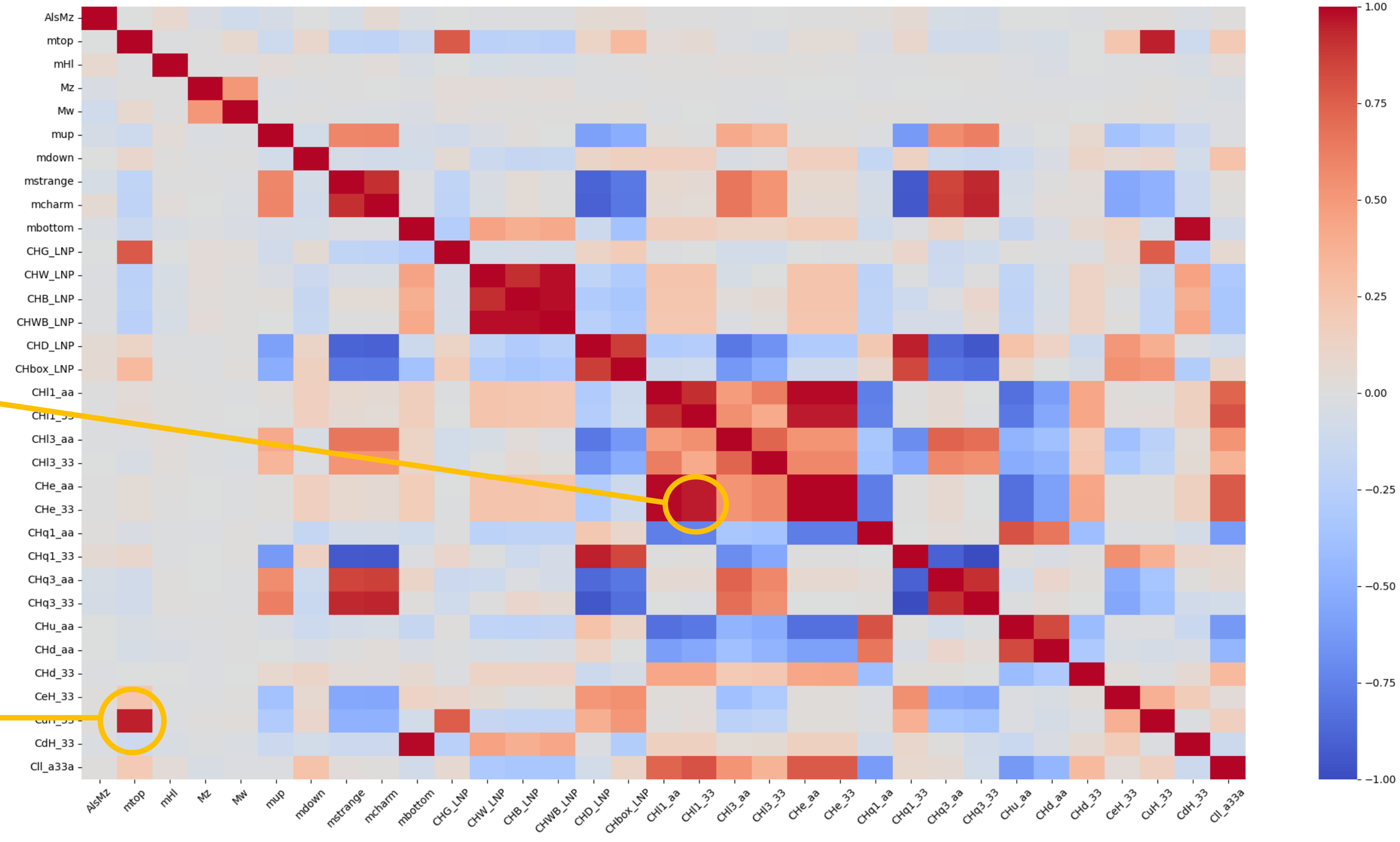
$$\frac{\Delta\sigma_i^{(1,2)}}{\sigma_i^{SM}} \quad \frac{\Delta\Gamma_j^{(1,2)}}{\Gamma_j^{SM}}$$

