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Resummation for heavy quark and jet cross sections

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Abstract

We review the resummation of threshold logarithms for heavy quark, dijet, direct photon, and W boson production cross sections in hadronic collisions. Beyond leading logarithms the resummed cross section is sensitive to the color exchange in the hard scattering. The resummation is formulated at next-to-leading logarithmic or higher accuracy in terms of anomalous dimension matrices which describe the factorization of soft gluons from the hard scattering. We give results for the soft anomalous dimension matrices at one loop for the full range of partonic subprocesses involved in heavy quark, dijet, direct photon, and W boson production. We discuss the general diagonalization procedure that can be implemented for the calculation of the resummed cross sections, and we give numerical results for top quark production at the Fermilab Tevatron. We also present analytical results for the one- and two-loop expansions of the resummed cross sections.

1 Introduction

The calculation of a large number of hadronic cross sections in perturbative Quantum Chromodynamics (QCD) relies on factorization theorems [1, 2, 3, 4] which separate the short-distance hard scattering, calculable in perturbation theory, from universal parton distribution functions that are determined from experiment. Factorization introduces a scale μ which separates the short- and long-distance physics; the factorization scale is often taken to be the same as the renormalization scale, though in principle these two quantities are independent. Leading order calculations for the cross sections exhibit a strong dependence on μ, but next-to-leading order (NLO) calculations diminish this scale dependence, and sensitivity to even higher orders is normally exhibited by the variation of the scale. NLO results exist for a wide range of processes including Drell-Yan [5], direct-photon [6], electroweak boson [7], Higgs [8], heavy quark [9], and jet production [10]. Results exist at two-loops or higher for some processes such as Drell-Yan production [11], e^+e^- annihilation [12], and deep inelastic scattering [13]. Calculations to even higher orders are formidable and in general one cannot make more precise predictions. Near threshold for the production of the observed final state, however, there are large logarithmic corrections which arise from soft gluon emission and which can be resummed to all orders in perturbation theory [14]. At threshold there is no phase space left for the emission of gluonic radiation and the cancellation of infrared divergences between virtual and real diagrams is incomplete. The resultant singular distributions were first resummed for the Drell-Yan process [15, 16]; this resummation is related to an earlier study of the transverse momentum distribution of hadron pairs in e^+e^- annihilation [17]. Here we will review the threshold resummation of Sudakov logarithms in heavy quark, dijet, direct photon, and W boson production in hadronic collisions, giving special attention to the resummation of next-to-leading logarithms.

The resummation of the leading theshold logarithms in heavy quark production cross sections [18, 19, 20, 21, 22, 23, 24] and inclusive differential distributions [25, 19, 21] is based on the analogy with the Drell-Yan process since the leading logarithms (LL) are universal. The top quark has finally been found at the Fermilab Tevatron [26, 27] (for a review see [28, 29]). The top's mass and production cross section are of great interest, and the precision of their measurements will increase in future runs, so it is important to have good theoretical predictions for the cross section in terms of the top mass; threshold effects are expected to play a significant role in these predictions. Threshold resummation will also be of relevance to bottom quark production at the HERA-B experiment at DESY.

A study of subleading logarithms in heavy quark production near threshold and an attempt to resum them was first presented in Ref. [19]. The resummation of next-to-leading logarithms (NLL) for QCD hard scattering and heavy quark production, in particular, was first put on a solid theoretical framework in [21, 30, 31]. It was found that at NLL one has to take into account the color exchange in the hard scattering. The resummation of the cross section is achieved by refactorizing the cross section into hard components associated with the hard scattering, center-of-mass distributions for the colliding partons, and a soft function that describes coherent [32] noncollinear soft gluon emission. The soft function is a matrix in color space and it satisfies a renormalization group equation in terms of soft anomalous dimensions. The resummation is then given in terms of the angular-dependent eigenvectors and eigenvalues of the anomalous dimension matrix. Applications to top and bottom quark production at a fixed center-of-mass scattering angle, $\theta = 90^\circ$ (where the anomalous dimension matrix is diagonal), were discussed in Ref. [33]. More recently the full angle-integrated cross section was presented and numerical results given for top quark production at the Tevatron [34]. A different formalism [35] which avoids the angular dependence in the resummed corrections has appeared recently for the heavy quark total cross section only (and also for direct photon production [36]). The formalism of Refs. [21, 30, 31] and its extensions that we review in this paper is more general since it keeps an explicit angular dependence; it can thus also be applied easily to differential distributions [37].

Natural extensions of the NLL resummation formalism to the hadroproduction of dijets have been presented in Refs. [38, 39]. In addition, rapidity gaps in high-p_T dijet events have been studied by resumming logarithms of the energy flow into the central rapidity interval [40]. The resummation formalism has also been applied to single-particle inclusive cross sections [41] (including direct photon production) and the electroproduction of heavy quarks [42]. Extensions to W + jet production are also straightforward [43].

The NLL resummation formalism that we review here concerns hadronic processes of the form

$$h_a(p_a) + h_b(p_b) \to T(Q^2, y, \chi) + X\,, \tag{1.1}$$

where h_a, h_b, are incoming hadrons and $T(Q^2, y, \chi)$ denotes the hadronic final state, such as a pair of heavy quarks or high-p_T jets, produced with large invariant mass Q and rapidity y. The variable χ represents the internal structure of the final state, for example the angle θ between the direction of the produced heavy quark, or jet, and the beam axis, or the rapidity interval between the two final-state jets. The cross section for such processes may be written in factorized form as a convolution of the perturbatively calculable hard-scattering partonic cross section, $H_{f_a f_b}$, with parton distributions $\phi_{f_i/h}$, at factorization scale μ, for parton f_i carrying a momentum fraction x_i of hadron h. Thus, we have:

$$\begin{aligned}\frac{d\sigma_{h_a h_b \to T}(S, Q^2, y, \chi)}{dQ^2\, dy\, d\chi} &= \sum_{f_a, f_b} \int \frac{dx_a}{x_a}\, \frac{dx_b}{x_b}\, \phi_{f_a/h_a}(x_a, \mu^2)\, \phi_{f_b/h_b}(x_b, \mu^2) \\ &\qquad \times H_{f_a f_b}\left(\frac{Q^2}{x_a x_b S}, y, \chi, \frac{Q}{\mu}, \alpha_s(\mu^2)\right)\,, \end{aligned} \tag{1.2}$$

where S is the center-of-mass (c.m.) energy squared of the incoming hadrons and we sum over all relevant partons f_a, f_b. Although the physical cross section cannot depend on the choice of factorization scheme (usually DIS or $\overline{\text{MS}}$) and factorization scale μ, at any fixed order there is such a dependence. $H_{f_a f_b}$ includes singular distributions which arise, as we mentioned before, from incomplete cancellations between cross sections with gluon emission and with virtual gluon corrections. These are "plus" distributions, of the form $[\ln^m(1-z)/(1-z)]_+$, $m \le 2n-1$ at nth order in the perturbative expansion, which are singular for $z = 1$, where

$$z = \frac{Q^2}{x_a x_b S} = \frac{Q^2}{s}\,, \tag{1.3}$$

with $s = x_a x_b S$ the invariant mass squared of the partons that initiate the hard scattering. We shall refer to $z = 1$ as "partonic threshold" or the "elastic limit". We note that by partonic threshold, we mean that the c.m. total energy of the incoming partons is just enough to produce a fixed final state, such as a pair of heavy quarks or jets; thus, heavy quarks, for example, are not necessarily produced at rest.

