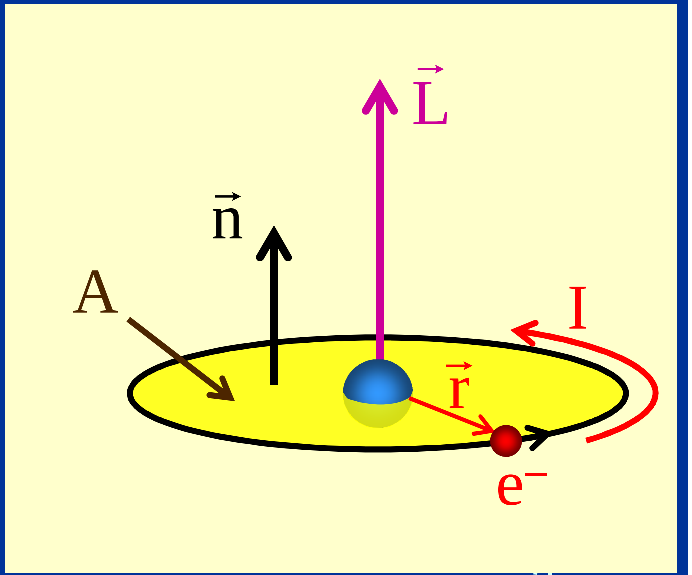
The Muon, g-2

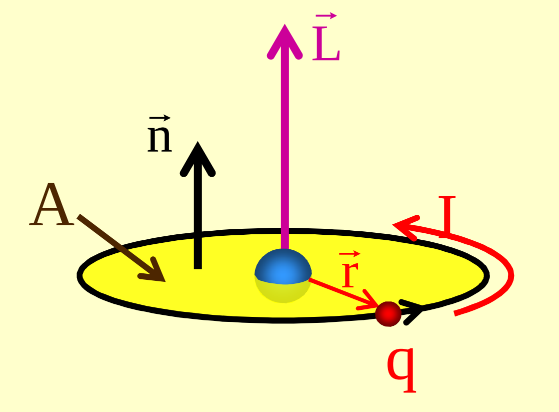
Atoms in magnetic field

- orbiting electron behaves like current loop
  - $\Rightarrow$ magnetic moment μ = current x area
  - interaction energy = $\boldsymbol{\mu} \cdot \mathbf{B}$ (both vectors!)
    $= \mu \cdot B_{||}$
  - loop current = $-ev/(2\pi r)$
  - angular momentum $L = mvr$
  - magnetic moment $= -\mu_B L/\hbar$
    $\mu_B = e\hbar/2m_e$ = "Bohr magneton"
  - interaction energy
    $= m\,\mu_B B_z$
    (m = z comp of L)



Gyromagnetic ratio

- orbiting charged particle behaves like current loop
  - $\Rightarrow$ magnetic moment μ = current x area
  - loop current = qv/(2πr)
  - angular momentum L = mvr
  - magnetic moment $\mu = IA = \frac{qv}{2\pi r}\cdot \pi r^2 = \frac{q}{2m}\cdot mvr = \frac{q}{2m}L = GL$
- Same formula is obtained for rotating body
  - (can be thought of as consisting of many small particles orbiting around axis)
- Def: gyromagnetic ratio
  - G= ratio between magnetic dipole moment and angular momentum
  - For classical particle $\frac{\mu}{L} = \frac{q}{2m}$

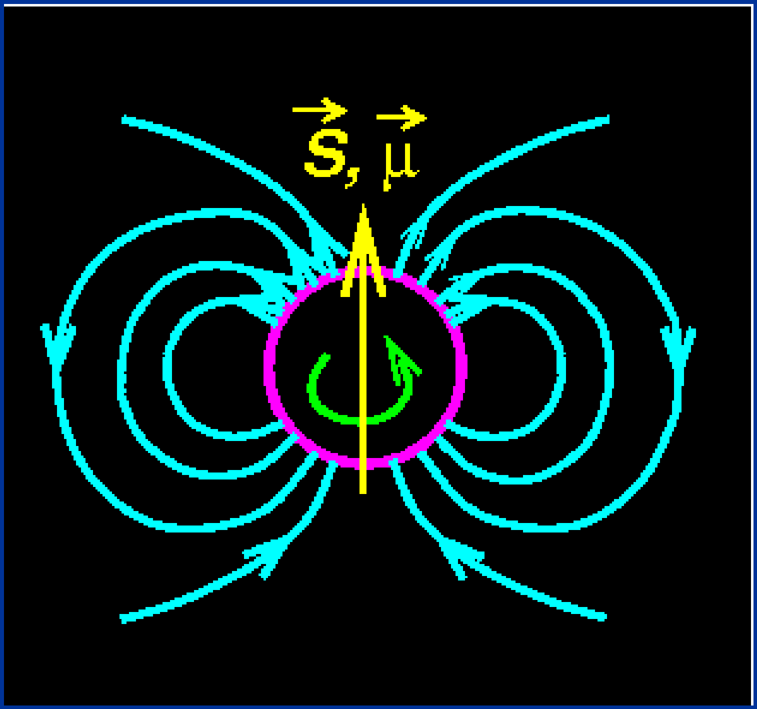


Gyromagnetic ratio for spin

- For microscopic particles, there may be additional factor g $\mu = g \cdot G \cdot L$
  - For classical body and orbital ang. mom., "g-factor" = 1
  - For isolated electron g=2 according to Dirac equation $\mu = g_e \cdot G \cdot S$, S= spin
  - QED predicts that g≠2 $g_e = 2(1+a) = 2(1+\frac{\alpha}{2\pi}+.....)$
  - a=(g-2)/2 ("anomaly") for the electron has been calculated and measured to very high precision (12 decimal places)

