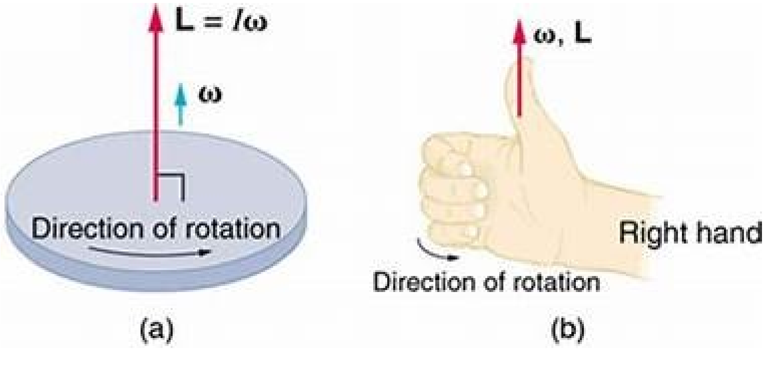
# Rotational Variables as Vectors

- Rotation in 2D easy, it's counterclockwise or clockwise
- We live in 3D: need to define axis of rotation
- Combine the two by defining angular variables as a vector that points along the axis
  - Which way along axis? **Right hand rule**



# Vector Torque

- Now $\Sigma\vec{\tau} = I\vec{\alpha}$

  $\vec{\tau} = \vec{r} \times \vec{F}$

- If axis along z, can still do 2D rotation problems with z component as +/-
  - +z points out of page (ccw rotation), -z points into page (cw rotation)

# Vector (Cross) Product

- Unlike scalar (dot) product, vector (cross) product gives a vector as the result
- Direction: perpendicular to two vectors
- How to calculate with components:
  - Option a: use algebra rules and

$$\hat{i}\times\hat{j}=-\hat{j}\times\hat{i}=\hat{k}$$
$$\hat{j}\times\hat{k}=-\hat{k}\times\hat{j}=\hat{i}$$
$$\hat{k}\times\hat{i}=-\hat{i}\times\hat{k}=\hat{j}$$

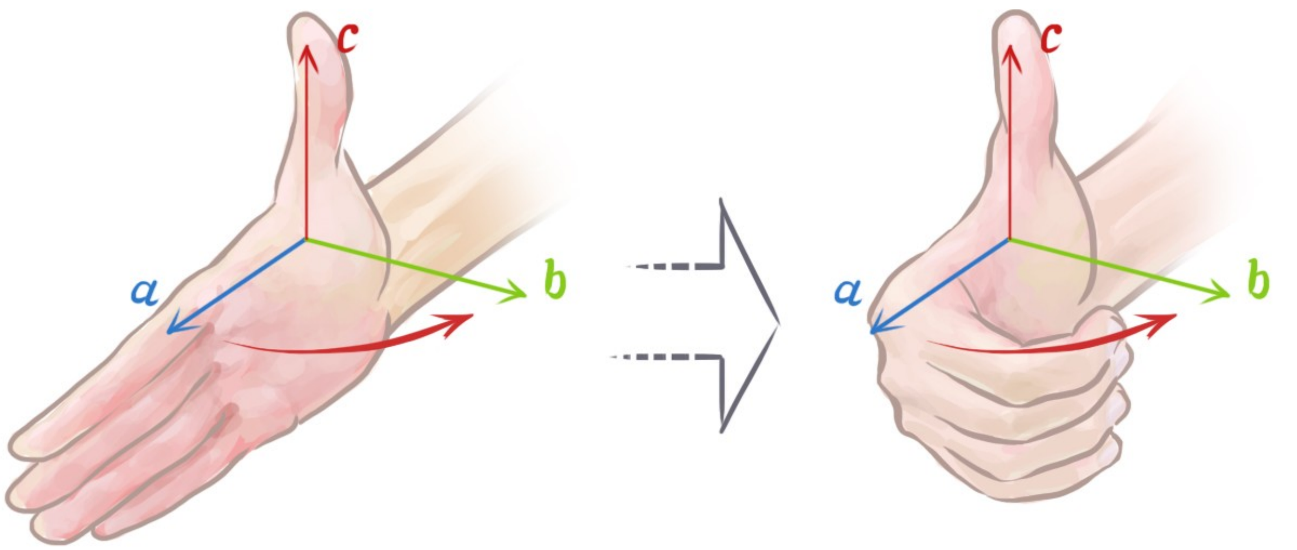
Not commutative!
Don't switch order!

  - Option b: general formula

$$\vec{A}\times\vec{B}=\left(A_y B_z-A_z B_y\right)\hat{i}+\left(A_z B_x-A_x B_z\right)\hat{j}+\left(A_x B_y-A_y B_x\right)\hat{k}$$

# Vector (Cross) Product

- If you know the plane of the two original vectors result has to be perpendicular
- Which perpendicular? Right hand rule again



# Angular Momentum

- Angular momentum will help us analyze problems involving rotation like momentum did for straight motion (translation)
  - Point object: $\vec{L}=\vec{r}\times\vec{p}=m\,\vec{r}\times\vec{v}$
  - Extended object: $\vec{L}=I\,\vec{\omega}$
- Angular momentum of an object/system will only change if there are external torques on it:

$$\sum\vec{\tau}_{ext}=\frac{d\vec{L}}{dt}$$

# Conservation of Angular Momentum

- If no external torques, total angular momentum is constant $\vec{L}_i = \vec{L}_f$
- Two ways to change angular momentum of an object:
  - Change its rotational velocity
  - Change its moment of inertia