Quantum physics
(quantum theory, quantum mechanics)

Part 1
Outline

- Introduction
- Problems of classical physics
- Black-body Radiation
  - experimental observations
  - Wien’s displacement law
  - Stefan – Boltzmann law
  - Rayleigh - Jeans
  - Wien’s radiation law
  - Planck’s radiation law
- photoelectric effect
  - observation
  - studies
  - Einstein’s explanation
- Quantum mechanics
  - Features
  - postulates
- Summary
- Quantum Physics
Question: What do these have in common?

- lasers
- solar cells
- transistors
- computer chips
- CCDs in digital cameras
- Ipods
- superconductors
- ........

Answer:

- They are all based on the quantum physics discovered in the 20th century.
“Classical” vs “modern” physics

- Speed
  - Far less than $3 \times 10^8$ m/s
  - Comparable to $3 \times 10^8$ m/s

- Size
  - Far larger than $10^{-9}$ m
  - Near or less than $10^{-9}$ m

- Classical Mechanics
- Relativistic Mechanics
- Quantum Mechanics
- Quantum Field Theory
Why Quantum Physics?

“Classical Physics”:
- developed in 15\textsuperscript{th} to 20\textsuperscript{th} century;
- provides very successful description “macroscopic phenomena, i.e. behavior of “every day, ordinary objects”
  - motion of trains, cars, bullets,....
  - orbit of moon, planets
  - how an engine works,..
  - Electrical and magnetic phenomena
- subfields: mechanics, thermodynamics, electrodynamics,

“There is nothing new to be discovered in physics now. All that remains is more and more precise measurement.”

--- William Thomson (Lord Kelvin), 1900
Why Quantum Physics? – (2)

- **Quantum Physics:**
  - developed early 20th century, in response to shortcomings of classical physics in describing certain phenomena (blackbody radiation, photoelectric effect, emission and absorption spectra…)
  - describes microscopic phenomena, e.g. behavior of atoms, photon-atom scattering and flow of the electrons in a semiconductor.

- **Quantum Mechanics**
  - a collection of postulates based on a huge number of experimental observations.
  - Extremely successful theory
Quantum Physics

- QP is “weird and counterintuitive”
  - “Those who are not shocked when they first come across quantum theory cannot possibly have understood it” (Niels Bohr)
  - “Nobody feels perfectly comfortable with it “ (Murray Gell-Mann)
  - “I can safely say that nobody understands quantum mechanics” (Richard Feynman)

- But:
  - QM is the most successful theory ever developed by humanity
  - underlies our understanding of atoms, molecules, condensed matter, nuclei, elementary particles
  - Crucial ingredient in understanding of stars, …
Classical Point of View – Newtonian Mechanics

- Motion of particles: particle trajectories.
- Particle:
  - indivisible mass point object with well-defined properties (observables) that can be measured,
  - Observables (e.g. position and momentum) specify the state of the particle
  - All properties of a particle can be known to infinite precision.
- System:
  - collection of particles which interact among themselves via internal forces, and/or with the outside world via external forces.
  - State of a system = collection of the states of the particles that comprise the system.
- Conclusions:
  - trajectory \(\rightarrow\) state descriptor of Newtonian physics,
  - Evolution of the state \(\rightarrow\) Use Newton's second law
  - Causality: \(\rightarrow\) Two identical systems with the same initial conditions, subject to the same measurement will yield the same result.
Quantum Point of View

- Quantum particles can act as both particles and waves
  ⇒ Wave – particle duality
- Quantum state:
  - conglomeration of several possible outcomes of measurement of physical properties
  - ⇒ Quantum mechanics uses the language of probability theory (random chance)
- Observer is part of the system which is studied:
  - observer cannot observe a microscopic system without altering some of its properties.
- Quantization:
  - Many observables are "quantized", i.e. spectrum of possible values is not continuous:
    - Energy, angular momentum,
Heisenberg Uncertainty Principle

- For a microscopic particle, one cannot precisely specify the values of a particle's position and its momentum, i.e.

\[ \Delta x \cdot \Delta p_x \geq \frac{\hbar}{2} = \frac{1}{2} \frac{h}{2\pi} \]

- “Uncertainties” of our knowledge of position and momentum cannot be arbitrarily small.