Summary of μ_S

$$\vec{\mu}_s = g_s \left(\frac{e\hbar}{2m} \right) \vec{s}$$



the moment consists of 2 parts

$$\mu = (1 + a) \frac{e\hbar}{2m}$$

Dirac + Pauli moment

the anomaly $a = \left(\frac{g-2}{2} \right);$ **or** $g = 2(1 + a)$

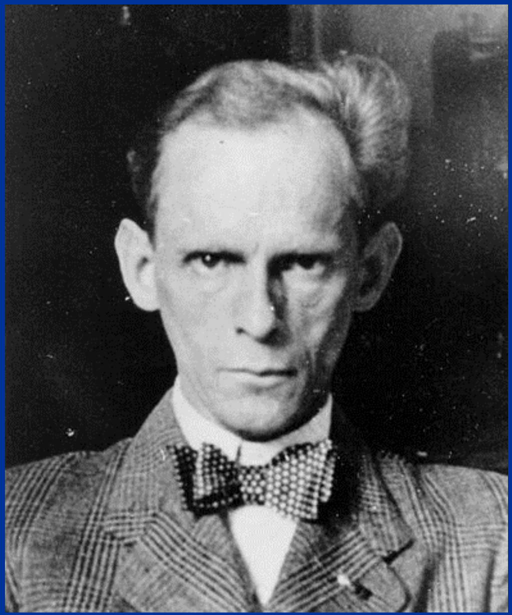
SM predicts a

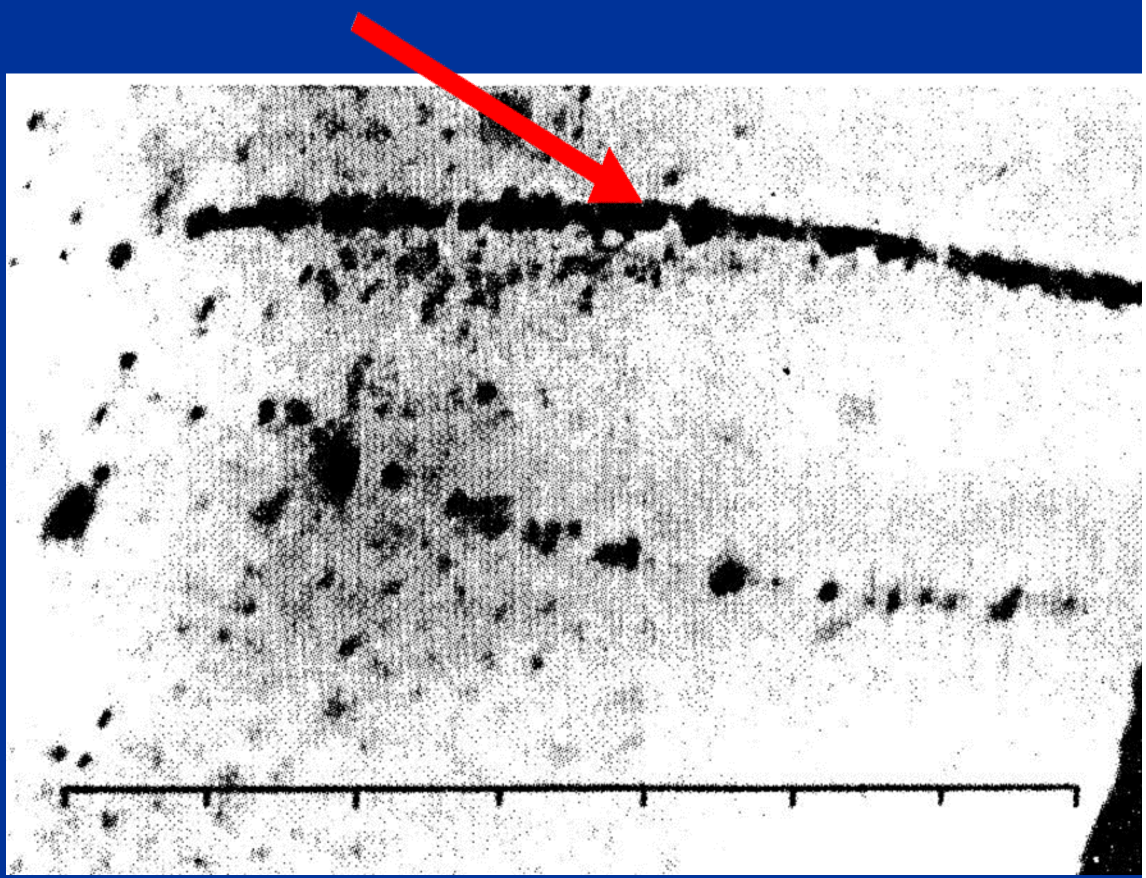
The Muon

First observed in cosmic rays in 1933

- Paul Kunze, Z. Phys. 83, 1 (1933)

"a particle of uncertain nature"





Identified in 1936

Study of cosmic rays by Seth Neddermeyer and Carl Anderson

MAY 15, 1937 PHYSICAL REVIEW VOLUME 51

Note on the Nature of Cosmic-Ray Particles

SETH H. NEDDERMEYER AND CARL D. ANDERSON
California Institute of Technology, Pasadena, California

(Received March 30, 1937)

MEASUREMENTS[1] of the energy loss of particles occurring in the cosmic-ray showers have shown that this loss is proportional massive than protons but more penetrating than electrons obeying the Bethe-Heitler theory, we have taken about 6000 counter-tripped photo-

Confirmed by: Street & Stevenson, Nishina, Tekeuchi & Ichimiya

NOVEMBER 1, 1937 PHYSICAL REVIEW VOLUME 52

LETTERS TO THE EDITOR

Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the eighteenth of the preceding month, for the second issue, the third of the month. Because of the late closing dates for the section no proof can be shown to authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Communications should not in general exceed 600 words in length.

New Evidence for the Existence of a Particle of Mass Intermediate Between the Proton and Electron

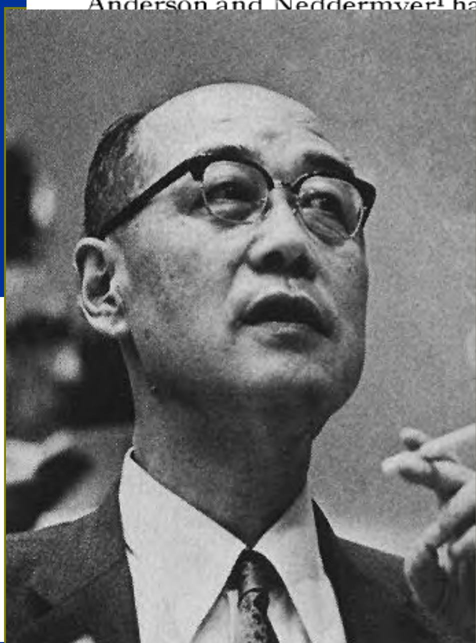
Anderson and Neddermyer[1] have shown that, for energies

between those of the proton and electron. If this is true, it should be possible to distinguish clearly such a particle from an electron or proton by observing its track density

J. C. Street
E. C. Stevenson

Research Laboratory of Physics,
Harvard University,
Cambridge, Massachusetts,
October 6, 1937.

[1] Anderson and Neddermeyer, Phys. Rev. **50**, 263 (1936).
[2] Street and Stevenson, Phys. Rev. **51**, 1005 (1937).
[3] Neddermeyer and Anderson, Phys. Rev. **51**, 885 (1937).



PHYSICAL REVIEW VOLUME 52

On the Nature of Cosmic-Ray Particles

Y. Nishina, M. Takeuchi, and T. Ichimiya
Institute of Physical and Chemical Research, Tokyo

(Received August 28, 1937)

It took 10 years to conclude that the muon interacted too weakly with matter to be the "Yukawa" particle which was postulated to carry the nuclear force

The Standard Model (Our Periodic Table)

Leptons

e	μ	τ
ν_e	ν_μ	ν_τ

Interact weakly through the

Electroweak gauge bosons

γ	Z^0	$W^\pm$

Quarks

u	c	t
d	s	b

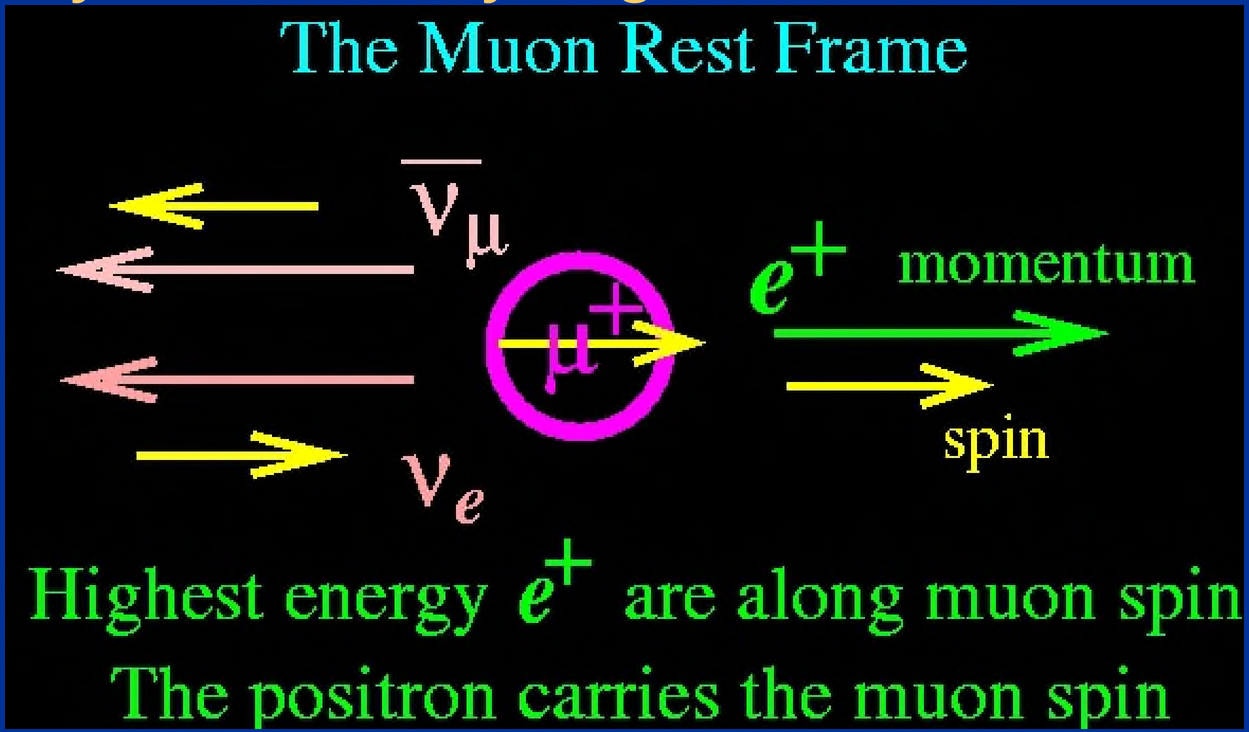
Interact strongly through the gluons **g**

Spin, helicity

- As all matter particles, leptons are spin ½ particles
- "helicity"
  - =projection of spin on momentum direction (component of spin in mom dir.)
- Left and right-handedness:
  - "left-handed" fermion: spin is in the opposite direction to their momentum
  - "right-handed" fermion: spin is in the same direction as their momentum
- According to SM: only left-handed fermions and right-handed antifermions participate in weak interactions
  - "right-handed e^+, μ^+, τ^+ and "left-handed e^-, μ^-, τ^- feel weak int.
  - (corresponding statements for neutrinos and quarks)

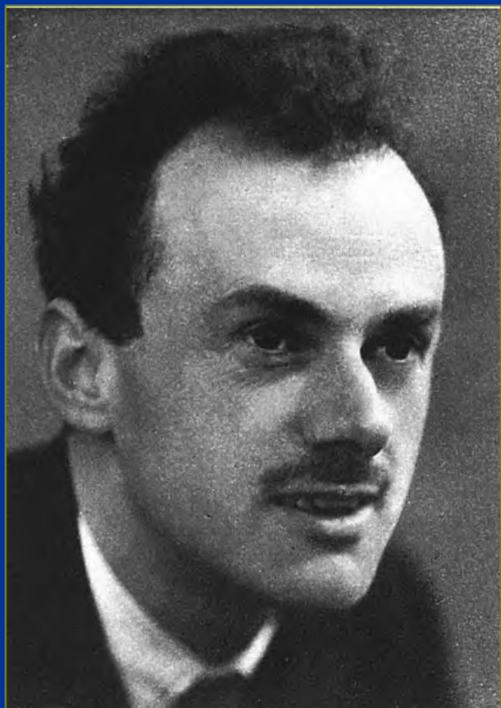
Death of the Muon

- Decay is self analyzing



- The highest energy $e^{\pm}$ from $\mu^{\pm}$ decay carry information on the spin direction.

