

# Making Things Rotate

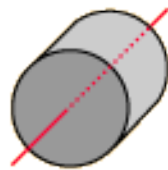
- Like a net force makes objects accelerate, a net torque makes them want to rotate (angular acceleration):  $\Sigma \vec{\tau} = I \vec{\alpha}$ 
  - I know I promised review on angular variables/vectors & cross product, but not needed for this homework
- Just like more mass means less acceleration, more **moment of inertia** means less angular acceleration

# Moment of Inertia

- To calculate moment of inertia, need to know axis about which object(s) will rotate
  - If not constrained to rotate about some axis, will naturally rotate about center of mass
- Point mass:  $I = mr^2$
- Solid object with size/shape:  $I = \int r^2 dm$

# Precomputed Moments of Inertia

Solid cylinder or disc, symmetry axis



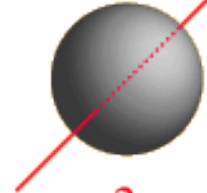
$$I = \frac{1}{2}MR^2$$

Hoop about symmetry axis



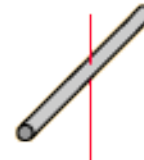
$$I = MR^2$$

Solid sphere



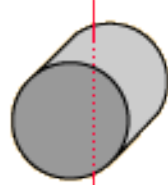
$$I = \frac{2}{5}MR^2$$

Rod about center



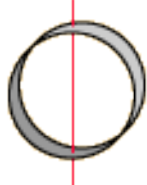
$$I = \frac{1}{12}ML^2$$

$$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$$



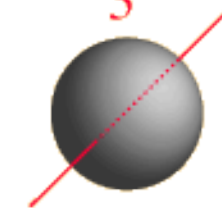
Solid cylinder, central diameter

$$I = \frac{1}{2}MR^2$$



Hoop about diameter

$$I = \frac{2}{3}MR^2$$



Thin spherical shell

$$I = \frac{1}{3}ML^2$$



Rod about end

# Parallel Axis Theorem

- Trick if you know moment of inertia about axis through center of mass, but want a different, parallel axis:

$$I = I_{cm} + M d^2$$

# Rotational Kinetic Energy

- Rotating objects have kinetic energy

$$K = \frac{1}{2} I \omega^2$$

- Total energy of rotating (and moving) object:

$$K = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \omega^2$$

- Trick with rolling objects: use  $v_{CM} = R\omega$

$$K = \frac{1}{2} v_{cm}^2 \left( M + \frac{I}{R^2} \right)$$