Global fit: Higgs-boson Observables

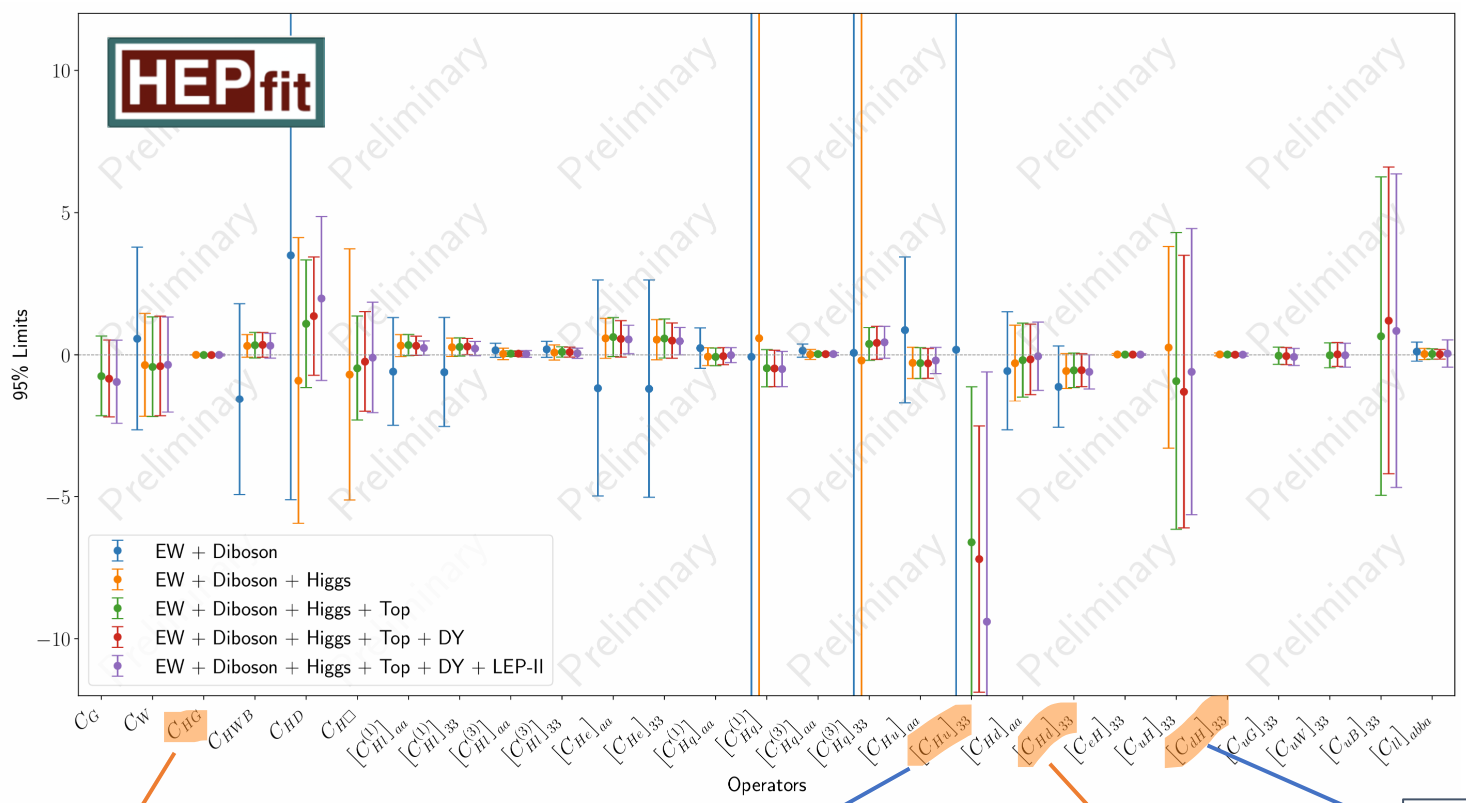
- Higgs-boson inclusive and fiducial $\mu' s$ measured by ATLAS and CMS
- STXS improve constraining power
- $C_W, C^{(1)}_{Hq,33}, C^{(3)}_{Hq,33}, C_{Hu,33}, C_{uW,33}, C_{HB,33}$ unconstrained by EWPO+Higgs
- **Correlation on WCs:**