Theory of Magnetic and Electric Dipole Moments



- Dirac equation
  - Relativistic generalization of Schrödinger equation
  - Predicts spin of electron
  - Predicts antiparticles (positron)
  - Predicts $g_e = 2$

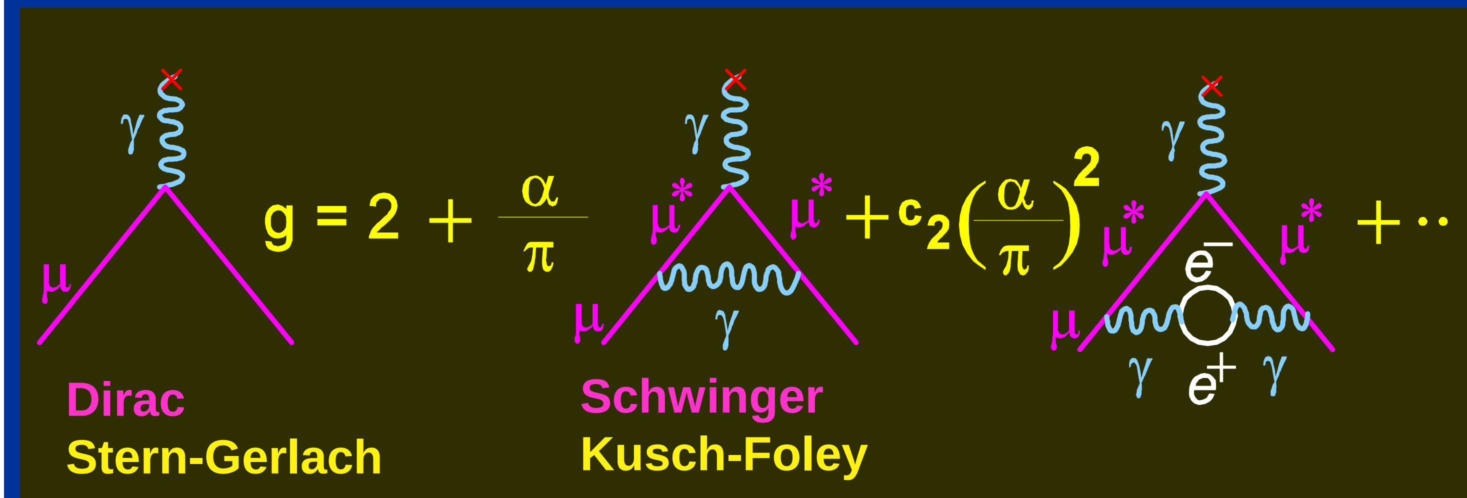
Proc. R. Soc. (London) **A117**, 610 (1928)

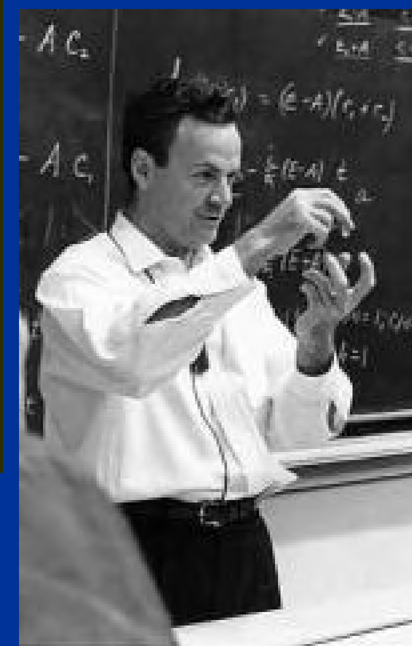
The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

Quantum electrodynamics: radiative corrections change g from its Dirac value of 2. corrections symbolically expressed as “Feynman diagrams”



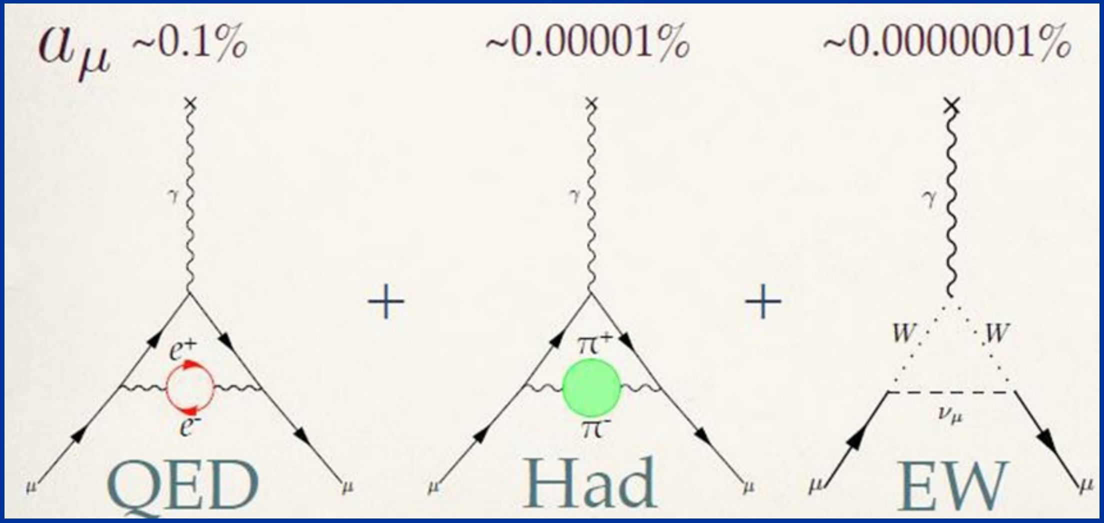


perturbation expansion:

$$a(\mathrm{QED}) = \frac{1}{2}\frac{\alpha}{\pi} + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