The exponentiation of singular distributions is derived in terms of moments of the cross section with respect to

$$\tau = \frac{Q^2}{S} \,. \tag{1.4}$$

Under moments, the convolution in Eq. (1.2) becomes a product of moments of the parton distributions and the partonic cross section. The exponentiated cross section in moment space must be inverted back to momentum space to derive the physical cross section. There are a number of different prescriptions (and an ensuing debate in the literature) for implementing this inversion and avoiding the Landau pole that have been proposed for the Drell-Yan process [44, 45, 46] and heavy quark production [18, 22, 23]. These prescriptions do not of course exhaust all possibilities. Here, we will mostly be concerned with the resummed cross section in moment space.

In Section 2 we present the resummation formalism for heavy quark production. In Sections 3 and 4 we give explicit results for the soft anomalous dimension matrices in the channels $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$, relevant to heavy quark production. Some technical details of the calculation are given in the Appendix. We also give one- and two-loop expansions of the resummed cross section. The anomalous dimension matrices are in general not diagonal. In Section 5 we discuss the diagonalization procedure and present some numerical results for top quark production at the Fermilab Tevatron. In Section 6 we discuss threshold resummation for dijet production and consider the additional complications due to the final state jets. In Sections 7, 8, and 9, we give results for the anomalous dimension matrices for the many subprocesses relevant to dijet production. In Section 10 we briefly discuss resummation for direct photon and W boson production in the context of single-particle inclusive kinematics. We conclude with a summary.

2 Threshold resummation for heavy quark production

In this section we review the general formalism for the resummation of threshold singularities for heavy quark production. We first write the factorized form of the cross section and identify singular distributions in it near threshold. Then we refactorize the cross section into functions associated with gluons collinear to the incoming quarks, non-collinear soft gluons, and the hard scattering. Resummation follows from the renormalization properties of these functions and is given in terms of anomalous dimension matrices for each partonic subprocess involved.

2.1 Factorized cross section

We consider the production of a pair of heavy quarks of momenta p_1, p_2, in collisions of incoming hadrons h_a and h_b with momenta p_a and p_b,

$$h_a(p_a) + h_b(p_b) \to \bar{Q}(p_1) + Q(p_2) + X\,, \tag{2.1}$$

with total rapidity y and scattering angle θ in the pair center-of-mass frame. The production cross section can be written in a factorized form as

$$\begin{aligned}\frac{d\sigma_{h_a h_b \to Q\bar{Q}}(S,Q^2,y,\theta)}{dQ^2\,dy\,d\cos\theta} \quad = \quad & \sum_{f\bar{f}=q\bar{q},gg} \int \frac{dx_a}{x_a}\,\frac{dx_b}{x_b}\,\phi_{f/h_a}(x_a,\mu^2)\,\phi_{\bar{f}/h_b}(x_b,\mu^2) \\ & \times H_{f\bar{f}}\left(\frac{Q^2}{x_a x_b S}, y, \theta, \frac{Q}{\mu}, \alpha_s(\mu^2)\right)\,,\end{aligned} \tag{2.2}$$

where $S = (p_a + p_b)^2$, $Q^2 = (p_1 + p_2)^2$, and we sum over the two main production partonic subprocesses, $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$. The short-distance hard scattering $H_{f\bar{f}}$ is a smooth function only away from the edges of partonic phase space. The parton distribution functions are given in terms of the partonic momentum fractions and the factorization scale μ, which separates long-distance from short-distance physics.

By using the observation of [47] that we can treat the total rapidity of the heavy quark pair as a constant, equal to its value at threshold, we can rewrite the above cross section as

$$\begin{aligned}\frac{d\sigma_{h_a h_b \to Q\bar{Q}}(S,Q^2,y,\theta)}{dQ^2\,dy\,d\cos\theta} \quad = \quad & \sum_{f\bar{f}=q\bar{q},gg} \int_\tau^1 dz \int \frac{dx_a}{x_a}\,\frac{dx_b}{x_b}\,\phi_{f/h_a}(x_a,\mu^2) \\ & \times \phi_{\bar{f}/h_b}(x_b,\mu^2)\,\delta\left(z - \frac{Q^2}{x_a x_b S}\right)\,\delta\left(y - \frac{1}{2}\ln\frac{x_a}{x_b}\right) \\ & \times \hat{\sigma}_{f\bar{f}\to Q\bar{Q}}\left(1-z, \frac{Q}{\mu}, \theta, \alpha_s(\mu^2)\right)\,,\end{aligned} \tag{2.3}$$

where in addition we have introduced an explicit integration over $z = Q^2/s$, and a simplified hard scattering function $\hat{\sigma}$.

In general, $\hat{\sigma}$ includes "plus" distributions with respect to $1-z$, with singularities at nth order in α_s of the type

$$-\frac{\alpha_s^n}{n!}\left[\frac{\ln^m(1-z)}{1-z}\right]_+, \qquad m \le 2n-1\,. \tag{2.4}$$

These "plus" distributions are defined by their integrals with any smooth functions $\mathcal{F}(z)$ (such as parton distribution functions) by

$$\begin{aligned}\int_y^1 dz\left[\frac{\ln^m(1-z)}{1-z}\right]_+ \mathcal{F}(z) \;&=\; \int_y^1 dz\left[\frac{\ln^m(1-z)}{1-z}\right][\mathcal{F}(z)-\mathcal{F}(1)] \\ &\quad -\mathcal{F}(1)\int_0^y dz\left[\frac{\ln^m(1-z)}{1-z}\right]. \end{aligned} \tag{2.5}$$

All distributions of this kind have been resummed for the Drell-Yan production cross section at leading and nonleading logarithms [15, 16]. Here we will review the more recent work on resummation for heavy quark production including next-to leading logarithms [21, 30, 31].

In order to calculate the hard-scattering function $\hat{\sigma}_{f\bar{f}\to Q\bar{Q}}$, we consider the infrared regularized parton-parton scattering cross section, which factorizes as the hadronic cross section. After we integrate over rapidity, we have

$$\begin{aligned}\frac{d\sigma_{f\bar{f}\to Q\bar{Q}}(S,Q^2,\theta,\epsilon)}{dQ^2\,d\cos\theta} \;&=\; \int_\tau^1 dz\int\frac{dx_a}{x_a}\frac{dx_b}{x_b}\,\phi_{f/f}(x_a,\mu^2,\epsilon)\,\phi_{\bar{f}/\bar{f}}(x_b,\mu^2,\epsilon) \\ &\quad\times\delta\left(z-\frac{Q^2}{x_ax_bS}\right)\hat{\sigma}_{f\bar{f}\to Q\bar{Q}}\left(1-z,\frac{Q}{\mu},\theta,\alpha_s(\mu^2)\right). \end{aligned} \tag{2.6}$$

The argument ϵ represents the universal collinear singularities. We note that the leading power as $z\to 1$ comes entirely from the flavor diagonal parton distributions $\phi_{f/f}(x_a,\mu^2,\epsilon)$ and $\phi_{\bar{f}/\bar{f}}(x_b,\mu^2,\epsilon)$.

If we take Mellin transforms of the above equation, the convolution becomes a simple product of the moments of the parton distributions and the hard scattering function $\hat{\sigma}$:

$$\begin{aligned}&\int_0^1 d\tau\tau^{N-1}\frac{d\sigma_{f\bar{f}\to Q\bar{Q}}(S,Q^2,\theta,\epsilon)}{dQ^2\,d\cos\theta} \\ &\quad=\tilde{\phi}_{f/f}(N,\mu^2,\epsilon)\,\tilde{\phi}_{\bar{f}/\bar{f}}(N,\mu^2,\epsilon)\,\hat{\sigma}_{f\bar{f}\to Q\bar{Q}}(N,Q/\mu,\theta,\alpha_s(\mu^2))\,, \end{aligned} \tag{2.7}$$

with the moments given by $\tilde{\sigma}(N) = \int_0^1 dz\, z^{N-1}\hat{\sigma}(z)$, $\tilde{\phi}(N) = \int_0^1 dx\, x^{N-1}\phi(x)$. We then factorize the initial-state collinear divergences into the parton distribution functions, expanded to the same order in α_s as the partonic cross section, and thus obtain the perturbative expansion for the infrared-safe hard-scattering function, $\hat{\sigma}$.