- Position and momentum are “incompatible variables”.

- Heisenberg uncertainty principle strikes at the very heart of the classical physics, the particle trajectory.
Correspondence Principle

- Classical physics works well for macroscopic objects
- For consistency, classical physics must be limit of QP for distances/sizes >> atomic scale
- In general: new theory should reproduce the results of older well-established theories (which become limiting cases) in those domains where the old theories work.
- “correspondence principle” (Niels Bohr, 1913, 1920)
  - Quantum mechanical description of systems must reproduce classical physics in the limit of large distances, (e.g. large quantum numbers in atoms)
  - Non-classical phenomena (uncertainty, wave-particle duality,..) must become undetectable
Particle-Wave Duality -

- behavior of a "microscopic" particle is very different from that of a classical particle:
  - in some experiments it resembles the behavior of a classical wave (not localized in space)
  - in other experiments it behaves as a classical particle (localized in space)

- Corpuscular theories of light:
  - treat light as though it were composed of particles, but can not explain diffraction and interference
  - Maxwell's theory of electromagnetic radiation:
    - can explain these two phenomena, which was the reason why the corpuscular theory of light was abandoned.
Particle-Wave Duality

- **Waves as particles:**
  - Max Planck’s explanation of black-body radiation (1899)
    - assumed that the molecules of the cavity walls, described using a simple oscillator model, can only exchange energy in quantized units.
  - Einstein’s photon hypothesis (1905)
    - energy in an electromagnetic field is not spread out over a spherical wavefront, but instead is localized in individual clumps - quanta.
    - Each quantum of frequency $f$ (“photon”) has a discrete amount of energy $E=hf$ and momentum $p=hf/c = h/\lambda$.
    - travels through space with speed of light,
    - used to explain the photoelectric effect, later confirmed by the X-ray experiments of Compton.

- **Particles as waves**
  - Louis de Broglie matter waves
    - Wavelength associated with particle $\lambda = h/p$
  - Double-slit experiment
    - use electron gun instead of using a light source
    - electrons are diffracted by the slit and then interfere in the region between the slits and the detector.
Thermal radiation

- thermal radiation = e.m. radiation emitted by a body by virtue of its temperature
- spectrum is continuous, comprising all wavelengths
- thermal radiation formed inside body by random thermal motions of its atoms and molecules, repeatedly absorbed and re-emitted on its way to surface ⇒ original character of radiation obliterated ⇒ spectrum of radiation depends only on temperature, not on identity of object
- amount of radiation actually emitted or absorbed depends on nature of surface
- good absorbers are also good emitters (Kirchhoff)
  - [http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/absrad.html](http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/absrad.html)
  - [http://casswww.ucsd.edu/archive/public/tutorial/Planck.html](http://casswww.ucsd.edu/archive/public/tutorial/Planck.html)
  - [http://panda.unm.edu/Courses/Finley/P262/ThermalRad/ThermalRad.html](http://panda.unm.edu/Courses/Finley/P262/ThermalRad/ThermalRad.html)
  - [http://www.youtube.com/watch?v=CDncSyDvpdQ](http://www.youtube.com/watch?v=CDncSyDvpdQ)
warm bodies emit radiation
Blackbody Radiation

- Known since centuries: when a material is heated, it radiates heat and its color depends on its temperature
- Emission spectrum depends on the material
- Example: heating elements of a stove:
  - Dark red: 550°C
  - Bright red: 700°C
  - Then: orange, yellow and finally white (really hot!)