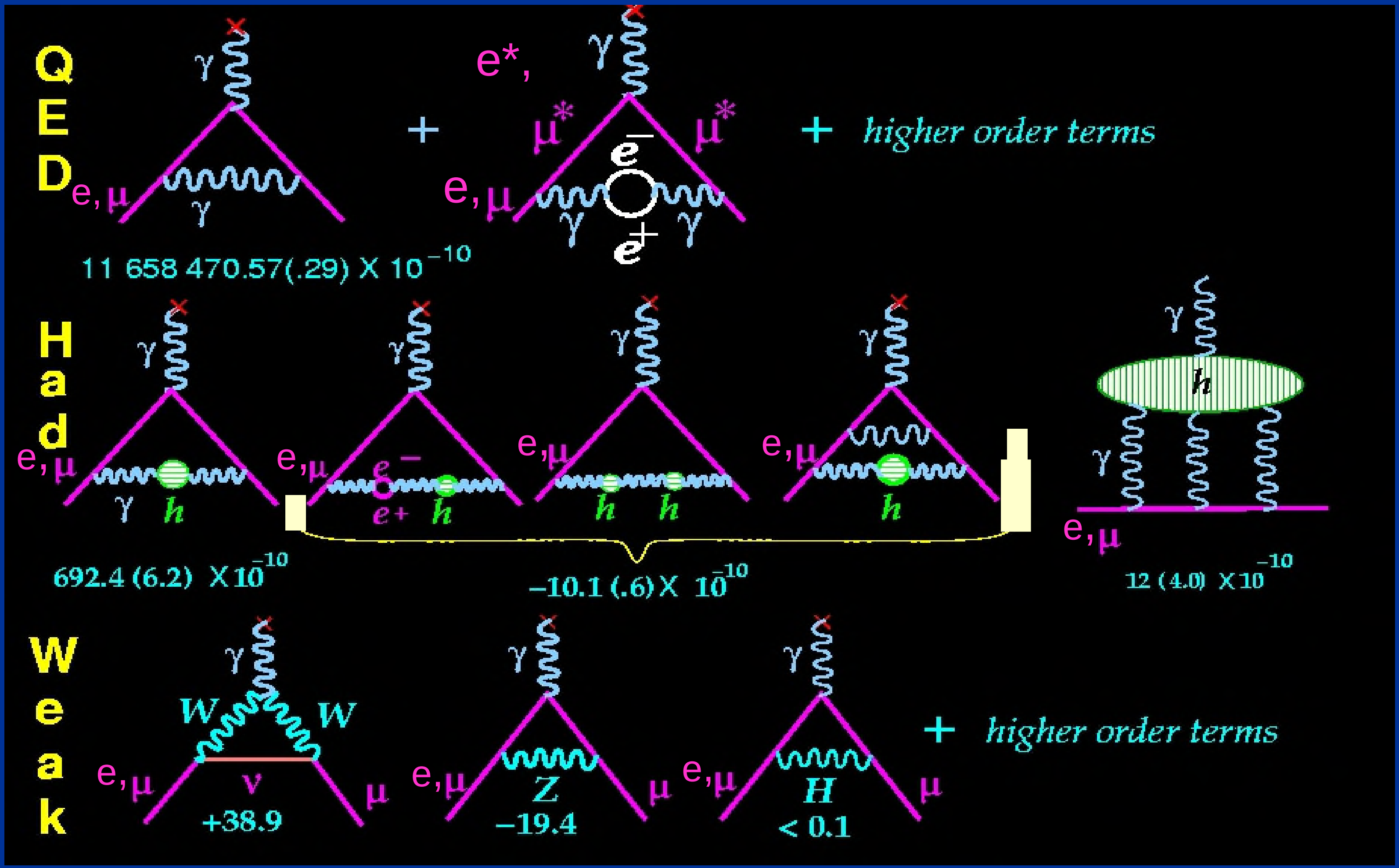
g-2 for muons

- Calculation somewhat more complicated:
  - Muon mass 200 times electron mass
  - Some corrections depend on mass rations – bigger contributions for muon



The SM Value for electron, muon and tau anomalies

$$a_\mu(\mathrm{SM}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{hadronic}) + a_\mu(\mathrm{weak})$$



e vs. μ : relative contribution of heavier things $\left(\frac{m_\mu}{m_e}\right)^2 \simeq 42,000$

Theory calculation

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{had} + a_\mu^{EW}$$

	Value ($\times 10^{-11}$) units
QED ($\gamma + \ell$)	$116\,584\,718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077_\alpha$
HVP(lo) [47]	$6\,923 \pm 42$
HVP(lo) [48]	$6\,949 \pm 43$
HVP(ho) [48]	-98.4 ± 0.7
HLbL	105 ± 26
EW	154 ± 1
Total SM [47]	$116\,591\,802 \pm 42_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}}\ (\pm 49_{\text{tot}})$
Total SM [48]	$116\,591\,829 \pm 43_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}}\ (\pm 45_{\text{tot}})$

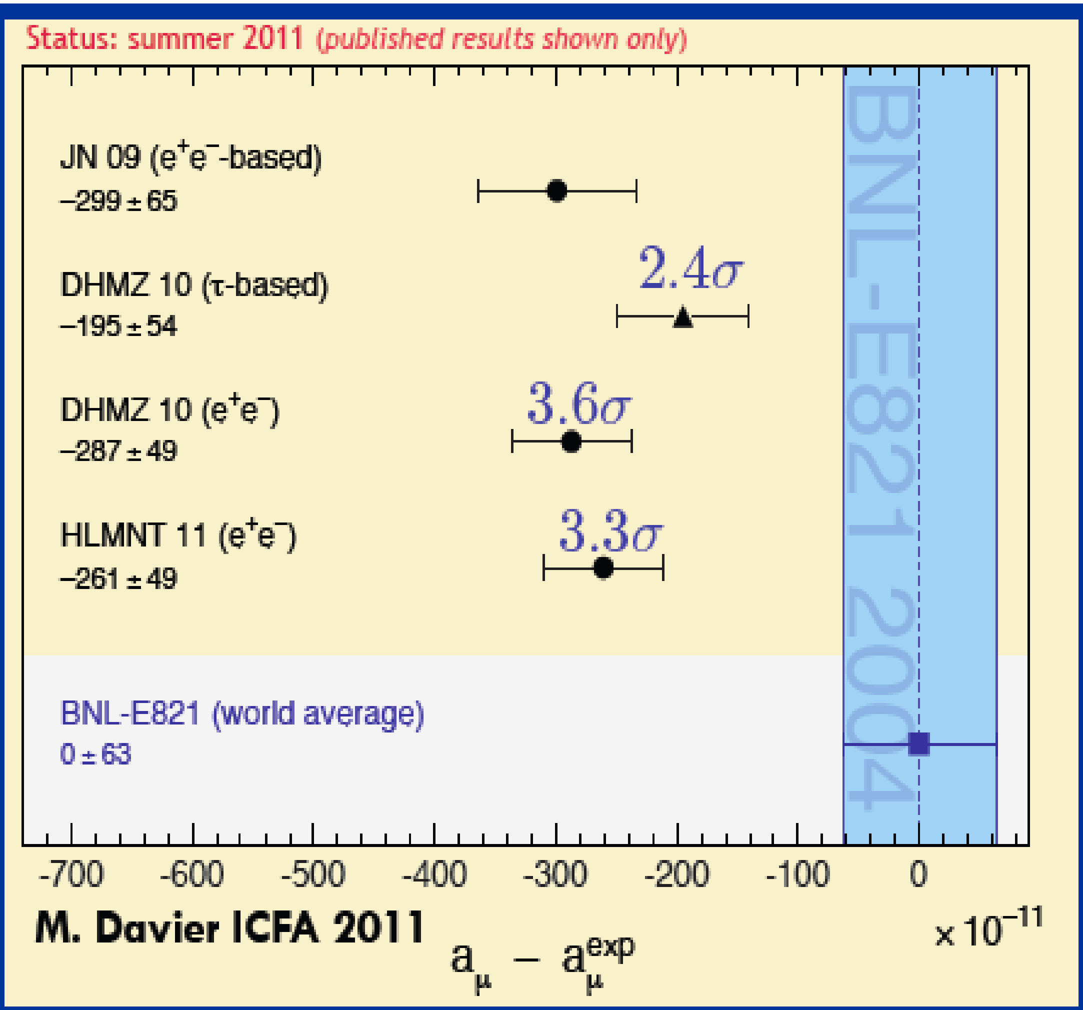
[47] DHMZ, Eur.Phys.J.C72:1874 (2012)
[48] HLMNT, J.Phys.G38,085003 (2011)

$\Delta a_\mu = (286 \pm 80) \times 10^{-11}$ [47]

$\Delta a_\mu = (260 \pm 78) \times 10^{-11}$ [48]

- Dominating theoretical uncertainties are hadronic components
- Most from low energy non-perturbative QCD regime
- The hadronic vacuum polarization (HVP) is related to the cross section for hadron production $e^+e^- \rightarrow$ hadrons
- The hadronic light by light (HLbL) is model specific (cannot be determined from data directly), much less known (25% error)
- Lattice QCD is starting to get involved, could be a big help

Comparison



(3 – 3.6) σ difference depending on HVP LO contribution

- If the discrepancy between the theory and the experimental result sustains, it can point to new physics
- More importantly, Δa_μ tightly constraints new physics models and has significant implications to interpret any new phenomena

Spin Motion in a Magnetic Field

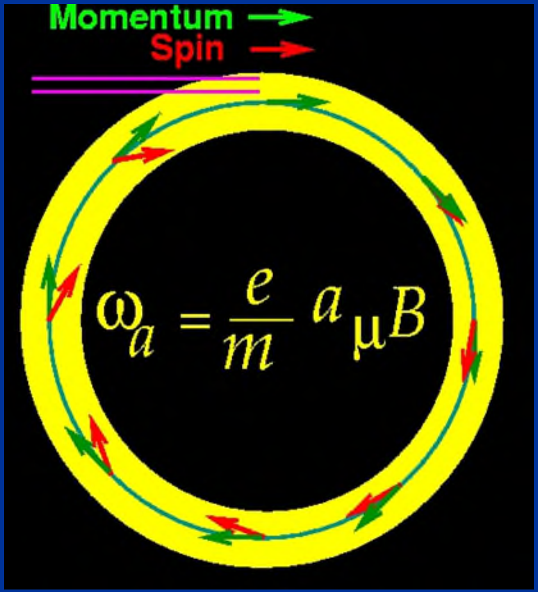
- charged (q) particle moving in a magnetic field: momentum turns with cyclotron frequency ω_C, spin turns with ω_S

$$\omega_C = -\frac{qB}{\gamma m}, \quad \omega_S = -g\frac{qB}{2m} - (1-\gamma)\frac{qB}{\gamma m}$$

