Welcome to Math Methods!

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Course Purpose

- Goal: mastery of basics of undergraduate math curriculum to prepare for physics grad courses
- Ideally also cover advanced math topics of interest for physicists

Course Philosophy

- Math course for physicists? (Physicist vs. Mathematician)
- Calculations pretty straightforward compared to physics puzzles—try together while we’re learning

Topics

- Small, interactive class: we will go at a challenging pace, but nobody left behind
- Speed and topics covered depend on what *you* want and need
  - questionnaire

Grade Breakdown

- 15%: In class (participation, possible quizzes)
- 60%: Homework
- 10%: Midterm
- 15%: Final

Official Grade Boundaries

		>= 92	A	88 - 92	A-
84 - 88	B+	80 - 84	B	76 - 80	B-
72 - 76	C+	68 - 72	C	64 - 68	C-
52 - 64	D			<= 52	F

Unofficial Grade Boundaries

- A: Generally mastered material
- B: Some areas need improvement
- C: You blew something off

Homework Policy

- Weekly, due at beginning of class
- Groups allowed, work is your own
- No use of computer (Maple, Mathematica, etc.) unless specified

Today (?):

- Polynomial equations & roots
- Trigonometric identities

Polynomial Equations

- Frequently-encountered type of function: **polynomial**, a sum of powers of the dependent variable with coefficients
  - Straightforward to compute
  - Appears as limits of more general functions
  - Max power of dependent variable: **degree** of polynomial

Roots of Polynomials

- Often, want to invert polynomial equation
  - Ex: what time is particle at position x, given x(t)?
  - Write equation as y(x) = 0, solutions known as roots
  - Fundamental theorem of algebra: can always write polynomial as product of roots
    - Some roots may be complex, repeated

Class ex:

- The roots of a polynomial are $t=1,3,-1$. Find $x(t)$ in the standard form.

Finding Roots

- Straightforward methods for solving n=1 (linear) and n=2 (quadratic)
- More complicated general solutions for n=3 (cubic) and n=4 (quartic)
- Lots of indirect ways of finding how many real roots, properties

Numbers of Roots

- Number of complex roots always same as degree of polynomial
  - Some may be repeated
  - Not all are real

Tricks for Numbers of Real Roots

- Degree is even: limits at $\pm\infty$ same, have to cross axis even number of times
- Degree is odd: limits at $\pm\infty$ opposite, have to cross axis odd number of times

Where is a root?

- Try setting dependent variable to zero, ±∞, look at whether result is positive or negative
  - A root must exist if crossing from negative↔positive

More Tricks

- Set x=0: it’s a root if no constant term
- Set x=1: it’s a root if coefficients all add to zero
- Set x=-1: it’s a root if coefficients add to zero with sign flips

I Found a Root! Now What?

- Might be able to get is multiplicity: try taking derivatives
- Can start to factor polynomial

Example

- Find the roots of $f(x) = 2x^3 + 3x^2 - 3x - 2$

Trigonometric Functions

- Basic functions: sin, cos

- “Derived” functions: tan x = sin x/cos x
  cot x = 1/tan x, sec x = 1/cos x, csc x = 1/sin x

- Reflection properties: sin(-x) = - sinx, cos(-x) = cos x

Trig Identities

- As a practical matter, can always find a table easily:

TRIGONOMETRIC IDENTITIES

Co-function Identities

$\sin\theta = \cos(\pi/2-\theta)$
$\sec\theta = \csc(\pi/2-\theta)$
$\tan\theta = \cot(\pi/2-\theta)$

Negative Angle Identities

$\sin(-\theta) = -\sin\theta$ $\quad \csc(-\theta) = -\csc\theta$
$\cos(-\theta) = \cos\theta$ $\quad \sec(-\theta) = \sec\theta$
$\tan(-\theta) = -\tan\theta$ $\quad \cot(-\theta) = -\cot\theta$

Addition and Subtraction Identities

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

Double-Angle Identities

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Product Identities

$\sin A\cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$

$\cos A\sin B = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$

$\cos A\cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$

$\sin A\sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$

Supplement Angle Identities

$\sin(\pi-\theta) = \sin\theta$ $\quad \csc(\pi-\theta) = \csc\theta$
$\cos(\pi-\theta) = -\cos\theta$ $\quad \sec(\pi-\theta) = -\sec\theta$
$\tan(\pi-\theta) = -\tan\theta$ $\quad \cot(\pi-\theta) = -\cot\theta$

$\sin(\pi+\theta) = -\sin\theta$ $\quad \csc(\pi+\theta) = -\csc\theta$
$\cos(\pi+\theta) = -\cos\theta$ $\quad \sec(\pi+\theta) = -\sec\theta$
$\tan(\pi+\theta) = \tan\theta$ $\quad \cot(\pi+\theta) = \cot\theta$

Quotient Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$ $\quad \csc\theta = \dfrac{1}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Half-Angle Identities

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

$\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

Sum Identities

$\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$

$\sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$

$\cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$

$\cos A - \cos B = -2\sin\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$

Important Identities

- Most important: $\cos^2\theta + \sin^2\theta = 1$

- Very useful: angle addition

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$