We note that under moments divergent distributions in $1-z$ produce powers of $\ln N$:

$$\int_0^1 dz\, z^{N-1} \left[\frac{\ln^m(1-z)}{1-z}\right]_+ = \frac{(-1)^{m+1}}{m+1} \ln^{m+1} N + \mathcal{O}\left(\ln^{m-1} N\right) . \tag{2.8}$$

The hard scattering function $\hat{\sigma}$ still has sensitivity to soft-gluon dynamics through its $1-z$ dependence (or the N dependence of its moments). We may now refactorize (moments of) the cross section into N-independent hard components H_{IL}, which describe the truly short-distance hard-scattering, center-of-mass distributions ψ, associated with gluons collinear to the incoming partons, and a soft gluon function S_{LI} associated with non-collinear soft gluons. I and L are color indices that describe the color structure of the hard scattering. Then we write the refactorized cross section as [31]

$$\begin{aligned} \int_0^1 d\tau \tau^{N-1} \frac{d\sigma_{f\bar{f}\to Q\bar{Q}}(\tau, Q^2, \theta, \epsilon)}{dQ^2\, d\cos\theta} &= \sum_{IL} H_{IL}\left(\frac{Q}{\mu}, \theta, \alpha_s(\mu^2)\right) \\ \times\, \tilde{S}_{LI}\left(\frac{Q}{N\mu}, \theta, \alpha_s(\mu^2)\right)\; \tilde{\psi}_{f/f}\left(N, \frac{Q}{\mu}, \epsilon\right)\; \tilde{\psi}_{\bar{f}/\bar{f}}&\left(N, \frac{Q}{\mu}, \epsilon\right) + \mathcal{O}(1/N) \,. \end{aligned} \tag{2.9}$$

This factorization is illustrated in Fig. 1, for the process

$$f(p_a) + \bar{f}(p_b) \to \bar{Q}(p_1) + Q(p_2)\,, \tag{2.10}$$

in which $f\bar{f}$ represents a pair of light quarks or gluons that annihilate or fuse, respectively, into a heavy quark pair.

The hard-scattering function takes contributions from the amplitude and its complex conjugate,

$$H_{IL}\left(\frac{Q}{\mu}, \theta, \alpha_s(\mu^2)\right) = {h^*}_L\left(\frac{Q}{\mu}, \theta, \alpha_s(\mu^2)\right)\; h_I\left(\frac{Q}{\mu}, \theta, \alpha_s(\mu^2)\right) . \tag{2.11}$$

The center-of-mass distribution functions ψ absorb the universal collinear singularities associated with the initial-state partons in the refactorized cross

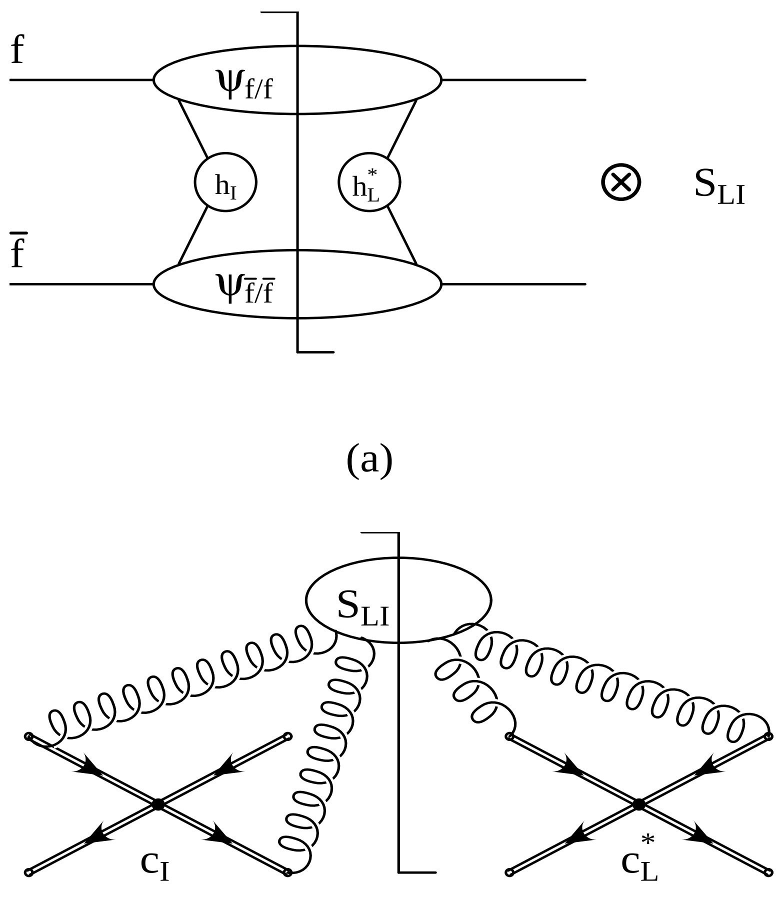


Fig. 1. (a) Factorization for heavy quark production near partonic threshold. (b) The soft-gluon function S_{LI}, in which the vertices c_I, c_L^* link ordered exponentials.

section. They differ from standard light-cone parton distributions by being defined at fixed energy, rather than light-like momentum fraction. We define them by analogy to the light-cone parton distributions ϕ via the matrix elements, evaluated in $n \cdot A = 0$ axial gauge in the partonic c.m. frame,

$$\begin{aligned}
\psi_{q/q}(x, 2p_0/\mu, \epsilon) &= \frac{1}{2\pi 2^{3/2}} \int_{-\infty}^{\infty} dy_0 \; \mathrm{e}^{-ixp_0y_0} \langle q(p)|\bar{q}(y_0, \vec{0}) \frac{1}{2} v \cdot \gamma q(0)|q(p)\rangle \,, \\
\psi_{\bar{q}/\bar{q}}(x, 2p_0/\mu, \epsilon) &= \frac{1}{2\pi 2^{3/2}} \\
&\quad \times \int_{-\infty}^{\infty} dy_0 \; \mathrm{e}^{-ixp_0y_0} \langle \bar{q}(p)|\mathrm{Tr}\left[\frac{1}{2} v \cdot \gamma q(y_0, \vec{0}) \bar{q}(0)\right] \; |\bar{q}(p)\rangle \,, \\
\psi_{g/g}(x, 2p_0/\mu, \epsilon) &= \frac{1}{2\pi 2^{3/2} p^+} \\
&\quad \times \int_{-\infty}^{\infty} dy_0 \; \mathrm{e}^{-ixp_0y_0} \langle g(p)|v_\mu F^{\mu\perp}(y_0, \vec{0}) v_\nu F^{\nu}{}_{\perp}(0)|g(p)\rangle \,,
\end{aligned} \tag{2.12}$$

where the vector v is light-like in the opposite direction from p^μ, and the argument ϵ denotes the collinear singularities that ψ absorbs. The moments of ψ can then be written as products of moments of the light-cone parton distributions ϕ and an infrared safe function.