- Spin turns relative to momentum with ω_a

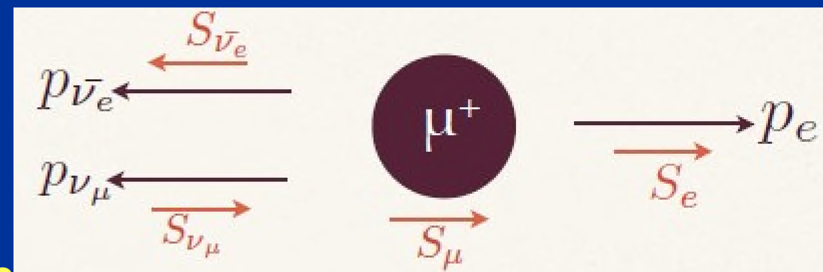
$$\omega_a = \omega_C - \omega_S = -\left(\frac{g-2}{2}\right)\frac{qB}{m} = -a\frac{qB}{m}$$

- Measure the frequency difference between the spin and momentum precession



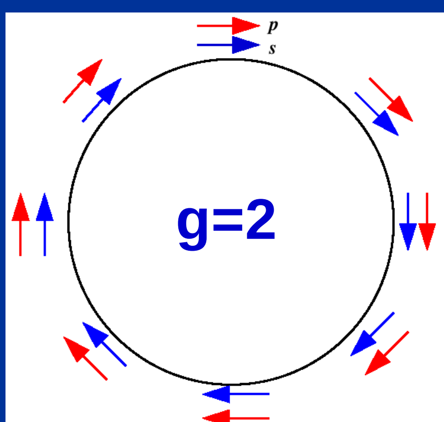
How to measure?

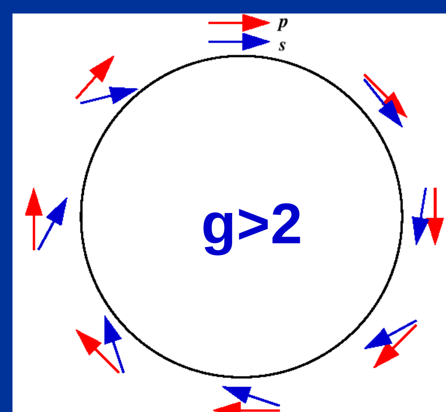
- **Put (polarized) muons in a magnetic field and measure precession freq**
- **Get muon spin direction from decayed electrons**
- **a_μ ~ difference between precession frequency and cyclotron frequency**



$$\omega_C = \frac{eB}{m}, \quad \omega_S = g\frac{eB}{2m}$$

$$\omega_a = \omega_C - \omega_S = a_\mu \frac{eB}{m}$$





A slight complication…

The magic muon momentum

- **Muons make horizontal circular movement under influence of magnetic field B, what about vertical movement?**
  - **Need to use electrostatic quadrupoles to confine muons vertically, this brings additional complication**

$$\vec{\omega} = \frac{e}{m}[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})(\vec{\beta} \times \vec{E})]$$

- **How to measure E?**
  - **No need! choose γ = 29.3, then coefficient vanishes!**
  - **γ = 29.3 means p_μ= 3.09 GeV (magic momentum)**

$$\omega_a = a_\mu \frac{eB}{mc}$$

The magic muon momentum

- Muons make horizontal circular movement under influence of magnetic field B, what about vertical movement?
- Need to use electrostatic quadrupoles to confine muons vertically, this brings additional complication

$$\vec{\omega} = \frac{e}{m}[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})(\vec{\beta} \times \vec{E})]$$

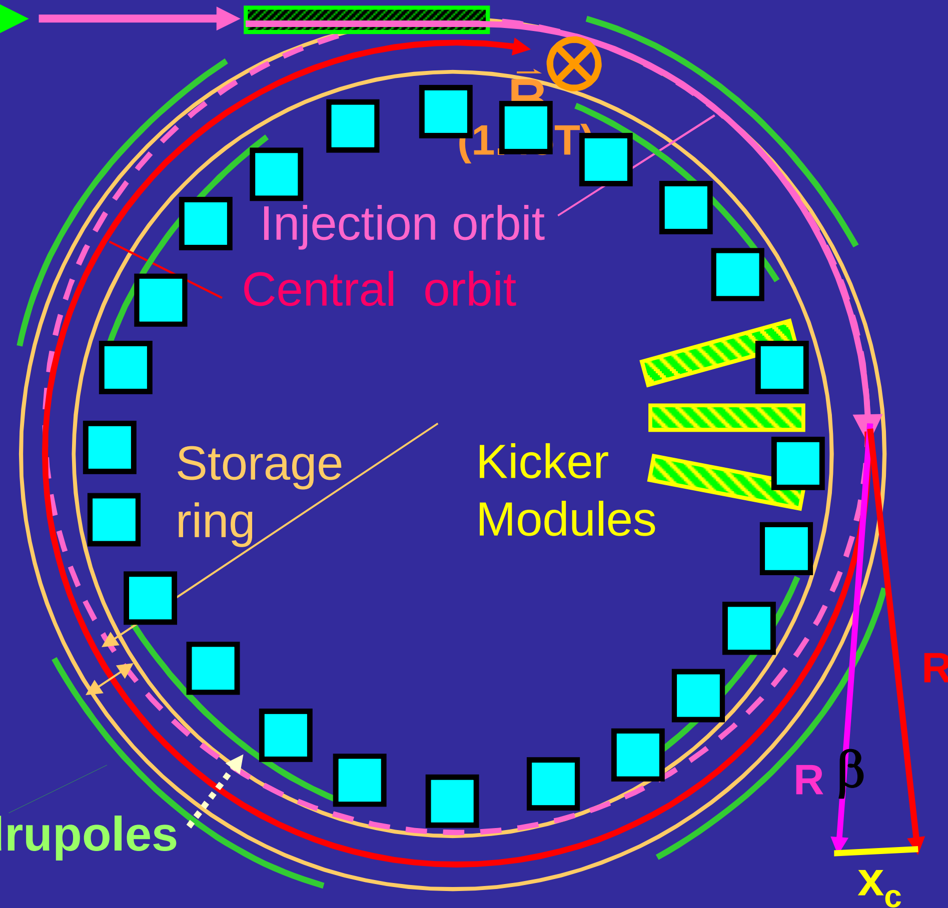
Experimental Technique AGS experiment

Proton bunches
from accelerator

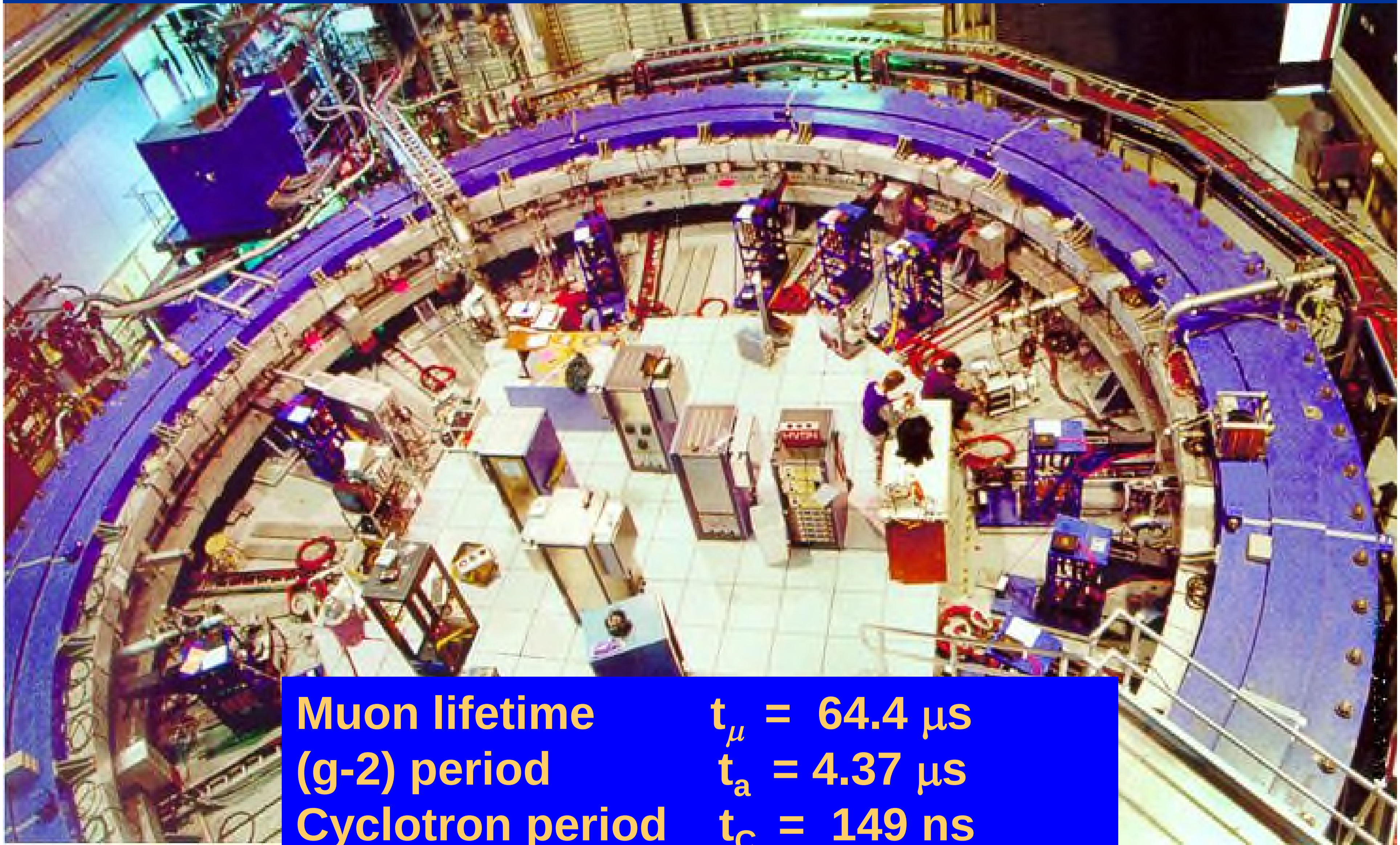
$x_c \approx 77$ mm
$\beta \approx 10$ mrad
$B\cdot dl \approx 0.1$ Tm