- Theoretical description: simplifications necessary
  -- Idealization: Blackbody
A material is constantly exchanging heat with its surrounding (to remain at a constant temperature):

- It absorbs and emits radiations
- Problem: it can reflect incoming radiations, which makes a theoretical description more difficult (depends on the environment)

Idealization: black body

blackbody is a perfect absorber:

- Incoming radiation is totally absorbed and none is reflected
Black-body radiation

- **“Black body”**
  - **perfect absorber**
    - ideal body which absorbs all e.m. radiation that strikes it, any wavelength, any intensity
    - such a body would appear black $\Rightarrow$ “black body”
  - **must also be perfect emitter**
    - able to emit radiation of any wavelength at any intensity -- “black-body radiation”

- **“Hollow cavity” (“Hohlraum”) kept at constant $T$**
  - hollow cavity with small hole in wall is good approximation to black body (Kirchhoff 1859)
  - thermal equilibrium inside, radiation can escape through hole, looks like black-body radiation
  - Kirchhoff’s challenge to theorists: calculate spectrum of hollow cavity radiation (black body radiation)
Blackbody radiation – Stefan-Boltzmann

- “Global description”, i.e without frequency dependence:
  - Descriptions successful, i.e. in agreement with observations
- Joseph Stefan (1879) first measured temperature dependence of the total amount of radiation emitted by blackbody at all wavelengths and found it varied with absolute temperature
- Ludwig Boltzmann: theoretical explanation
- total power output of black body: Stefan-Boltzmann law:
  - For an “ideal” radiator (“black body”),
    total emitted power (per unit emitting area), \( P/A \)
    \[
    P/A = \sigma \cdot T^4
    \]
    \( \sigma = 5.672 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \)
    (Josef Stefan, Ludwig Boltzmann 1879, 1884)
    [http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/stefan.html](http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/stefan.html)
- Note that the power per unit area radiated by blackbody depends only on the temperature, and not of other characteristic of the object, such as its color or the material, of which it is composed.
  An object at room temperature (300 K) will double the rate at which it radiates energy as a result of temperature increase of only 57°.
Black body radiation – Wien’s law

- Wien’s displacement law (1893)
  peak vs temperature:
  \[ \lambda_{\text{max}} \cdot T = C, \quad C = 2.898 \cdot 10^{-3} \text{ m} \cdot \text{K} \]
  inverse relationship between the wavelength \( \lambda_{\text{max}} \) of the peak of the emission of a black body and its temperature when expressed as a function of wavelength

  - [hyperphysics.phy-astr.gsu.edu/hbase/quantum/wien2.html](http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/wien2.html)
  - [http://webphysics.davidson.edu/faculty/dmb/blackbody/Wiendemo.html](http://webphysics.davidson.edu/faculty/dmb/blackbody/Wiendemo.html)

- **Example**: The wavelength at the peak of the spectral distribution for a blackbody at 4300 K is 674 nm (red). At what temperature would the peak be 420 nm (violet)?

  **Solution**: From Wien’s law, we have

  \[ \lambda_1 T_1 = \lambda_2 T_2 \]

  \[ (674 \times 10^{-9} \text{m})(4300 \text{ K}) = (420 \times 10^{-9} \text{m})(T_2) \]

  \[ T_2 = 6900 \text{ K} \]
Wien’s displacement law

- Intensity $\rho(\lambda, T) =$ power radiated per unit area per unit wavelength at a given temperature
- Maximum of the distribution shifts to smaller wavelengths as the temperature increases
- Wavelengths for visible light: 400 to 700 nm, UV < 400 nm, IR > 700 nm
Wien’s displacement law

- is used to
  - determine the surface temperatures of stars by analyzing their radiation.
  - also used to map out the variation in temperature over different regions of the surface of an object. Map = “thermograph”

- Example: *thermograph* can be used to detect cancer because cancerous tissue results in increased circulation which produce a slight increase in skin temperature.
Attempts to get radiation spectrum

- Wilhelm Wien (1896)
  \[ \rho(f, T) = af^3 \exp(-bf/T), \]
  \((a \text{ and } b \text{ constants})\).