2.2 Eikonal cross section

The soft function S_{LI} represents the coupling of soft gluons to the partons in the hard scattering process. This coupling may be described by ordered exponentials [48], or eikonal or Wilson lines, written as [38, 39]

$$\Phi_\beta^{(f)}(\lambda_2, \lambda_1; x) = P \exp\left(-ig \int_{\lambda_1}^{\lambda_2} d\lambda \; \beta \cdot A^{(f)}(\lambda\beta + x)\right) \,, \tag{2.13}$$

where the gauge field $A^{(f)}$ is a matrix in the representation of the parton flavor f of the gauge group $SU(3)$, β is the four-velocity of the corresponding parton, and P is an operator that orders group products in the same sense as ordering in the variable λ.

The color tensors of the hard scattering connect together the eikonal lines to which soft gluons couple. We can construct an eikonal operator describing soft-gluon emission as

$$w_I(x)_{\{c_i\}} = \Phi^{(f_2)}_{\beta_2}(\infty,0;x)_{c_2,d_2}\ \Phi^{(f_1)}_{\beta_1}(\infty,0;x)_{c_1,d_1}$$
$$\times\,(c_I)_{d_2 d_1, d_b d_a}\ \Phi^{(f_a)}_{\beta_a}(0,-\infty;x)_{d_a,c_a}\Phi^{(f_b)}_{\beta_b}(0,-\infty;x)_{d_b,c_b}\,, \qquad (2.14)$$

with c_I a color tensor. Then, we may write a dimensionless eikonal cross section, which describes the emission of soft gluons by the eikonal lines as

$$\sigma^{(\text{eik})}_{LI}\left(\frac{wQ}{\mu},\theta,\alpha_s(\mu^2),\epsilon\right) = \sum_\xi \delta\left(w - w(\xi)\right)$$
$$\times \langle 0|\bar{T}\left[(w_L(0))^\dagger_{\{c_i\}}\right]|\xi\rangle\,\langle\xi|T\left[w_I(0)_{\{c_i\}}\right]|0\rangle\,, \quad (2.15)$$

where ξ is a set of intermediate states which contribute to the weight, w. The moments of $\sigma^{(\text{eik})}$ can be factorized, like the moments of the full cross section, into moments of the soft gluon function, S, times moments of jet functions, j_{IN}, analogous to the ψ's, which absorb the collinear divergences of the incoming eikonal lines. Hence S is free of these collinear divergences. Thus, we may write

$$\tilde{\sigma}^{(\text{eik})}_{LI}\left(\frac{Q}{N\mu},\theta,\alpha_s(\mu^2),\epsilon\right) = \tilde{S}_{LI}\left(\frac{Q}{N\mu},\theta,\alpha_s(\mu^2)\right)$$
$$\times\,\tilde{j}^{(f_a)}_{\text{IN}}\left(\frac{Q}{N\mu},\alpha_s(\mu^2),\epsilon\right)\,\tilde{j}^{(f_b)}_{\text{IN}}\left(\frac{Q}{N\mu},\alpha_s(\mu^2),\epsilon\right)\,, \quad (2.16)$$

where the incoming eikonal jet functions are given by products of eikonal lines as [31, 38]

$$j^{(f_i)}_{\text{IN}}\left(\frac{w_i Q}{\mu},\alpha_s(\mu^2),\epsilon\right) = \frac{Q}{2\pi}\int_{-\infty}^{\infty} dy_0\ \mathrm{e}^{-iw_iQy_0}$$
$$\times\,\langle 0|\ \mathrm{Tr}\Big\{\ \bar{T}[\Phi^{(f_i)\dagger}_{\beta_i}(0,-\infty;y)]\ T[\Phi^{(f_i)}_{\beta_i}(0,-\infty;0)]\ \Big\}\ |0\rangle\,, \qquad (2.17)$$

with $y^\nu = (y_0,\vec{0})$ a vector at the spatial origin.

2.3 Resummation

Now, comparing Eqs. (2.7) and (2.9), we see that the moments of the heavy-quark production cross section are given by

$$\hat{\sigma}_{f\bar{f}\to Q\bar{Q}}(N) = \left[\frac{\tilde{\psi}_{f/f}(N,Q/\mu,\epsilon)}{\tilde{\phi}_{f/f}(N,\mu^2,\epsilon)}\right]^2 \sum_{IL} H_{IL}\left(\frac{Q}{\mu},\theta,\alpha_s(\mu^2)\right)$$

$$\times \tilde{S}_{LI}\left(\frac{Q}{N\mu},\theta,\alpha_s(\mu^2)\right)\,, \tag{2.18}$$

where $f\bar{f}$ denotes $q\bar{q}$ or gg, the sum over I and L is over color tensors, and we have used the fact that $\psi_{q/q}=\psi_{\bar{q}/\bar{q}}$ and $\phi_{q/q}=\phi_{\bar{q}/\bar{q}}$. Each of the factors in Eq. (2.18) is gauge and factorization scale dependent. The constraint that the product of these factors is independent of the choice of gauge and factorization scale results in the exponentiation of logarithms of N in ψ/ϕ and S_{LI} [49, 50]. We now proceed to discuss exponentiation for each factor.

2.3.1 Parton distributions

The first factor, $(\psi_{f/f}/\phi_{f/f})^2$, in Eq. (2.18) is "universal" between electroweak and QCD-induced hard processes, and was computed first with $f=q$ for the Drell-Yan cross section [15]. First, let's present the resummed expression for the ratio ψ/ϕ, with $\mu=Q$. We have

$$\frac{\tilde{\psi}_{f/f}(N,1,\epsilon)}{\tilde{\phi}_{f/f}(N,Q^2,\epsilon)}=R_{(f)}\left(\alpha_s(Q^2)\right)\,\exp\left[E^{(f)}(N,Q^2)\right]\,, \tag{2.19}$$

where

$$\begin{aligned}E^{(f)}\left(N,Q^2\right) \;=\; & -\int_0^1 dz\frac{z^{N-1}-1}{1-z}\,\left\{\int_{(1-z)^2}^{(1-z)^{m_S}}\frac{d\lambda}{\lambda}A^{(f)}\left[\alpha_s(\lambda Q^2)\right]\right.\\ & \left.+B^{(f)}\left[\alpha_s((1-z)^{m_s}Q^2)\right]+\frac{1}{2}\nu^{(f)}\left[\alpha_s((1-z)^2Q^2)\right]\right\}\end{aligned} \tag{2.20}$$

and $R_{(f)}(\alpha_s)$ is an N-independent function of the coupling, which can be normalized to unity at zeroth order. The parameter m_S and the resummed coefficients $B^{(f)}$ depend on the factorization scheme. This scheme-dependence must be compensated for by differences in the parton distributions themselves. In the DIS and $\overline{\mathrm{MS}}$ factorization schemes we have $m_S=1$ and $m_S=0$, respectively.

$A^{(f)}, B^{(f)}$ and $\nu^{(f)}$ are finite functions of their arguments and below we give expressions for them needed at next-to-leading order accuracy in $\ln N$. We have [15, 16]

$$A^{(f)}(\alpha_s)=C_f\left(\frac{\alpha_s}{\pi}+\frac{1}{2}K\left(\frac{\alpha_s}{\pi}\right)^2\right)\,, \tag{2.21}$$

with $C_f = C_F = (N_c^2 - 1)/(2N_c)$ for an incoming quark, and $C_f = C_A = N_c$ for an incoming gluon, with N_c the number of colors. Also

$$K = C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} n_f \,, \tag{2.22}$$

where n_f is the number of quark flavors [51]. $B^{(f)}$ is given for quarks in the DIS scheme by

$$B^{(q)}(\alpha_s) = -\frac{3}{4} C_F \, \frac{\alpha_s}{\pi} \,, \tag{2.23}$$

while it vanishes in the $\overline{\text{MS}}$ scheme for quarks and gluons. Note that the DIS scheme is normally applied to quarks only, but extended definitions for gluons are possible [52]. Finally, the lowest-order approximation to the scheme-independent $\nu^{(f)}$ is [31]

$$\nu^{(f)} = 2C_f \, \frac{\alpha_s}{\pi} \,. \tag{2.24}$$

The above results were for $\mu = Q$. To change the scale μ, we need the renormalization group behavior of the parton distributions ψ and ϕ.