- Muon polarization
- Muon storage ring
- injection & kicking
- focus with Electric Quadrupoles
- 24 electron calorimeters

$$\vec{\omega}_a = -\frac{e}{m} a_\mu \vec{B}$$

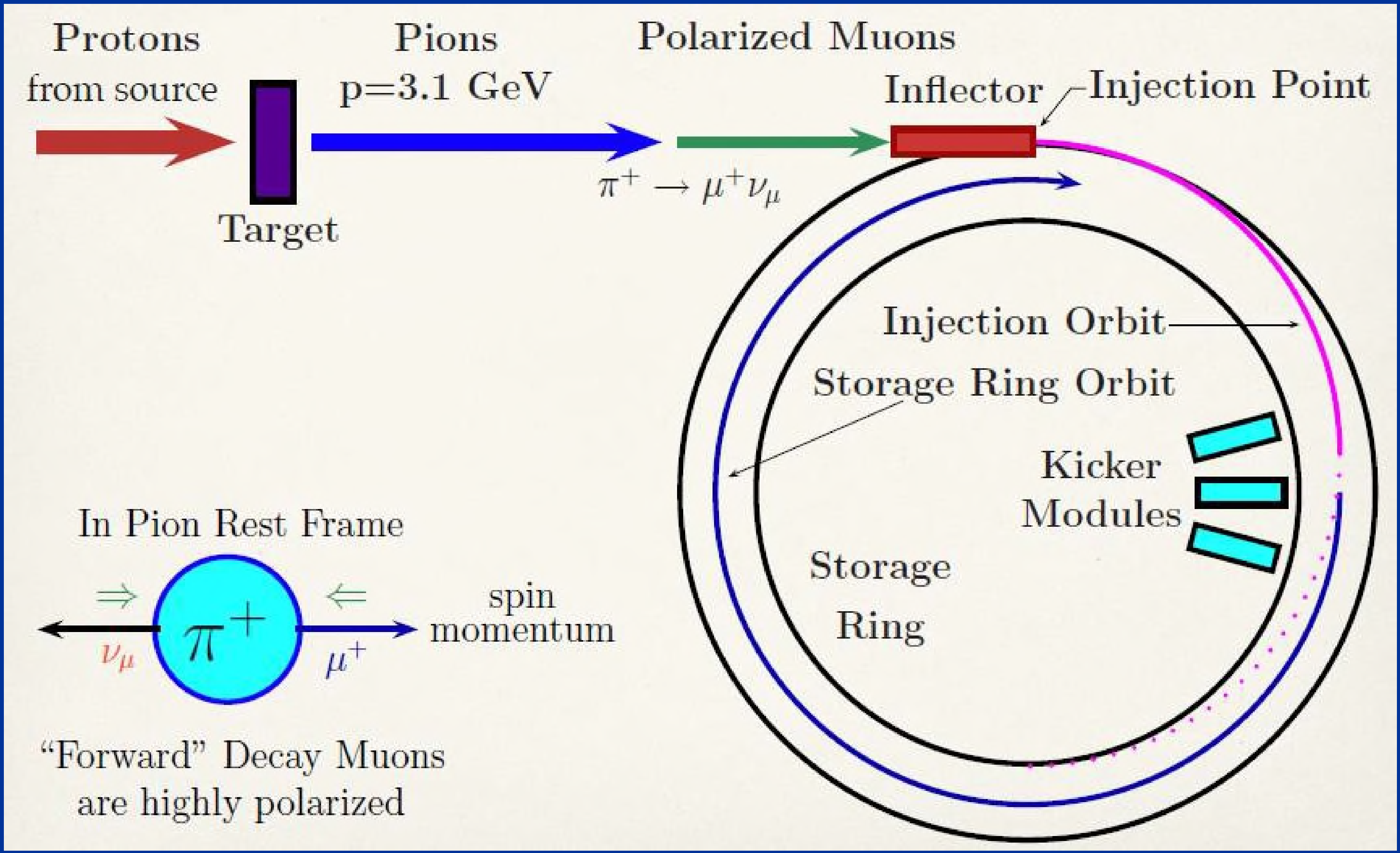


muon (g-2) storage ring at BNL

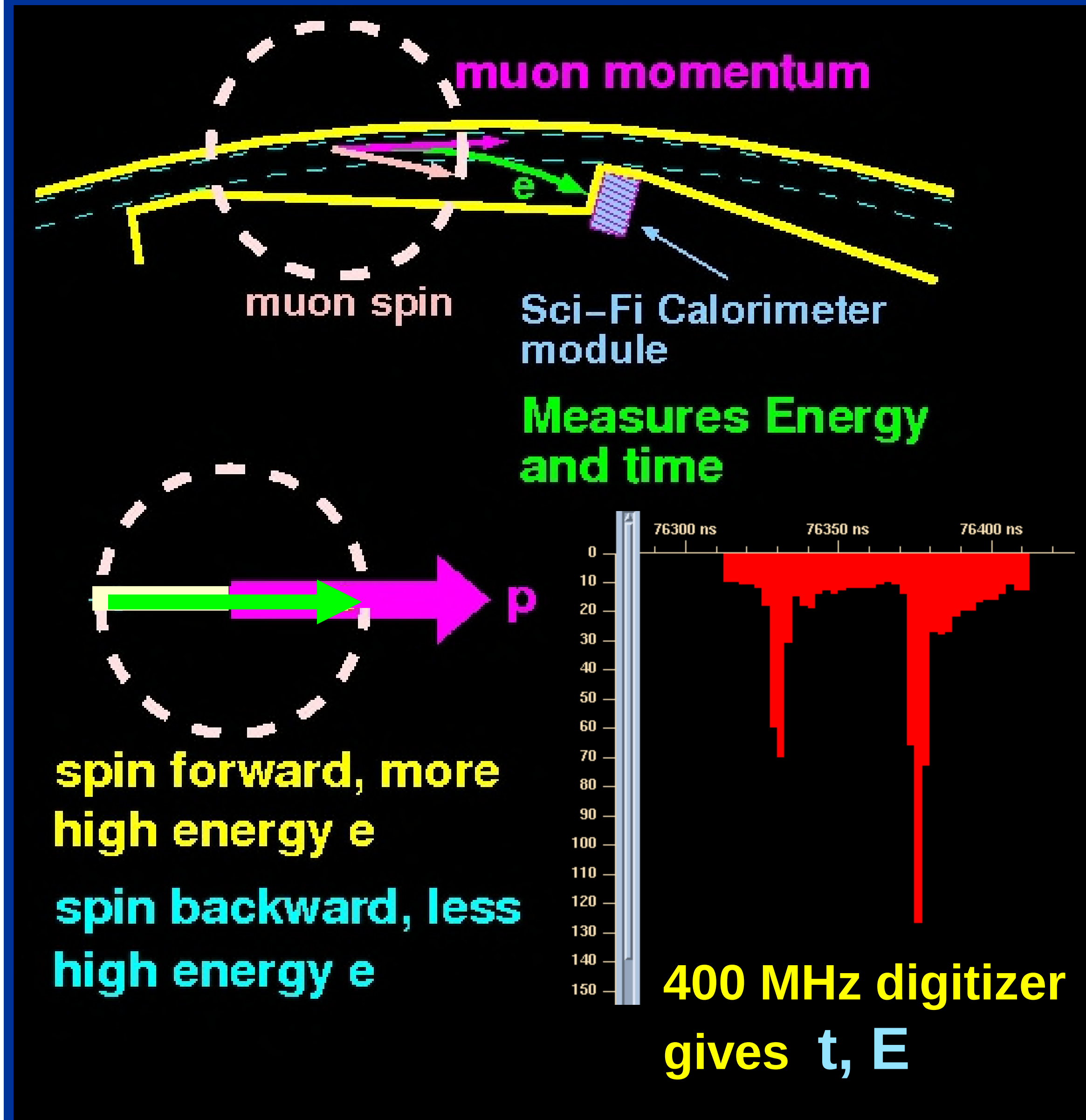


Muon lifetime t_μ **= 64.4 μs**
(g-2) period t_a **= 4.37 μs**
Cyclotron period t_C **= 149 ns**