  - OK for high frequency but fails for low frequencies
    - http://bado-shanai.net/map%20of%20physics/mopWienslaws.htm
    - http://physics.info/planck/

- Rayleigh-Jeans Law (1900)
  \[ \rho(f, T) = af^2T \]  \((a = \text{constant})\)

  - (constant found to be \(= 8\pi k/c^3\) by James Jeans, in 1906)
  - OK for low frequencies, but “ultra – violet catastrophe” at high frequencies, i.e. intensity grows \(\propto f^2 \rightarrow \infty\) for \(f \rightarrow \infty\) (corresponding to limit of wavelength \(\rightarrow 0\))

  - http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/rayj.html
    - http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html
Ultraviolet catastrophe

Blackbody Radiation

Classical theory (Rayleigh-Jeans)
Planck’s radiation law

- Planck’s assumption:
  - Radiation in the cavity emitted and absorbed by some sort of oscillators in the walls
  - Used Boltzmann's statistical methods to derive formula for the energy of the oscillators

- Two important modifications to classical theory
  - The oscillators can only have certain discrete energies given by \( E = n hf \) (\( n = \text{integer}, f = \text{frequency of oscillation}, h = \text{constant} \))
  - The oscillators can absorb or emit energy only in discrete multiples of the fundamental quantum energy given by \( \Delta E = hf \)
Planck’s radiation formula

- Multiplying average oscillator energy by the number of oscillators per unit volume in the interval $d\lambda$ given by $n(\lambda) = 8\pi c\lambda^{-4}$ (the number of modes of oscillation per unit volume), finally obtain the energy distribution function for the radiation in cavity:

$$u(\lambda, T) = \frac{8\pi}{\lambda^{5}} \left( \frac{hc}{e^{\frac{hc}{\lambda kT}} - 1} \right)$$

- And for blackbody radiation:

$$\rho(f, T) = \frac{8\pi f^2}{c^3} \frac{hf}{e^{\frac{hf}{kT}} - 1}$$
Black-body radiation spectrum

Measurements of Lummer and Pringsheim (1900)
Quantization

- Blackbody emission spectrum explained by introducing quantized energy transfers, which resolves the ultraviolet catastrophe
  - low wavelength $\Leftrightarrow$ high-frequency ($f=\frac{c}{\lambda}$)
  - at small wavelength, the energy needed to fill up the oscillator states increases
  - due to the Boltzmann factor, the probability of the high-energy states to be occupied decreases rapidly, faster than the increase from the Rayleigh genes formula,
  - no ultraviolet catastrophe
- this new idea very strongly disputed,
- Planck not happy with it, tried to find a way to make $\hbar \to 0$, without success
Consequences of Planck’s hypothesis

- oscillator energies $E = nh\nu$, $n = 0,1,...$;
  - $h = 6.626 \times 10^{-34}$ Js = $4.13 \times 10^{-15}$ eV·s now called Planck’s constant
  - oscillator’s energy can only change by discrete amounts, absorb or emit energy in small packets – “quanta”;
    - $E_{\text{quantum}} = hf$

- average energy of oscillator
  - $<E_{\text{osc}}> = hf/(e^x - 1)$ with $x = hf/kT$;
  - for low frequencies get classical result $<E_{\text{osc}}> = kT$, $k = 1.38 \times 10^{-23}$ J·K$^{-1}$
Electron

- **Cathode rays:**
  - During 2nd half of 19th century, many physicists do experiments with "discharge tubes", i.e. evacuated glass tubes with "electrodes" at ends, electric field between them (HV)
  - Johann Hittorf (1869): discharge mediated by rays emitted from negative electrode ("cathode") -- "Kathodenstrahlen" "cathode rays"
  - Study of cathode rays by many physicists – find
    - cathode rays appear to be particles
    - cast shadow of opaque body
    - deflected by magnetic field
    - negative charge
  - Eventually realized cathode rays were particles which appeared much lighter than atoms -- named them electrons
Photoelectric effect

