Last Time

- Inhomogeneous methods

Today

- Series solutions

Problems with Methods so Far

- Most methods we’ve discussed require we “undo” derivatives somehow
  - solutions better be elementary functions or integrals involving them
  - requires tricks or simple enough equations (constant coefficients)
- What if solution isn’t easily related to elementary functions?

Try Series

- Perhaps the solution isn't an elementary function, but does have a Taylor series expansion—can we find the series directly?
- Try substituting in series expansion for $x(t)$, $x'(t)$, etc. everywhere, get series equation
  - Ex: $x' = x$
- For series to be equal, coefficients of each power must be equal

Recurrence Relations

- Generically, one always ends up with an equation relating c_i, c_{i-1}, ... c_{i-n} for order n
- Called **recurrence relation**
- Can solve to find pattern of coefficients, get solution series

Class Ex

- Find a recurrence relation for $x'' + x = 0$, about $t=0$.

More on Recurrence Relations

- Generally a coefficient depends on more than one previous one
- May be easier to build series by substituting in initial conditions (you know x(0), x'(0), etc.)
- Can get arbitrary solution by starting series with arbitrary constants: use first n terms

Will Series Work?

- Series will exist and converge near expansion point as long as ODE non-singular around expansion point
- Ex: for general second order homogeneous

$$x''+p(t)x'+q(t)x=0$$

  p, q must have Taylor series expansions

Regular Singular Points

- Many ODEs have p, q that are not analytic
  - Often because x'' has variable coefficient
- Can fix as long as p, q not too bad
- Assuming t=0 is bad expansion point:
  - Rewite $p = u/t$, $q = v/t^2$
    - if u, v are analytic (series expansions), proceed with new series for x with "extra" power

Class Ex

- Find the series solutions about $t=0$ of $4t\, x'' + 2\, x' + x = 0$.

Next Time

- Hermitian operators and Sturm-Liouville theory