Students use momentum conservation, energy conservation and two-dimensional vector addition to calculate the mass of the heaviest of the six known quarks. They gather data from data plots from the DØ experiment at Fermilab. The events were chosen carefully; all of the decay products moved in a plane perpendicular to the beam. This makes the vector addition much simpler.

STANDARDS
National Science Education Standards (U.S. National Research Council)
- Physical Science Content Standard B:
  - As a result of this activity . . . students should develop an understanding of:
    - Conservation of energy and increase in disorder.
    - Interactions in energy and matter.

LEARNING OBJECTIVES
Students will know and be able to:
- Apply momentum conservation to real-life problems.
- Calculate the invariant mass of an object at rest.
- Use energy conservation to determine the mass of an object undergoing decay.

PRIOR KNOWLEDGE
Students must be able to add vectors in two dimensions and be able to use energy and momentum units common to particle physics. (Momentum–eV/c, energy–eV/c²)

BACKGROUND MATERIAL
These event displays are real data. However, most high school students think of data as numbers, perhaps columns of numbers. Use the event displays to prompt a discussion of data forms and the fact that they can use this real data to calculate the top mass. The students are teams doing a “double-blind” analysis of top-quark event data. They are members of a collaboration trying to see if the selected event candidates all have the same mass. If they do, then the detector has been properly calibrated and can be used to search for more top quark events.

IMPLEMENTATION
Available online at (http://tinyurl.com/y8gtukk), this activity was developed by physics teacher Bob Grimm, and can be used as a more traditional teacher-introduced-and-led activity or as an online student-directed activity. In either case, students use printed event images, ruler and protractor to analyze the data. This activity requires averaging as many as ten data points per class to yield a remarkably accurate result. A class analyzes DØ top-quark event images, one real and three Monte Carlo, chosen for near-planar collisions. Students use a ruler and protractor to add particle momentum vectors, discover the MAGNITUDE of the missing neutrino momentum and calculate the mass of the top quark.

When Bob used this activity, his students were studying conservation laws, so the activity fit right in. They didn't know about quarks, etc., but they spent some time, before the lab day, looking at overheads (taken right from the website) that describe the top quark experiment. So, with their particle questions addressed, they were able to focus on the vector addition portion of the activity. This is a good investment; on lab day, the students' questions about vectors and momentum were not clouded by questions about particles. This is not to say they did not have questions about
particles. Indeed, the questions they had were fairly sophisticated. But they did not seem distracted by details of the experiment.

Bob recommends keeping in mind the following "I" ideas while leading your students through this activity:

• Invest time in describing the experiment.

• Ignore errors in the direction of the neutrino momentum vector.

• Integrate your students' results. Averaging a large set of data is critical.

• Indistinguishable units. Near the speed of light, mass = momentum = energy.

The website has a wealth of teacher background information, teaching suggestions and all the images you will need—except for rulers and protractors. We provide a printable copy of the student page and a short cut to the URL for printing data: [http://tinyurl.com/yaj338g](http://tinyurl.com/yaj338g)

The key to this solution is to determine the “missing P_t” (transverse momentum) carried away in these events by a neutrino. Since the detector can’t see neutrinos—they barely interact with matter—the students have to look at all of the momentum recorded in the event and then apply momentum conservation to determine what is needed to make the system’s net momentum zero. Recall that energy and momentum are equal at these energies.

**ASSESSMENT**
Consider asking the students questions such as:

• What did we calculate the top quark mass to be? How does the value compare with the value physicists use?

• How close are these values to each other? Are there outliers? Are they all valid measurements? How do you *know*?

• If there are outliers, talk with the class about what the differences in analysis might be.

• Why can we use vector addition to find the top mass with these events?

• How does our calculation rely on conservation laws?

• How do physicists determine the missing energy from these events? How do they know that their value is correct?

• How do you use the experimental data to determine the mass of the top quark? Are energy and mass the same thing?

• Why can’t the detector see the neutrinos?

• How many neutrinos are created in the event? How do you *know*?
CALCULATE THE TOP QUARK MASS

E = mc² Used in the Creation of the Most Massive Quark Yet Discovered!
Analysis of DZero Data From Fermi National Accelerator Laboratory

Today you will use Einstein's famous equation and experimental data collected in 1995 from a special event in Fermilab's DØ experiment that is two-dimensional rather than three-dimensional to determine the mass of the top quark, the most massive quark discovered.

A Top-Antitop Quark Event from the DZero Detector at Fermilab

While this event looks complex at first, it may be summarized easily:
- That a proton and antiproton collide to create a top-antitop pair that exists for a very short time.
- Almost immediately the very massive top and antitop decay into the constituents that are known to be their signature. These include:
  - Four "jets" (large blasts of particles) that are the result of decays of W bosons and some less massive quarks. It is important to note that one of the jets will often contain a low-energy or "soft" muon. The soft muon helps identify the jet as a bottom quark jet.
  - A muon and a neutrino. (You can see them in the upper right part of the diagram.)

The diagram shows the collision for the event labeled Run 92704 Event 14022. Other top-antitop event displays can be represented by similar diagrams but may not have exactly the same debris, going in the directions shown here.
Data from DØ events are displayed in images like this one for event Run 92704 Event 14022. It shows the recorded momentum (in GeV/c) of the particle debris that came from the collision. Your class has four event displays.

Can you identify the constituents of the top-antitop signature in data display? Look closely; the only information given about the neutrino is the magenta tower indicating its direction. While scientists can predict with confidence that a neutrino comes out of the collision, DØ cannot detect it. Still, a careful consideration of the momentum before the collision and after the collision may give you a clue as to how much momentum the neutrino has!

What do we know?
1. Momentum is conserved.
2. The total momentum of the system is zero before the proton and antiproton collide.
3. Momentum is a vector.
4. These 2-D events largely occurred in the plane of the paper on which the event display is printed.
5. Physicists know that with a careful choice of units, it is possible to equate momentum and energy in a way that is similar to the way mass and energy are related. Specifically, it may be shown that the momentum of the collision debris has the same numerical value as the energy or mass of the particles. In other words, $E \text{ (in GeV)} = p \text{ (in GeV)} = m \text{ (in GeV)}$ (GigaelectronVolts). This shows, then, that the total energy that came from the two top quarks that were formed is equal to the numerical sum of all the momenta discovered in the collision.

What tools do we need for our analysis?
Ruler, protractor, pencil to make a momentum vector diagram

What are your claims? What is your evidence?

<table>
<thead>
<tr>
<th>Momentum, Energy or Mass</th>
<th>Jet 1</th>
<th>Jet 2</th>
<th>Jet 3</th>
<th>Jet 4</th>
<th>Muon</th>
<th>Soft Muon</th>
<th>Neutrino</th>
</tr>
</thead>
</table>

- What is the numerical value of the neutrino momentum?
- What do you calculate for the top mass?
- How do physicists determine missing energy from these events?
- How do they know that their value is correct?
- How do you use the experimental data to determine the top mass?
- Are energy and mass the same thing?
- Could you use this method to calculate the top mass in a 3-D event?