The distribution ψ, and each of its moments, renormalizes multiplicatively, because it is the matrix element of a product of renormalized operators. Thus, it obeys the renormalization group equation

$$\mu \frac{d\tilde{\psi}_{f/f}(N, Q/\mu, \epsilon)}{d\mu} = 2\gamma_f(\alpha_s(\mu^2)) \; \tilde{\psi}_{f/f}(N, Q/\mu, \epsilon) \,, \tag{2.25}$$

where γ_f is the anomalous dimension of the field of flavor f, which is independent of N.

The evolution of the light-cone distribution $\tilde{\phi}_{f/f}$ with the factorization scale μ depends on the factorization scheme that we choose, such as $\overline{\text{MS}}$ or DIS. The simplest case is the $\overline{\text{MS}}$ factorization scheme. In this scheme, the moments of ϕ obey the renormalization group equation

$$\mu \frac{d\tilde{\phi}_{f/f}(N, \mu^2, \epsilon)}{d\mu} = 2\gamma_{ff}(N, \alpha_s(\mu^2)) \; \tilde{\phi}_{f/f}(N, \mu^2, \epsilon) \,, \tag{2.26}$$

where γ_{ff} is the anomalous dimension of the color-diagonal splitting function for flavor f [53]. Since only color-diagonal splitting functions are singular as $x \to 1$, only the flavor-diagonal evolution survives in the large N limit.

Then the generalization of Eq. (2.19) is

$$\begin{aligned}\frac{\tilde{\psi}_{f/f}(N,Q/\mu,\epsilon)}{\tilde{\phi}_{f/f}(N,\mu^2,\epsilon)} &= R_{(f)}\left(\alpha_s(\mu^2)\right)\ \exp\left[E^{(f)}(N,Q^2)\right] \\ \times \exp\left\{-2\int_\mu^Q \frac{d\mu'}{\mu'}\ \left[\gamma_f(\alpha_s(\mu'^2)) - \gamma_{ff}(N,\alpha_s(\mu'^2))\right]\right\}. \end{aligned} \tag{2.27}$$

2.3.2 Renormalization of the hard and soft functions

Next, we discuss resummation for the soft function. The soft matrix S_{LI} depends on N through the ratio $Q/(N\mu)$, and it requires renormalization as a composite operator. Its N-dependence, then, can be resummed by renormalization group analysis [54, 55, 56, 57]. However, the product $H_{IL}S_{LI}$ of the soft function and the hard factors needs no overall renormalization, because the UV divergences of S_{LI} are balanced by construction by those of H_{IL}. Thus, we have [31]

$$\begin{aligned} H_{IL}^{(0)} &= \prod_{i=a,b} Z_i^{-1}\ \left(Z_S^{-1}\right)_{IC} H_{CD}\left[\left(Z_S^\dagger\right)^{-1}\right]_{DL}, \\ S_{LI}^{(0)} &= (Z_S^\dagger)_{LB} S_{BA} Z_{S,AI}, \end{aligned} \tag{2.28}$$

where $H^{(0)}$ and $S^{(0)}$ denote the unrenormalized quantities, Z_i is the renormalization constant of the ith incoming partonic field external to h_I, and $Z_{S,LI}$ is a matrix of renormalization constants, which describe the renormalization of the soft function, including mixing of color structures. Z_S is defined to include the wave function renormalization necessary for the outgoing eikonal lines that represent the heavy quarks.

From Eq. (2.28), we see that the soft function S_{LI} satisfies the renormalization group equation [31]

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right)\ S_{LI} = -(\Gamma_S^\dagger)_{LB}S_{BI} - S_{LA}(\Gamma_S)_{AI}\,, \tag{2.29}$$

where Γ_S is an anomalous dimension matrix that is calculated by explicit renormalization of the soft function. In a minimal subtraction renormalization scheme and with $\epsilon = 4 - n$, where n is the number of space-time dimensions, the matrix of anomalous dimensions at one loop is given by

$$\Gamma_S(g) = -\frac{g}{2}\frac{\partial}{\partial g}\mathrm{Res}_{\epsilon\to 0} Z_S(g,\epsilon)\,. \tag{2.30}$$

Explicit results for the soft anomalous dimension matrices Γ_S for the partonic subprocesses involved in heavy quark production will be presented in the next two sections.

The renormalization group equation (2.29) is, in general, a matrix equation. Its solution is

$$\begin{aligned}
&\mathrm{Tr}\left\{H\left(\frac{Q}{\mu},\theta,\alpha_s(\mu^2)\right)\ \tilde{S}\left(\frac{Q}{N\mu},\theta,\alpha_s(\mu^2)\right)\right\} \\
&= \mathrm{Tr}\left\{H\left(\frac{Q}{\mu},\theta,\alpha_s(\mu^2)\right)\ \bar{P}\exp\left[\int_\mu^{Q/N}\frac{d\mu'}{\mu'}\,{\Gamma_S}^\dagger\left(\alpha_s(\mu'^2)\right)\right]\right. \\
&\left.\times\ \tilde{S}\left(1,\theta,\alpha_s\left(Q^2/N^2\right)\right)\ P\exp\left[\int_\mu^{Q/N}\frac{d\mu'}{\mu'}\,\Gamma_S\left(\alpha_s(\mu'^2)\right)\right]\right\}\,, \qquad (2.31)
\end{aligned}$$

where the trace is taken in color space, so that

$$\mathrm{Tr}[H\tilde{S}] = H_{IL}\tilde{S}_{LI} = h_I h_L^* \tilde{S}_{LI}\,. \qquad (2.32)$$

At lowest order, $\tilde{S}_{LI} = \mathrm{Tr}[c_L^\dagger c_I]$. The symbols P and $\bar{P}$ refer to path ordering in the same sense as the integration variable μ' and against it, respectively.

2.3.3 Resummed cross section

Using Eqs. (2.18), (2.27), and (2.31), the resummed heavy quark cross section in moment space is then

$$\begin{aligned}
\tilde{\sigma}_{f\bar{f}\to Q\bar{Q}}(N) \;=\; & R^2_{(f)}(\alpha_s(\mu^2))\ \exp\left\{2\left[E^{(f_i)}(N,Q^2)\right.\right. \\
& \left.\left. -\,2\int_\mu^Q\frac{d\mu'}{\mu'}\ \left[\gamma_{f_i}(\alpha_s(\mu'^2))-\gamma_{f_if_i}(N,\alpha_s(\mu'^2))\right]\right]\right\} \\
& \times\ \mathrm{Tr}\left\{H\left(\frac{Q}{\mu},\theta,\alpha_s(\mu^2)\right)\ \bar{P}\exp\left[\int_\mu^{Q/N}\frac{d\mu'}{\mu'}\,\Gamma_S^\dagger\left(\alpha_s(\mu'^2)\right)\right]\right. \\
& \left.\times\ \ \tilde{S}\left(1,\theta,\alpha_s(Q^2/N^2)\right)\ P\exp\left[\int_\mu^{Q/N}\frac{d\mu'}{\mu'}\,\Gamma_S\left(\alpha_s(\mu'^2)\right)\right]\right\}\,. \qquad (2.33)
\end{aligned}$$

