Last Time

- Polynomials and roots
- Important trig identities

This Time

- Trig examples
- Partial fractioning
- Substitution integrals
- Integration by Parts
- Differentiation under the integral

Important Identities

- Most important: $\cos^2\theta+\sin^2\theta=1$

- Very useful: angle addition

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$
$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

Class ex

- We have $x = 5\cos(3t+3)$, $y = 2\sin(3t+3)$. Find t as a function of x and y, and a relationship between x and y that is not dependent on t.

Example

- Solve $A \sin\theta + B \cos\theta = C$ for θ, when A,B,C given.

Partial Fractioning

- We probably remember how to combine fractions (4^{th} grade)
  - Or algebraically (high school)
- Other direction is very useful—called partial fractioning

Example: Product of Linear Functions in Denominator

$$\frac{4x+2}{(x+1)(x+2)} = \frac{?}{x+1} + \frac{?}{x+2}$$

Numerators should be constants

Too-complicated Numerator

- What if there are too many terms to match in numerator? Ex: $\frac{x^3+3x^2+2x+1}{(x-3)(x+2)}=?$
  - Try dividing, working on remainder

$$\frac{x^3+3x^2+2x+1}{(x-3)(x+2)}=Ax+B+\frac{Cx+D}{(x-3)(x+2)}$$

Repeat Factors

$$\frac{x-4}{(x+1)(x-2)^2}=\frac{A}{x+1}+\frac{B}{(x-2)^2}\,?$$

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$$\frac{x-4}{(x+1)(x-2)^2}=\frac{A}{x+1}+\frac{Bx+C}{(x-2)^2}$$

Application: Integration

- One of most important/simplest integrals

$$\int dx \frac{1}{x} = \ln|x| + C$$

- Class ex: Find antiderivative of $\frac{1}{x(1-x)}$

Substitution

- Frequently changing variables makes problems easier
- Substitution: write dependent variable as function of new one x(u)

- Pointers:
  - Use trig functions/identities
  - Try to find variables u where du/dx is a factor
  - Variables with physical significance frequently happen to be the right ones!
- Class ex: integrate $\int_{x_1}^{x_2} dx \frac{x^2}{\left(x^3+4\right)^2}$

Integration by Parts

- In physics, we frequently run into integrals of products of things
- Products are easy for differentiation → reverse for integration

  - Tip: Find a product easy to antidifferentiate multiplied by one that gets simpler with differentiation

Class ex

- A particle has wavefunction $\psi(x) = e^{-x/2}$ for $x>0$, $\psi(x) = 0$ for $x<0$.
  Find the expectation value of its position.