New Kinds of Ordered Matter and New Kinds of Quantum Machines

Nick Bonesteel, Dept. of Physics & NHMFL, Florida State University
Two Big Questions

(1) What are the different ways matter can become ordered?

The classics…

Crystals

Magnets

Superconductors

~ 30 years ago, a new kind of order was discovered…

Quantum Hall States  Spin Liquids  Topological Insulators
Two Big Questions

(2) What are the fundamental limits on the machines we can build?

Today’s state of the art: The digital computer

Fundamental building block: The Bit

0 or 1

~ 20 years ago, people realized a qualitatively new kind of computer was possible: A quantum computer

Fundamental building block: The Qubit

\[ \alpha |0\rangle + \beta |1\rangle \]

http://web.physics.ucsb.edu/~martinisgroup/
Early Digital Memory

Stone
Early Digital Memory

= 0
Early Digital Memory

= 1
Early Digital Memory

The iStone

= 1
Early Digital Memory

The iStone: 1 bit
Early Digital Memory

The iStone 4: ~ 20 bits
Modern Digital Memory

The iPhone 4: \( \sim 2.6 \times 10^{11} \) bits
Modern Digital Memory

The iPod: $\sim 1.4 \times 10^{12}$ bits
Modern Digital Memory

http://en.wikipedia.org/wiki/Hard_disk_drive
Magnetic Order

A spin-1/2 particle: ●

“spin up”

“spin down”
Magnetic Order

A spin-1/2 particle:  

“spin up”

“spin down”

Many spin-1/2 particles:
Magnetic Order

A spin-1/2 particle:  

“spin up”  

“spin down”  

Magnetic Order
Magnetic Order

A spin-1/2 particle:  

“spin up”  

“spin down”  

Magnetic Order = 0
Magnetic Order

A spin-1/2 particle:

“spin up”  “spin down”

Magnetic Order = 1
Another Kind of Order

A valence bond:

\[ \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \]
Another Kind of Order

A valence bond:

\[
\begin{align*}
&= \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)
\end{align*}
\]
Another Kind of Order

A valence bond:

\[ = \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow) \]

Many spin-1/2 particles:
Another Kind of Order

A valence bond:

\[ \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow) \]
Another Kind of Order

A valence bond:

\[ \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow) \]
Another Kind of Order

A valence bond:

$$= \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)$$
Another Kind of Order

A valence bond:

\[
\frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)
\]
Another Kind of Order

A valence bond:

\[
\frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)
\]
Another Kind of Order

A valence bond:

\[
\begin{align*}
\text{bond} & = \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \\
\end{align*}
\]
Another Kind of Order

A valence bond:

$$\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$
Another Kind of Order

A valence bond:

\[ \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \]
Another Kind of Order

A valence bond:

\[ \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow) \]

Quantum superposition of valence-bond states. A “spin liquid.”
Another Kind of Order

A valence bond:

\[
\frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow)
\]
Another Kind of Order

A valence bond:

\[ \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \]
Another Kind of Order

A valence bond:

\[ \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \]

Even
Another Kind of Order

A valence bond:

\[
\frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow)
\]
Another Kind of Order

A valence bond:

\[ \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \]
Another Kind of Order

A valence bond:

\[
\frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)
\]
Another Kind of Order

A valence bond:

\[ \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \]
Is it a 0 or a 1?
Is it a 0 or a 1?
Is it a $|0\rangle$ or a $|1\rangle$?
Is it a $|0\rangle$ or a $|1\rangle$?
Is it a $|0\rangle$ or a $|1\rangle$?
Conventional Order: Excitations
Conventional Order: Excitations
Conventional Order: Excitations

Magnon = one spin flip: Total $S_z$ changes by $+1$
Topological Order: Excitations
Topological Order: Excitations
Topological Order: Excitations

Breaking a bond creates an excitation with $S_z = 1$
Breaking a bond creates an excitation with $S_z = 1$
Topological Order: Excitations

Breaking a bond creates an excitation with $S_z = 1$
Fractionalization

$S_z = 1$ excitation \textit{fractionalizes} into two $S_z = \frac{1}{2}$ excitations.
Particle Exchange in 2+1 Dimensions

Particle “world-lines” form braids in 2+1 (=3) dimensions
Topological Quantum Computing

\[ |\Psi_f\rangle = \begin{pmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MM} \end{pmatrix} |\Psi_i\rangle \]
Topological Quantum Computing

\[ |\Psi_f\rangle = \left( \begin{array}{ccc} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MM} \end{array} \right) |\Psi_i\rangle \]
Matrix depends only on the topology of the braid swept out by anyon world lines!

Kitaev ’97, Freedman, Larsen, and Wang ‘03
Where do things stand now?

Recent progress toward building a quantum computer, together with the growing body of experimental work on topologically ordered states is teaching us new and surprising things about how matter can order and what we can do with it.

Exciting times are ahead!