At the level of leading logarithms of N in the soft gluon function S, and therefore at next-to-leading logarithm of N in the cross section as a whole, we may simplify this result by choosing a color basis in which the anomalous dimension matrix Γ_S is diagonal, with eigenvalues λ_I for each basis color tensor labelled by I. Then, we have

$$\begin{aligned}\tilde{S}_{LI}\left(\frac{Q}{N\mu},\theta,\alpha_s(\mu^2)\right) &= \tilde{S}_{LI}\left(1,\theta,\alpha_s\left(\frac{Q^2}{N^2}\right)\right)\\ &\times\exp\left[-\int_{Q/N}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}[\lambda_I(\alpha_s(\bar{\mu}^2))+\lambda_L^*(\alpha_s(\bar{\mu}^2))]\right].\end{aligned} \tag{2.34}$$

Thus, in a diagonal basis, and with $\mu = Q$ and $R_{(f)}$ normalized to unity, we can rewrite the resummed cross section in a simplified form as

$$\begin{aligned}\tilde{\sigma}_{f\bar{f}\to Q\bar{Q}}(N) &= H_{IL}\left(\frac{Q}{\mu},\theta,\alpha_s(\mu^2)\right)\ \tilde{S}_{LI}\left(1,\theta,\alpha_s(Q^2/N^2)\right)\\ &\times\exp\left[E_{LI}^{(f\bar{f})}(N,\theta,Q^2)\right],\end{aligned} \tag{2.35}$$

where the exponent is

$$\begin{aligned}E_{LI}^{(f\bar{f})}(N,\theta,Q^2) &= -\int_0^1 dz\frac{z^{N-1}-1}{1-z}\left\{\int_{(1-z)^2}^{(1-z)^{m_S}}\frac{d\lambda}{\lambda}g_1^{(f\bar{f})}[\alpha_s(\lambda Q^2)]\right.\\ &\left.+g_2^{(f\bar{f})}[\alpha_s((1-z)^{m_S}Q^2)]+g_3^{(IL,f\bar{f})}[\alpha_s((1-z)^2Q^2),\theta]\right\},\end{aligned} \tag{2.36}$$

with the functions g_1, g_2, and g_3 defined by

$$\begin{aligned}g_1^{(f\bar{f})} = A^{(f)}+A^{(\bar{f})}\,, \quad g_2^{(f\bar{f})} = B^{(f)}+B^{(\bar{f})}\,,\\ g_3^{(IL,f\bar{f})} = -\lambda_I-\lambda_L^*+\frac{1}{2}\nu^{(f)}+\frac{1}{2}\nu^{(\bar{f})}\,.\end{aligned} \tag{2.37}$$

In the next two sections we present the soft anomalous dimension matrices for heavy quark production through light quark annihilation and gluon fusion, and we give one- and two-loop expansions of the resummed cross section.

3 Soft anomalous dimension matrix for the process $q\bar{q}\to Q\bar{Q}$

3.1 Eikonal diagrams and Γ_S for $q\bar{q} \to Q\bar{Q}$

We begin with the soft anomalous dimension matrix for heavy quark production through light quark annihilation,

$$q(p_a, r_a) + \bar{q}(p_b, r_b) \to \bar{Q}(p_1, r_1) + Q(p_2, r_2)\,, \tag{3.1}$$

where the p_i's and r_i's denote momenta and colors of the partons in the process. The anomalous dimension matrix for this process is 2×2, since the color indices I and L range over two values. We introduce the Mandelstam invariants

$$s = (p_a + p_b)^2\,, \quad t_1 = (p_a - p_1)^2 - m^2\,, \quad u_1 = (p_b - p_1)^2 - m^2\,, \tag{3.2}$$

with m the heavy quark mass, which satisfy $s + t_1 + u_1 = 0$ at partonic threshold. The UV divergent contribution to S_{LI} is the sum of graphs in Figs. 2(a) and 2(b). We give details of the calculation in the Appendix.

In our calculations we use Feynman rules for eikonal diagrams in axial gauge as shown in Fig. 3 (resummation can be performed in a covariant gauge as well [58]). We define dimensionless vectors v_i^μ by $p_i^\mu = Q v_i^\mu/\sqrt{2}$, which obey $v_i^2 = 0$ for the light incoming quarks and $v_i^2 = 2m^2/Q^2$ for the outgoing heavy quarks. The propagator for a quark, antiquark or gluon eikonal line is then given by

$$\frac{i}{\delta v \cdot q + i\epsilon}\,, \tag{3.3}$$

where $\delta = +1(-1)$ for the momentum q flowing in the same (opposite) direction as the dimensionless vector v. The interaction vertex for a quark or antiquark eikonal line is

$$-\,i g_s\,(T_F^c)_{ba}\,v^\mu \Delta\,, \tag{3.4}$$

with $\Delta = +1(-1)$ for a quark (antiquark). The T_F^c are the generators of $SU(3)$ in the fundamental representation.

For the determination of Γ_S an appropriate choice of color basis has to be made, e.g. singlet exchange in the s- and u-channels, or singlet and octet exchange in the s-channel. Here we give the result in a general axial gauge in the s-channel singlet-octet basis:

$$\begin{aligned} c_1 &= c_{\rm singlet} = \delta_{r_a r_b}\delta_{r_1 r_2}\,, \\ c_2 &= c_{\rm octet} = (T_F^c)_{r_b r_a}(T_F^c)_{r_2 r_1} = -\frac{1}{2N}c_{\rm singlet} + \frac{1}{2}\delta_{r_a r_2}\delta_{r_b r_1}\,. \end{aligned} \tag{3.5}$$