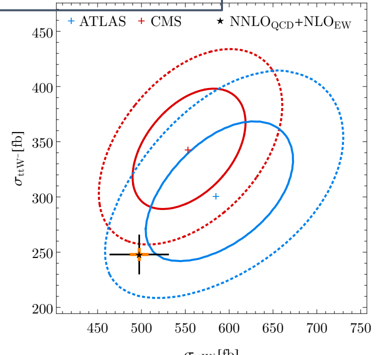
Global fit EW + Higgs + Top + …



- Increasing constraining power when adding classes of observables
- Increased correlation among WC
- RGE evolution increases relations among WCs

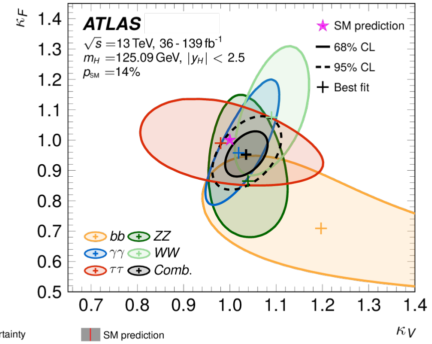
Highly constrained from ggH
RGE effects visible

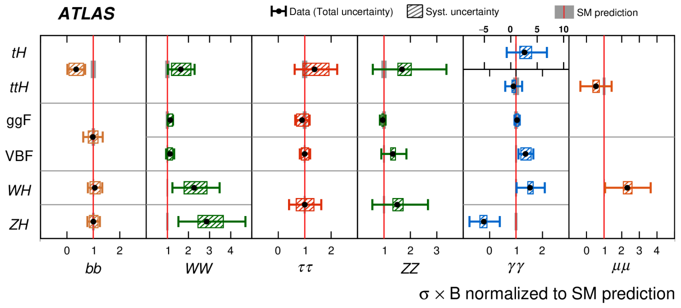
Effect of Vtt (V=Z,W,γ)