The experiment setup



To measure ω_a, we used Pb-scintillating fiber calorimeters.



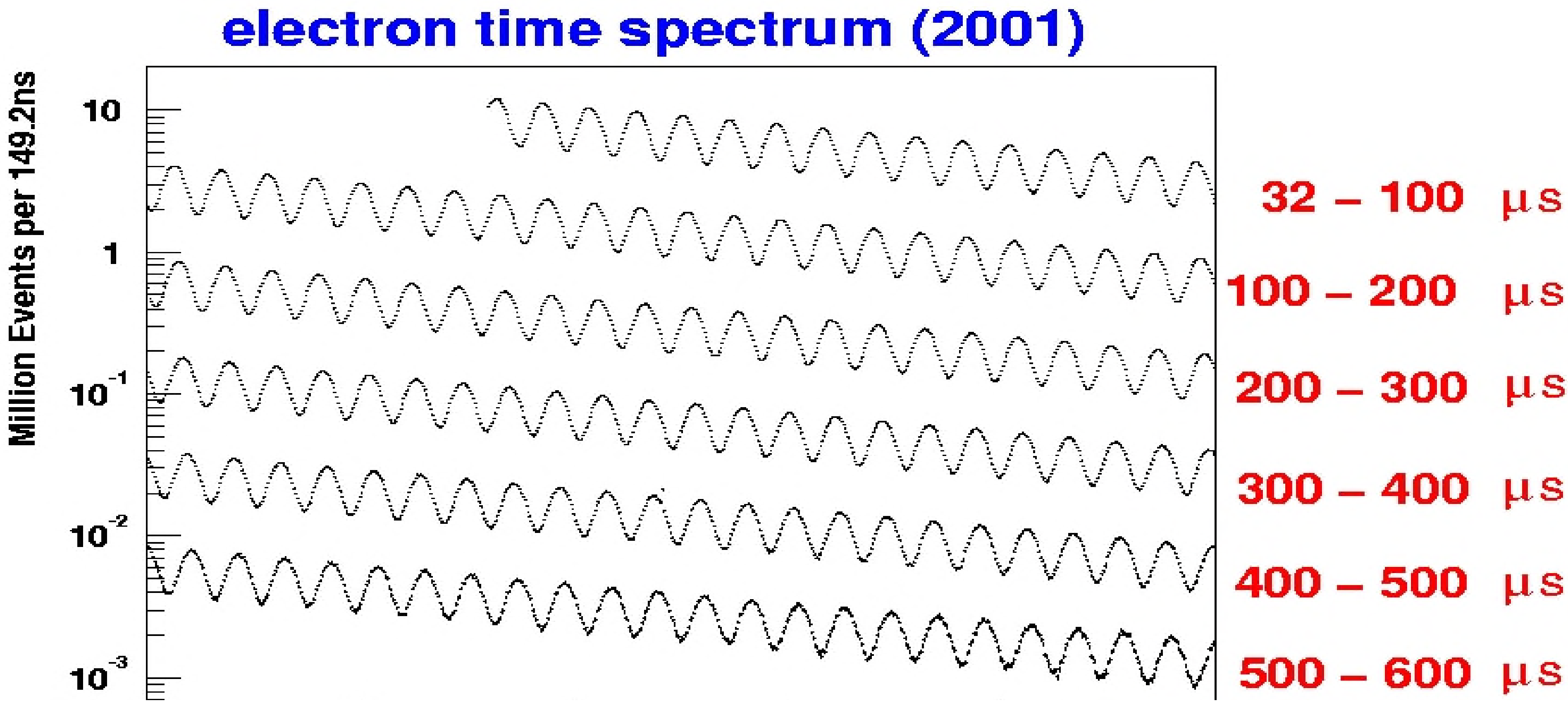
Count number of e^- with $E_e \geq 1.8$ GeV

BNLexpt 821:

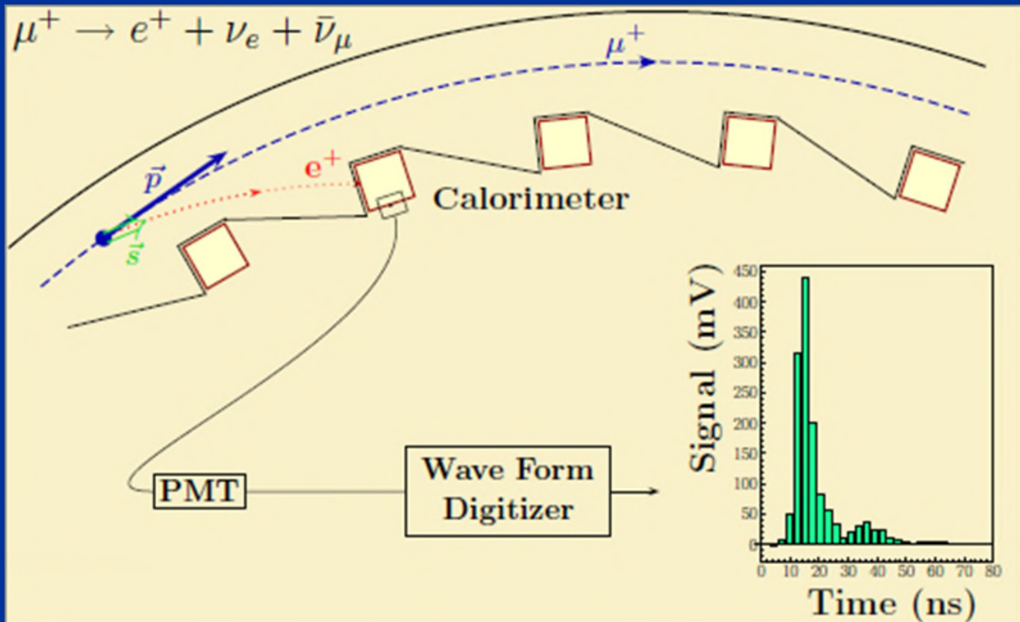
arrival-time spectrum of high-energy decay electrons

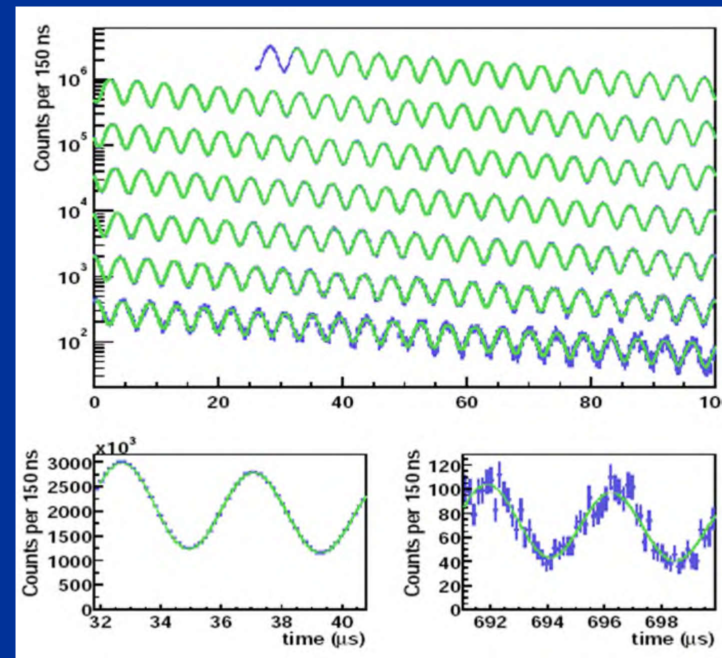
$$N(t) \simeq N(0)e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

$$4\times 10^9 e^-, E_{e^-} > 1.8 GeV$$



Fermilab expt: Measuring ω_a





- The integrated number of electrons (above Eth) modulated at ω_a
- Angular distribution of decayed electrons correlated to muon spin
- Five parameter fit to extract ω_a