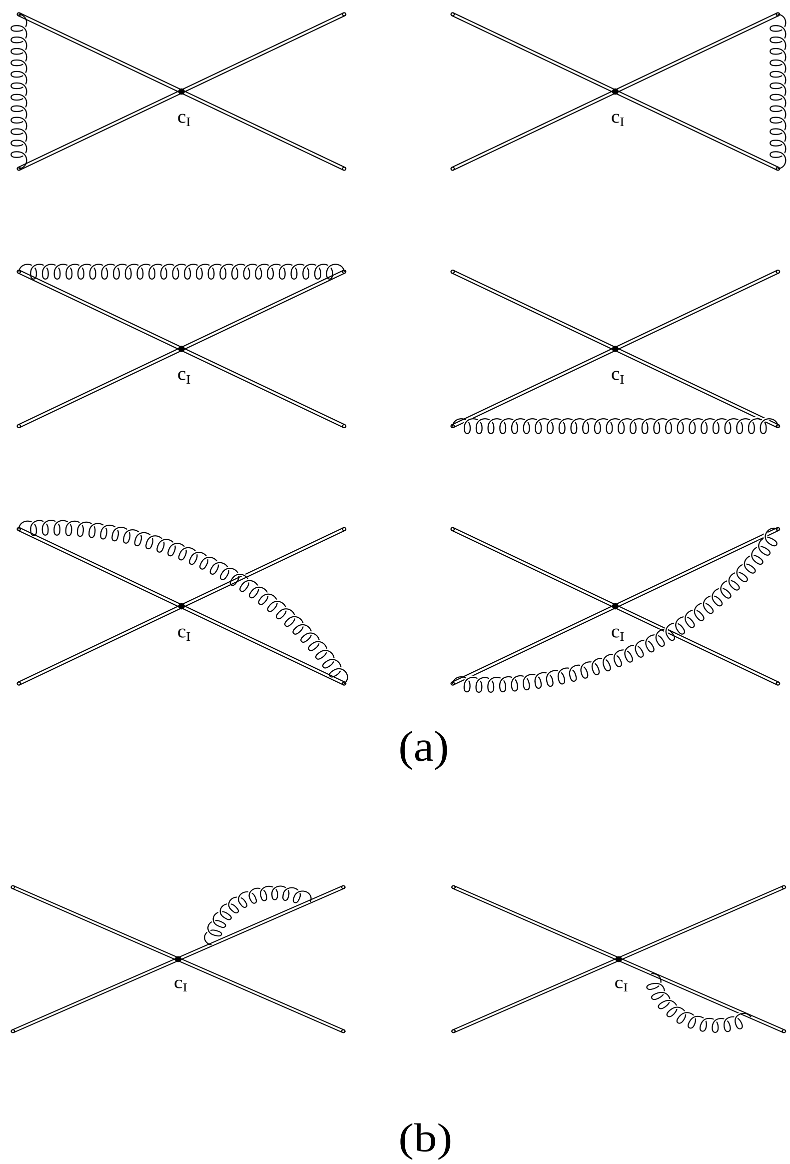


Fig. 2. One-loop corrections to S_{LI} for partonic subprocesses in heavy quark or dijet production: (a) eikonal vertex corrections; (b) eikonal self-energy graphs for heavy quark production.

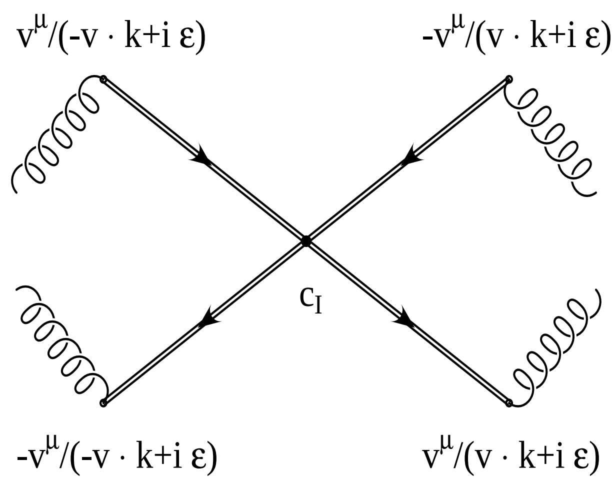


Fig. 3. Feynman rules for eikonal lines representing quarks, in the process $q\bar{q} \to Q\bar{Q}$. The gluon momentum flows out of the eikonal lines. Group matrices at the vertices are the same as for quark lines.

The results of our calculation are [21, 30, 31]

$$\Gamma_S = \left[\begin{array}{cc} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{array} \right] , \tag{3.6}$$

with

$$\begin{aligned}
\Gamma_{11} &= -\frac{\alpha_s}{\pi} C_F \left[L_\beta + \ln(2\sqrt{\nu_a \nu_b}) + \pi i \right] , \\
\Gamma_{21} &= \frac{2\alpha_s}{\pi} \ln\left(\frac{u_1}{t_1}\right) , \\
\Gamma_{12} &= \frac{\alpha_s}{\pi} \frac{C_F}{C_A} \ln\left(\frac{u_1}{t_1}\right) , \\
\Gamma_{22} &= \frac{\alpha_s}{\pi} \left\{ C_F \left[4 \ln\left(\frac{u_1}{t_1}\right) - \ln(2\sqrt{\nu_a \nu_b}) - L_\beta - \pi i \right] \right. \\
&\quad \left. + \frac{C_A}{2} \left[-3 \ln\left(\frac{u_1}{t_1}\right) - \ln\left(\frac{m^2 s}{t_1 u_1}\right) + L_\beta + \pi i \right] \right\} ,
\end{aligned} \tag{3.7}$$

where L_β is the velocity-dependent eikonal function

$$L_\beta = \frac{1 - 2m^2/s}{\beta} \left(\ln \frac{1-\beta}{1+\beta} + \pi i \right) , \tag{3.8}$$

with $\beta = \sqrt{1 - 4m^2/s}$. Note that here we haven't absorbed the function $\nu^{(f)}$, Eq. (2.24), in the results for Γ_S, as was done in Refs. [21, 30, 31]. The gauge dependence of the incoming eikonal lines is given in terms of

$$\nu_i \equiv \frac{(v_i \cdot n)^2}{|n|^2} \,. \tag{3.9}$$

This gauge dependence cancels against corresponding terms in the parton distributions. The gauge dependence of the outgoing heavy quarks is cancelled by the inclusion of the self-energy diagrams in Fig. 2(b) (see also the discussion in the Appendix).

Γ_S is diagonalized in this singlet-octet basis at absolute threshold, $\beta = 0$, and also for arbitrary β when the parton-parton c.m. scattering angle is $\theta = 90°$ (where $u_1 = t_1$), with eigenvalues that may be read off from Eq. (3.7).

In general, the eigenvalues of the anomalous dimension matrix, Eq. (3.6), are

$$\lambda_{1,2} = \frac{1}{2}\left[\Gamma_{11} + \Gamma_{22} \pm \sqrt{(\Gamma_{11} - \Gamma_{22})^2 + 4\Gamma_{12}\Gamma_{21}}\right] \,, \tag{3.10}$$

and the eigenvectors are

$$e_i = \begin{bmatrix} \frac{\Gamma_{12}}{\lambda_i - \Gamma_{11}} \\ 1 \end{bmatrix} \tag{3.11}$$

for each eigenvalue λ_i. These expressions will be useful when we discuss the diagonalization of the anomalous dimension matrices in Section 5.

3.2 One- and two-loop expansions of the resummed cross section for $q\bar{q} \to Q\bar{Q}$

We can expand the resummed heavy quark cross section to any fixed order in perturbation theory without having to diagonalize the soft anomalous dimension matrices. Such fixed-order expansions of resummed cross sections have also been done at one and two loops for the Drell-Yan process [59], Higgs production [60], and heavy quark electroproduction [42]. We will present one- and two-loop expansions of the resummed cross sections for direct photon and W boson production in Section 10.

First we give the one-loop expansion for $q\bar{q} \to Q\bar{Q}$. At one-loop the inverse Mellin transforms are trivial. The result is proportional to the Born cross section, $\sigma^B_{q\bar{q}\to Q\bar{Q}}$, and the one-loop contributions from $g_1^{(q\bar{q})}$ and $g_2^{(q\bar{q})}$, Eq. (2.37), and $\mathrm{Re}\,\Gamma_{22}$, the real part of Γ_{22}. In the DIS scheme the one-loop result is

$$\begin{aligned}
\hat{\sigma}^{\mathrm{DIS}\,(1)}_{q\bar{q}\to Q\bar{Q}}(1-z,m^2,s,t_1,u_1) &= \sigma^B_{q\bar{q}\to Q\bar{Q}}\frac{\alpha_s}{\pi}\left\{2C_F\left[\frac{\ln(1-z)}{1-z}\right]_+\right.\\
&+\left[\frac{1}{1-z}\right]_+\left[C_F\left(\frac{3}{2}+8\ln\left(\frac{u_1}{t_1}\right)-2-2\,\mathrm{Re}\,L_\beta+2\ln\left(\frac{s}{\mu^2}\right)\right)\right.\\
&\left.\left.+C_A\left(-3\ln\left(\frac{u_1}{t_1}\right)+\mathrm{Re}\,L_\beta-\ln\left(\frac{m^2 s}{t_1u_1}\right)\right)\right]\right\}, \qquad (3.12)
\end{aligned}$$

while in the $\overline{\mathrm{MS}}$ scheme

$$\begin{aligned}
\hat{\sigma}^{\overline{\mathrm{MS}}\,(1)}_{q\bar{q}\to Q\bar{Q}}(1-z,m^2,s,t_1,u_1) &= \sigma^B_{q\bar{q}\to Q\bar{Q}}\frac{\alpha_s}{\pi}\left\{4C_F\left[\frac{\ln(1-z)}{1-z}\right]_+\right.\\
&+\left[\frac{1}{1-z}\right]_+\left[C_F\left(8\ln\left(\frac{u_1}{t_1}\right)-2-2\,\mathrm{Re}\,L_\beta+2\ln\left(\frac{s}{\mu^2}\right)\right)\right.\\
&\left.\left.+C_A\left(-3\ln\left(\frac{u_1}{t_1}\right)+\mathrm{Re}\,L_\beta-\ln\left(\frac{m^2 s}{t_1u_1}\right)\right)\right]\right\}. \qquad (3.13)
\end{aligned}$$