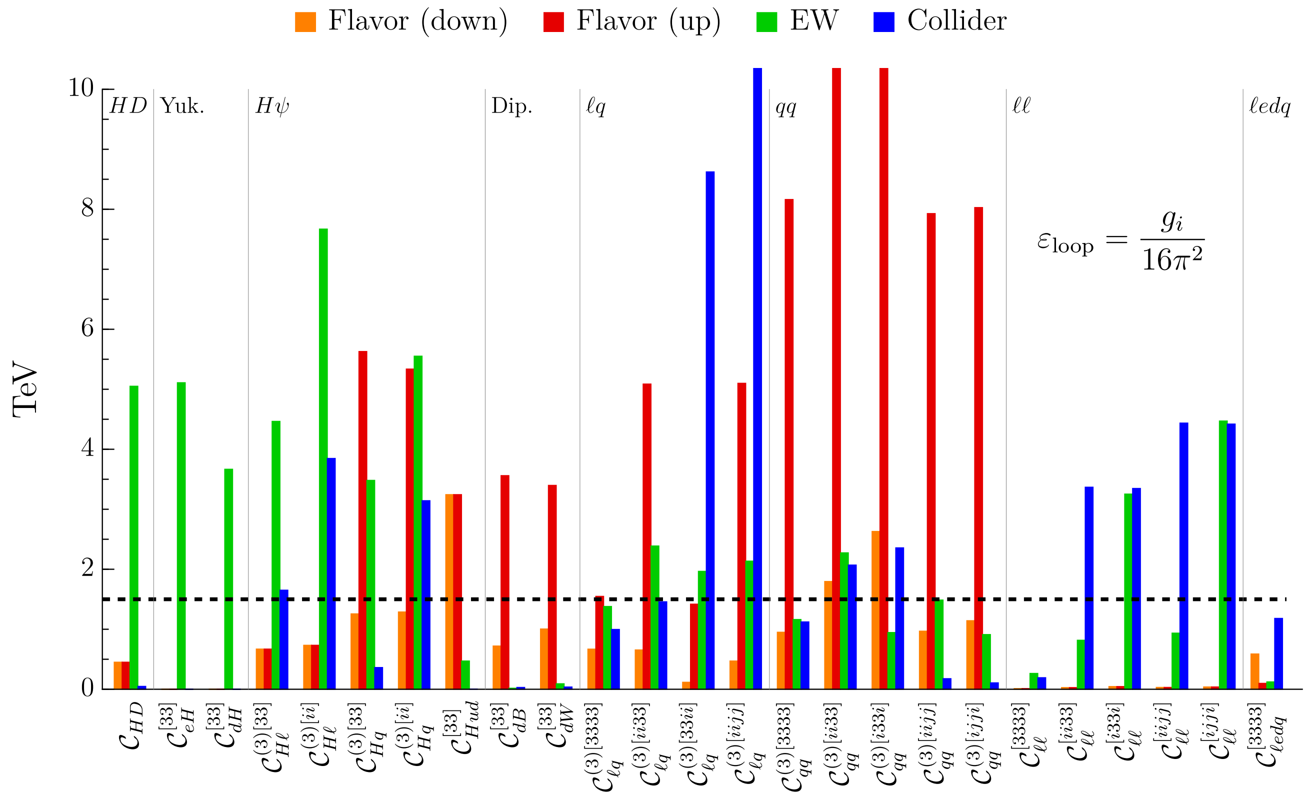
Driven by EW

Effect on H to bb



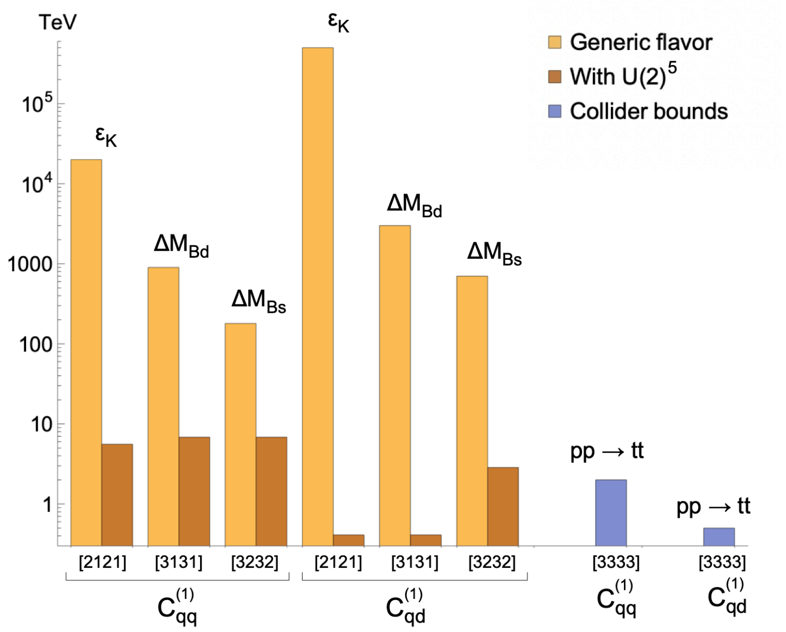


Adding flavor observables



[Allwicher et al., arXiv:2311.00020]

Impact on flavor assumption (see discussion of approximate symmetries in Lecture 2)



[Isidori and Wyler, arXiv:2303.16922]

Matching to UV models

- **Top-down**: quite powerful if guided by specific anomalies.
- Examples: $(g-2)_\mu$, $\mu \to e\gamma$, flavor anomalies

A model with leptoquarks

Sharpen the relation between low energy measurements and UV theories

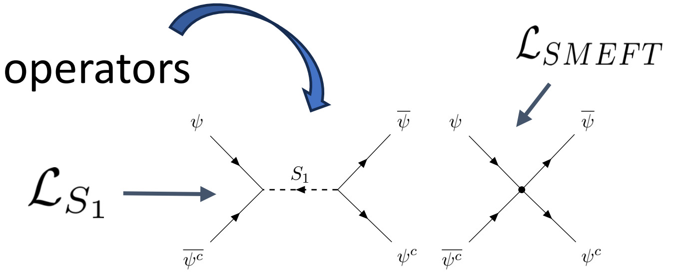
From Isidori and Wyler
arXiv:2303.16922

SM extension by a heavy colored scalar S_1 (leptoquark)

$$\mathcal{L}_{S_1} = \mathcal{L}_{SM} + (D_\mu S_1)^\dagger (D^\mu S_1) - M_S^2 S_1^+ S_1 - [\lambda^L_{pr}(\bar{q}^c_p \epsilon \ell_r) S_1 + \lambda^R_{pr}(\bar{u}^c_p e_r) S_1 + h.c.]$$