1887: Heinrich Hertz:

- In experiments on e.m. waves, unexpected new observation: when receiver spark gap is shielded from light of transmitter spark, the maximum spark-length became smaller

- Further investigation showed:
  - Glass effectively shielded the spark
  - Quartz did not
  - Use of quartz prism to break up light into wavelength components ⇒ find that wavelength which makes little spark more powerful was in the UV
Photoelectric effect— further studies

- Studies by Hallwachs, Lenard, Thomson:
  - Discharge effect observed only if negative charge
  - Shining light on surface causes negative particles to be emitted from surface
  - Energy of particles independent of light intensity
  - Energy of particles depends on color of light
  - Maximum wavelength beyond which no effect
  - Emitted particles same as the ones seen in “cathode ray” experiments
Photoelectric effect

- Light can eject electrons from a metal, but only if it's frequency is above a threshold frequency (characteristic for each material)
- Classically, for light as a wave, energy of light is proportional to the square of the amplitude
- If light = photon with energy $E=hf$, energy depends on frequency

![Photoelectric effect diagram](image-url)
The photoelectric effect provides evidence for the particle nature of light. It also provides evidence for quantization. If light shines on the surface of a metal, there is a point at which electrons are ejected from the metal. The electrons will only be ejected once the threshold frequency is reached. Below the threshold frequency, no electrons are ejected. Above the threshold frequency, the number of electrons ejected depend on the intensity of the light.
Explanation of photoelectric effect

- 1905: Albert Einstein (1879-1955) (Bern)
  - Gives explanation of observation relating to photoelectric effect:
    - Assume that incoming radiation consists of “light quanta” of energy $h\nu$ ($h =$ Planck’s constant, $\nu =$frequency)
    - $\Rightarrow$ electrons will leave surface of metal with energy
      $$E = h\nu - W$$
      $W =$ “work function” = energy necessary to get electron out of the metal
    - $\Rightarrow$ there is a minimum light frequency for a given metal (that for which quantum of energy is equal to work function), below which no electron emission happens
    - When cranking up retarding voltage until current stops, the highest energy electrons must have had energy $eV_{\text{stop}}$ on leaving the cathode
    - Therefore
      $$eV_{\text{stop}} = h\nu - W$$
Verification of Einstein’s explanation

  - Did not accept Einstein’s explanation
  - Tried to disprove it by precise measurements
  - Result: confirmation of Einstein’s theory, measurement of \( h \) with 0.5% precision

- 1923: Arthur Compton (1892-1962) (St. Louis):
  - Observes scattering of X-rays on electrons
Features of QP

- Quantum physics is basically the recognition that there is less difference between waves and particles than was thought before.

- Key insights:
  - "Wave-particle duality"
    - Light can behave like a particle.
    - Particles can behave like waves (or wave packets).
  - "Quantization of energy"
    - Waves gain or lose energy only in "quantized amounts".
  - Particles (e.g., electrons) are indistinguishable.
  - Detection (measurement) of a particle \( \Rightarrow \) wave will change suddenly into a new wave.
  - Quantum mechanical interference – amplitudes add.
  - QP is intrinsically probabilistic.
  - What you can measure is what you can know.
Summary

- Classical physics explanation of black-body radiation failed
- Planck's ad-hoc assumption of "energy quanta" of energy $E_{\text{quantum}} = h\nu$, modifying Wien's radiation law, leads to a radiation spectrum which agrees with experiment.
- Old generally accepted principle of "natura non facit saltus" violated
- Photoelectric effect can be explained if light is considered to be made of "light particles" -- photons
- Leads to development of a completely new way of looking at the microscopic world: Quantum Mechanics