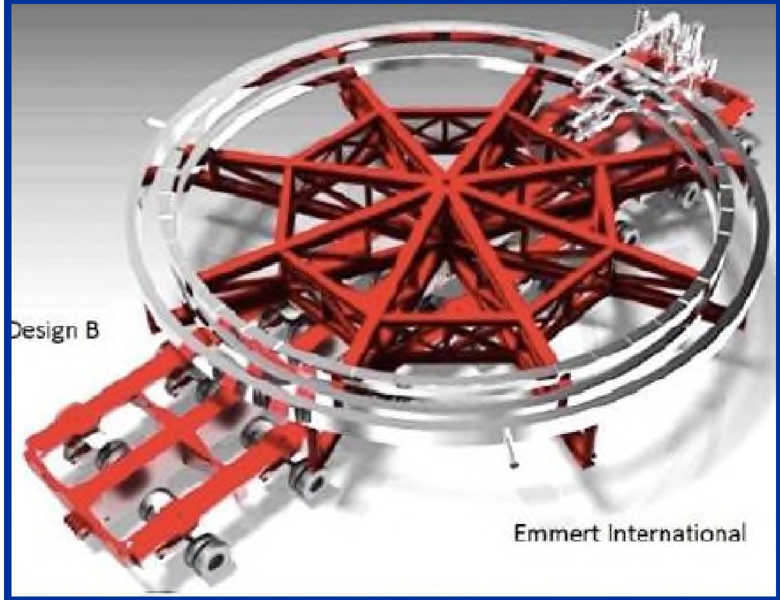
Fermilab Muon Campus

- Recycler
  - 8 GeV protons from Booster
  - Re-bunched in Recycler
  - New connection from Recycler to P1 line (existing connection is from Main Injector)
- Target station
  - Target
  - Focusing (lens)
  - Selection of magic momentum
- Beamlines / Delivery Ring
  - P1 to P2 to M1 line to target
  - Target to M2 to M3 to Delivery Ring
  - Proton removal
  - Extraction line (M4) to g-2 stub to ring in MC1 building

Fermilab Muon Campus

The ring has moved…

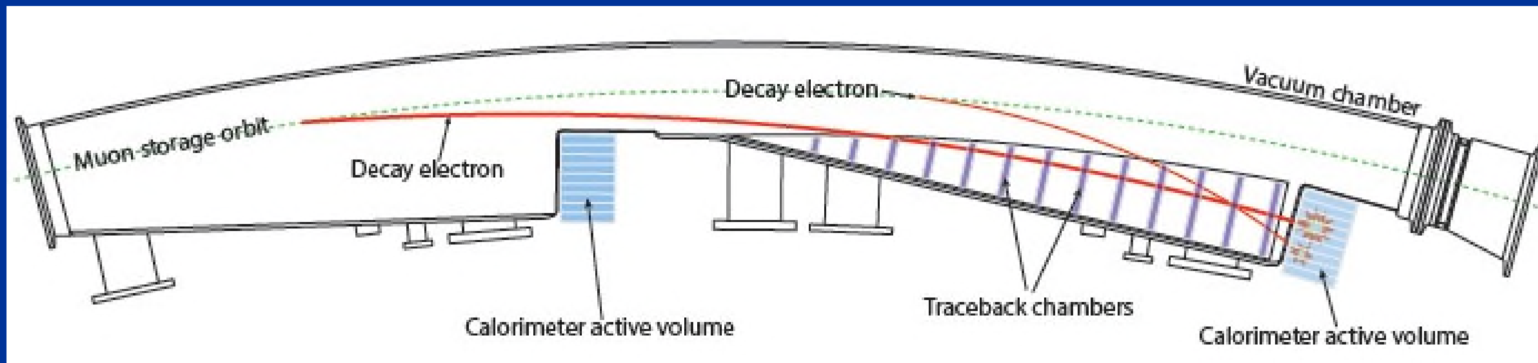
Disassembly

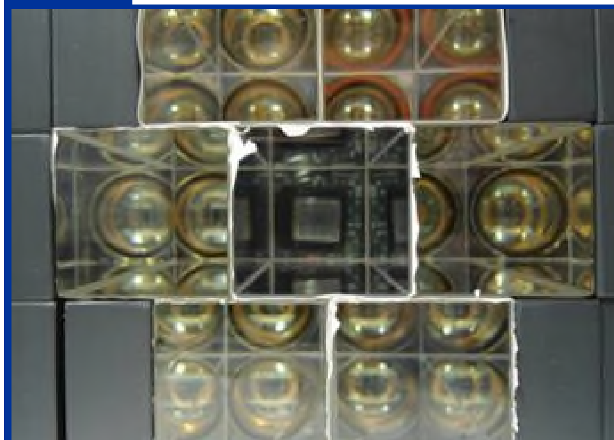


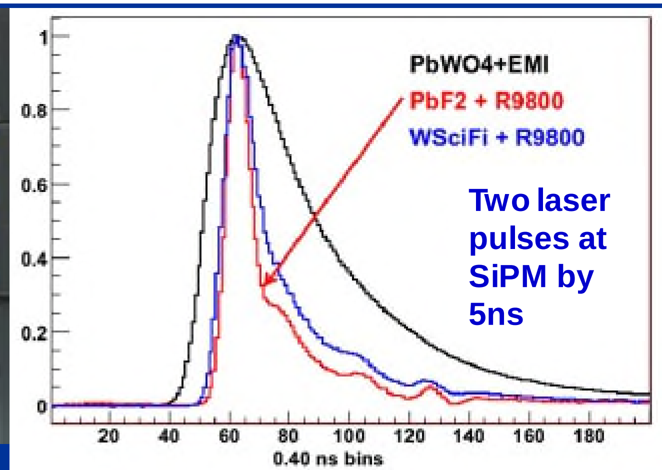
The Big Move

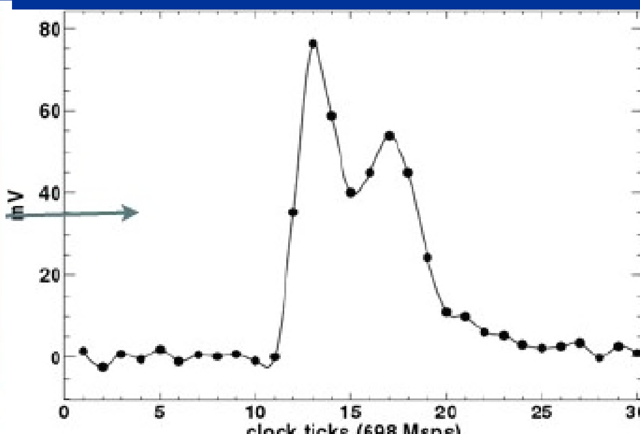
arrived at FNAL July 2013

Detector upgrade









- ❑ Segmented, fast response, crystal calorimeter
- ❑ Lead-fluoride Cherenkov crystal (PbF2) can reduce pileup
- ❑ Silicon photomultiplier (SiPM) directly on back of PbF2
- ❑ Not disturb magnetic field, avoid long lightguides

Summary

- The measurement of $e^{\pm}$ and $\mu^{\pm}$ magnetic dipole moments has been an important benchmark for the development of QED and the standard model of particle physics.
- Presently, difference between measured a_μ and the standard-model prediction (3.6 σ)
- theory situation is evolving
- a_μ has been particularly valuable in restricting physics beyond the standard model.
- will continue to be important in guiding the interpretation of the LHC data.

Summary – g-2 at Fermilab

- Flagship project within Fermilab muon campus
- Received Mission Need approval
- g-2 is extremely sensitive to new physics and high order calculations, correction
- Aiming to reduce experimental uncertainty by a factor of 4
- Theoretical uncertainty also expected to reduce by a factor of 2
- Could achieve 5.6 σ deviation with the same central value
- Great discovery potential and bright future in line with Fermilab muon / Project-X programs

References, acknowledgments

- Used slides from presentations by experts:
  - Lee Roberts http://g2pc1.bu.edu/~roberts/
  - Liang Li talk at SPSC2013 https://indico.cern.ch/event/234546/session/9/contribution/20/attachments/392218/545413/Muon_g-2_Experiment_at_Fermilab_SPCS2013.pdf
  - Hoecker&Marciano: g-2 summary at PDG http://pdg.lbl.gov/2013/reviews/rpp2013-rev-g-2-muon-anom-mag-moment.pdf