The **tree-level matching** projects on 4-fermion SMEFT operators

$$Q^{(1,3)}_{lq}, Q_{eu}, Q^{(1,3)}_{lequ} \longrightarrow C_i = C_i(\lambda^{L,R}_{pr})$$



The **one-loop matching** projects on dipole operators (among others)

$$[Q_{eB}]_{pr} = (\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu} \text{ and } [Q_{eW}]_{pr} = (\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W^I_{\mu\nu}$$

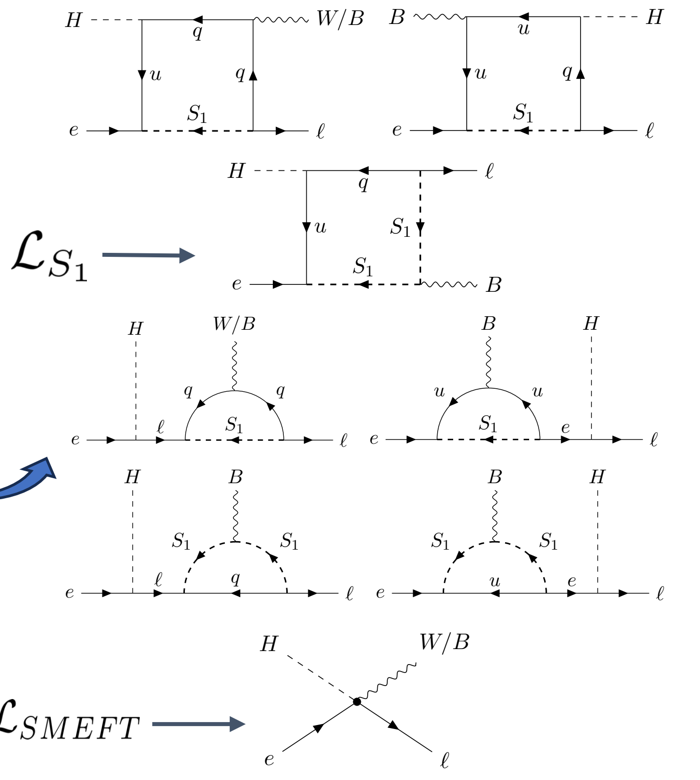
Which can be related to the photon dipole upon SSB

$$[Q_{e\gamma}]_{pr} = \frac{v}{\sqrt{2}} \bar{e}^L_p \sigma^{\mu\nu} e^R_r F_{\mu\nu}$$

$$\begin{pmatrix} [\mathcal{C}_{e\gamma}]_{pr} \\ [\mathcal{C}_{eZ}]_{pr} \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ -s_\theta & -c_\theta \end{pmatrix} \begin{pmatrix} [C_{eB}]_{pr} \\ [C_{eW}]_{pr} \end{pmatrix}$$

$\Rightarrow C_{e\gamma}(\lambda^{L,R}_{pr}, \ln(\mu_m^2/M_S^2))$

μ_m matching scale



A model with leptoquarks – cont'd

From **RGE evolution in the SMEFT** : $\boldsymbol{\mu_m \to \mu_W}$

$$[C_X]_{pr}(\mu_l) = [C_X]_{pr}(\mu_m) + \frac{1}{16\pi^2}\ln\left(\frac{\mu_l}{\mu_m}\right)[\beta_X]_{pr} \quad \to [C_{e\gamma}]_{pr}(\mu_W)$$

Notice: hidden in the RGE of $C_{e\gamma}$ is a strong dependence on the top Yukawa coupling y_t

From **RGE in the LEFT:** $\boldsymbol{\mu_W \to m_\mu}$

$$[C_{e\gamma}]_{pr}(\mu_W) \to [C_{e\gamma}]_{pr}(\mu < \mu_W) \;\; \text{e.g.} \;\; \mu \sim m_\mu$$

Most sensitive probes: $\boldsymbol{\mu \to e\gamma}$ and $\boldsymbol{(g-2)_\mu}$

$$\mathcal{B}(\mu^+ \to e^+\gamma) = \frac{m_\mu^3 v^2}{8\pi\Gamma_\mu}\frac{\left|[\mathcal{C}'_{e\gamma}]_{12}\right|^2 + \left|[\mathcal{C}'_{e\gamma}]_{21}\right|^2}{\Lambda^4} < 4.2 \times 10^{-13} \quad (90\%\ \mathrm{CL}),$$

