Last Time

- Complex vector spaces
- Function spaces

Today

- Operators on function spaces
  - Analogy with matrices

Linear Operators

- We’ve analyzed linear operators that take vectors in space to other vectors: matrices
- What kinds of things are linear operators on functions?

Matrix Elements

- Suppose we have a complete, orthonormal basis
- Label basis member with an index
- Can we write a 2-index “matrix” element that specifies the basis function expansion of the result of an operator?

Inner Products and Operators

- In quantum mechanics, wavefunctions are vectors in Hilbert space
- We get expectation values of operators by taking the inner product of a wavefunction and the result of an operator acting on it
- Bra-ket notation: sandwich the operator

Class Ex

- Work out the first matrix elements of the derivative operator for functions on [-1,1] in the normalized Legendre basis:

$$\phi_1 = 1, \phi_2 = \sqrt{3/2}\, x, \phi_3 = \sqrt{5/8}\left(3x^2 - 1\right)$$

The Unit Operator

- The matrix elements of the unit operator are simple (looks same as finite-dim vector case)

- If we have a complete set of basis functions, gives us a way to “insert” projections onto them (**completeness relation**)

  - Also an explicit way of writing operators in matrix elements

Next Time

- Fourier series
- Position as a basis
- Changes of basis