Last Time

- Sturm-Liouville Theory

Today

- Intro to PDEs
  - Definitions
  - Basic techniques and properties

Partial Differential Equations

- Differential equations also exist for functions of more than one variable u(x,y,...)
- If variables are independent, involves partial derivatives, called partial differential equation
- Ex: $\frac{\partial u}{\partial t} = c \frac{\partial u}{\partial x}$

Notation

- Can’t usually use prime derivative notation (which variable?!)
- Full Leibniz notation too cumbersome
- Common physics notation: $\frac{\partial}{\partial x} = \partial_x, \frac{\partial^2}{\partial x \partial y} = \partial_{xy}, \frac{\partial^2}{\partial x^2} = \partial_x^2 = \partial_{xx}$
- Common math notation: $\frac{\partial u}{\partial x} = u_x, \frac{\partial^2 u}{\partial x \partial y} = u_{xy}, \frac{\partial^2 u}{\partial x^2} = u_{xx}$
  - too easily confused with vector components in physics

Solving PDEs

- PDEs mix variables, hard to do antiderivatives or other techniques in two different directions at once
- Primary strategy for solving PDEs: turn them into ODEs by separating out variables

Example

- Solve $\partial_t u = c\,\partial_x u$
  by changing variables

Boundary Conditions for PDEs

- Due to derivatives, ODEs have arbitrary integration constants that need to be determined by points of new data (boundary or initial conditions)
- PDEs have arbitrary *functions* due to partial derivatives
  - require entire boundary *subspaces* to be specified for unique solution
  - Ex: 2D PDE requires specifying a function on a 1D space

Class Ex

- Solve the previous example for the boundary conditions
  - $u(x,t) = 2x$ along $t=0$
  - $u(x,t) = 2x$ along $x=t$
  - $u(x,t) = 2x$ along $x=-ct$

Characteristics

- Lines (or surfaces, etc.) where arbitrary function is a constant called **characteristic**
- PDE “evolves” along these lines
- Boundary conditions specified in a space that intersects characteristic more than once generally has no solution
  - definitely no solution if space is exactly along characteristic

Existence of PDE Solutions

- Even for linear PDEs, solutions not guaranteed unless boundary conditions compatible
- General form conditions need to take depends on PDE
  - Characteristics determine what's appropriate

Solving by Characteristics

- Characteristic lines/surfaces curved if PDE has non-constant coefficients
- Solving still involves finding change of variables, just a nonlinear one

Class Ex

- Find the characteristic line (and thus general solution) to $t\partial_t u - 2x\partial_x u = 0$

Next Time

- Inhomogeneous equations
- Higher-order PDE behavior
- Separation of Variables