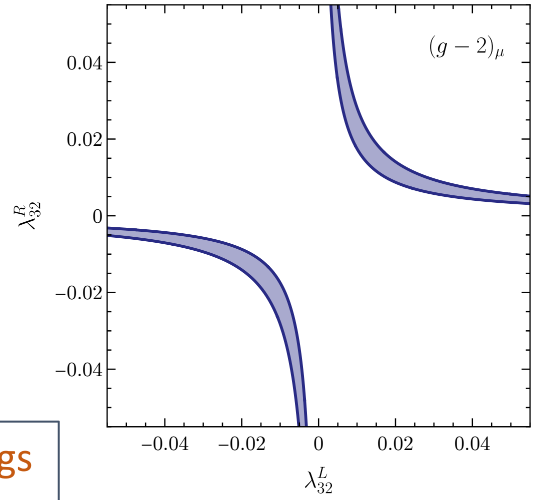
$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = -\frac{4m_\mu}{e}\frac{v}{\sqrt{2}}\frac{\mathrm{Re}[\mathcal{C}'_{e\gamma}]_{22}}{\Lambda^2} = (251 \pm 59) \times 10^{-11}$$

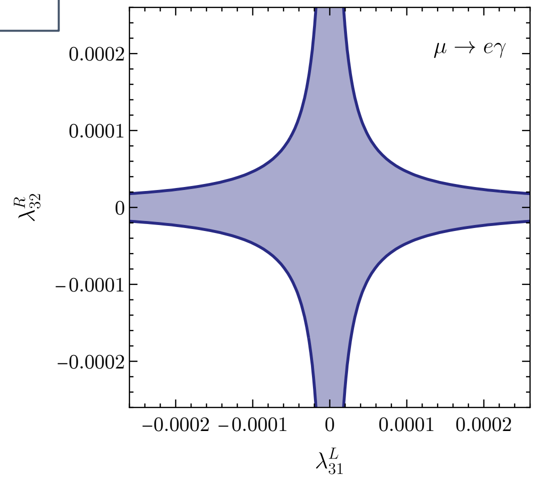
$$\left|\frac{[\mathcal{C}'_{e\gamma}]_{12(21)}}{\Lambda^2}\right| \lesssim 2.1 \times 10^{-10}\ \mathrm{TeV}^{-2},$$

$$\frac{\mathrm{Re}[\mathcal{C}'_{e\gamma}]_{22}}{\Lambda^2} \simeq -1.0 \times 10^{-5}\ \mathrm{TeV}^{-2}.$$

Back to the couplings of the UV model

$M_{S_1} = 2$ TeV





Another model with leptoquarks

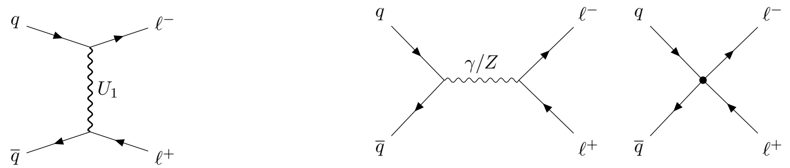
Connecting measurements at far apart scales

Model with one heavy vector (leptoquark) U_1

$$\mathcal{L}_{U_1} = \mathcal{L}_{SM} - \frac{1}{2}U^{\dagger}_{\mu\nu}U^{\mu\nu} + M_U^2 U^{\dagger}_{\mu}U^{\mu} + (U_\mu J^\mu + \text{h.c.}) \quad \text{where} \quad J^\mu = \frac{g_U}{\sqrt{2}}[\beta^L_{pr}(\bar{q}_p\gamma^\mu \ell_r) + \beta^R_{pr}(\bar{d}_p\gamma^\mu e_r)]$$

The tree level matching project on SMEFT 4-fermion operators

$$\mathcal{L}_{\rm W} = \mathcal{L}_{\rm SM} - \frac{g_U^2}{2M_U^2}\Big\{\frac{1}{2}\beta^L_{pr}\beta^{L*}_{st}\left([Q^{(1)}_{lq}]_{trps} + [Q^{(3)}_{lq}]_{trps}\right) + \beta^R_{pr}\beta^{R*}_{st}[Q_{ed}]_{trps} - \left(2\beta^R_{pr}\beta^{L*}_{st}[Q_{ledq}]_{trps} + \text{h.c.}\right)\Big\}$$



$\mathcal{L}_{U_1}$ $\qquad$ $\mathcal{L}_{SMEFT} \to \mathcal{L}_W$

affects Drell-Yan production: $pp \to \ell^+\ell^-$
In particular tail of $m_{\ell\ell}$ distribution (LHC)

These same operators contribute also to low-energy processes, such as $b \to c\ell\nu$ decays entering the R_D, R_{D^*} ratios

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)}$$

$$R_D = 0.356 \pm 0.029\,, \qquad R_D^{\rm SM} = 0.298(4)$$
$$R_{D^*} = 0.284 \pm 0.013\,, \qquad R_{D^*}^{\rm SM} = 0.254(5)$$

[HFLAV]

Another model with leptoquarks, cont'd

Described by the LEFT Lagrangian

$$\mathcal{L}_{b\to c} = -\frac{4\,G_F}{\sqrt{2}} V_{23}\Big[\,(1+\mathcal{C}^c_{LL})\,(\bar{c}_L\gamma^\mu b_L)(\bar{\tau}_L\gamma_\mu\nu_L) - 2\,\mathcal{C}^c_{LR}\,(\bar{c}_L b_R)(\bar{\tau}_R\nu_L)\,\Big]$$

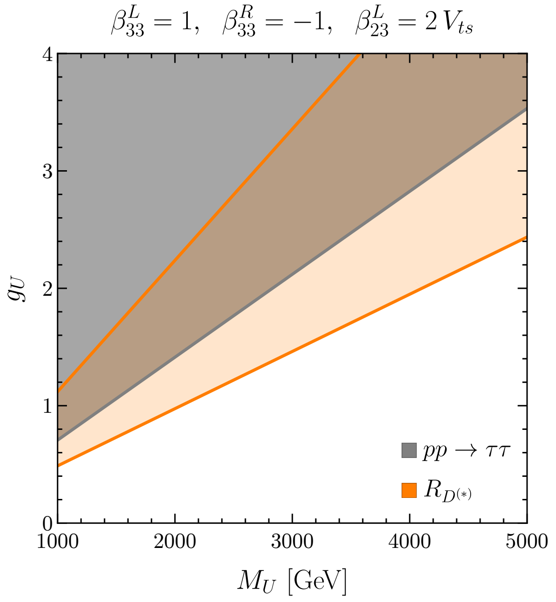
SMEFT to LEFT matching

$$\mathcal{C}^c_{LL} = -\frac{1}{\sqrt{2}G_F}\frac{1}{M_U^2}\sum_{k=1}^{3}\frac{[C^{(3)}_{lq}]_{33k3}V_{2k}}{V_{23}}\,,$$

$$\mathcal{C}^c_{LR} = \frac{1}{4\sqrt{2}G_F}\frac{1}{M_U^2}\sum_{k=1}^{3}\frac{[C^{*}_{ledq}]_{333k}V_{2k}}{V_{23}}\,,$$

After **running in the LEFT+SMEFT $\boldsymbol{\mu_b \to \mu_m}$** ($\mu_m \sim 1$TeV) perform combined fit with DY measurements of $m_{\ell\ell}$ distribution tail

In combination only a fraction of the parameter space is viable



SMEFT enables complementarity of low- and high-energy measurements

Conclusions

- The SM effective field theory can be a **powerful tool to explore the TeV scale** whose knowledge is crucial and still not complete.
- Effects of new physics can then be constrained using the **broad spectrum of precision measurements available from EW, Higgs, top, flavor physics** and more.
- The **SMEFT (→LEFT) framework** can be used to connect unknown physics at the UV scale (> 1 TeV) to the EW scale and below within a **systematic framework that allows some model independence.**
- With **increasing precision** in both theory and experiments, constraints **could start to show intriguing patterns and guide future explorations.**
- **In the presence of anomalies,** the SMEFT framework can connect them to a much broader phenomenology and offer a unique framework to their interpretation.

Some general references + refs. therein

- **General principles and broad spectrum of applications**

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THANK YOU!

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