Note that these expressions for the cross section are for fixed values of the $Q\bar{Q}$ invariant mass. There are approximate one-loop results for heavy quark production in the literature for a single-particle inclusive cross section where the singular behavior is given in terms of the variable $s_4 = s + t_1 + u_1$ [61] . Therefore there are small differences in the results for the two cross sections due to the different phase spaces. Nevertheless, the cross sections are kinematically equivalent in the limit $\beta \to 0$, where $s_4 = 2m^2(1-z)$.

If one uses single-particle inclusive kinematics in the resummation procedure [41], then one reproduces the results in Ref. [61] exactly. In addition one can add the Coulomb corrections to the one-loop expansions [19, 37] (these corrections also appear in Ref. [61]). Numerically the one-loop expansions of the NLL resummed cross section are very good approximations to the exact NLO cross sections at both the partonic and hadronic levels [34, 37].

One can expand the resummed cross section to higher orders and thus make predictions of perturbation theory at these higher orders near threshold. The inversion back to momentum space at any fixed order is straightforward.

For the two-loop expansion of the resummed cross section in the $\overline{\rm MS}$ scheme and with $\mu = Q$ we have

$$\hat{\sigma}^{\overline{\rm MS}\,(2)}_{q\bar{q}\to Q\bar{Q}}(1-z,m^2,s,t_1,u_1) \;=\; \sigma^B_{q\bar{q}\to Q\bar{Q}}\frac{\alpha_s^2}{\pi^2}\left\{8C_F^2\left[\frac{\ln^3(1-z)}{1-z}\right]_+ \right.$$
$$+\left[\frac{\ln^2(1-z)}{1-z}\right]_+ C_F\left[-\beta_0+12C_F\left(4\ln\left(\frac{u_1}{t_1}\right)-{\rm Re}\,L_\beta-1\right)\right.$$
$$\left.\left.+\,6C_A\left(-3\ln\left(\frac{u_1}{t_1}\right)-\ln\left(\frac{m^2 s}{t_1 u_1}\right)+{\rm Re}\,L_\beta\right)\right]\right\}\,. \quad (3.14)$$

Analogous results have been obtained in the DIS scheme, and also in single-particle inclusive kinematics in both schemes [37].

4 Soft anomalous dimension matrix for the process $gg \to Q\bar{Q}$

4.1 Eikonal diagrams and Γ_S for $gg \to Q\bar{Q}$

We continue with the soft anomalous dimension matrix for heavy quark production through gluon fusion,

$$g(p_a,r_a)+g(p_b,r_b)\to \bar{Q}(p_1,r_1)+Q(p_2,r_2)\,, \quad (4.1)$$

with momenta and colors labelled by the p_i's and r_i's, respectively. Here Γ_S is a 3×3 matrix. We use the same Mandelstam invariants as in the previous section, Eq. (3.2). The UV divergent graphs are the same as in Fig. 2, where now the incoming eikonal lines represent gluons. In our calculations we use the eikonal rules as shown in Fig. 4 (see also the previous section). The gluon eikonal vertex is

$$-\,g_s f^{abc} v^\mu \Delta\,, \quad (4.2)$$

where we read the color indices a,b,c anticlockwise, and where $\Delta=+1(-1)$ for the gluon located below (above) the eikonal line.

We make the following choice for the color basis:

$$c_1=\delta_{r_a r_b}\,\delta_{r_2 r_1},\quad c_2=d^{r_a r_b c}\,(T_F^c)_{r_2 r_1},\quad c_3=i f^{r_a r_b c}\,(T_F^c)_{r_2 r_1}\,, \quad (4.3)$$

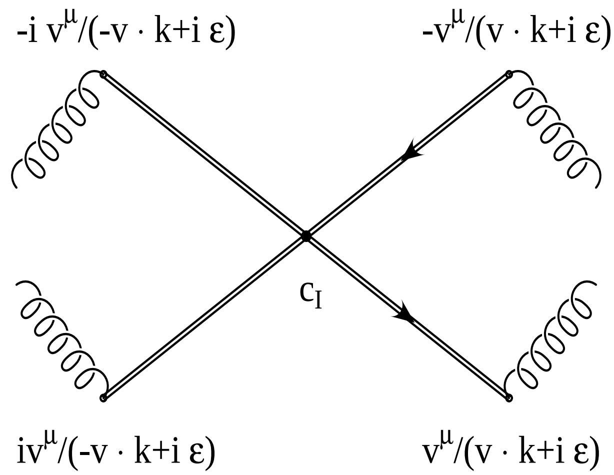


Fig. 4. Feynman rules for eikonal lines representing gluons, in the process $gg \to Q\bar{Q}$. The gluon momentum flows out of the eikonal lines. Group matrices at the vertices on the left side are those of three-gluon vertices; those on the right are as for quark lines.

where again the T_F^c are the generators of $SU(3)$ in the fundamental representation, and d^{abc} and f^{abc} are the totally symmetric and antisymmetric $SU(3)$ invariant tensors, respectively. The anomalous dimension matrix in this color basis and in a general axial gauge is given by [21, 31]

$$\Gamma_S = \begin{bmatrix} \Gamma_{11} & 0 & \frac{\Gamma_{31}}{2} \\ 0 & \Gamma_{22} & \frac{N_c}{4}\Gamma_{31} \\ \Gamma_{31} & \frac{N_c^2-4}{4N_c}\Gamma_{31} & \Gamma_{22} \end{bmatrix}, \tag{4.4}$$

where

$$\begin{aligned} \Gamma_{11} &= \frac{\alpha_s}{\pi}\left[-C_F(L_\beta+1)+C_A\left(-\frac{1}{2}\ln\left(4\nu_a\nu_b\right)+1-\pi i\right)\right], \\ \Gamma_{31} &= \frac{\alpha_s}{\pi}\ln\left(\frac{u_1^2}{t_1^2}\right), \\ \Gamma_{22} &= \frac{\alpha_s}{\pi}\left\{-C_F(L_\beta+1)+\frac{C_A}{2}\left[-\ln\left(4\nu_a\nu_b\right)+2+\ln\left(\frac{t_1u_1}{m^2s}\right)+L_\beta-\pi i\right]\right\}. \end{aligned} \tag{4.5}$$

Here we haven't absorbed the function $\nu^{(f)}$, Eq. (2.24), in the results for Γ_S, as was done in Refs. [21, 31]. Again we note that Γ_S is diagonalized in this basis at absolute threshold, $\beta = 0$, and also for arbitrary β, when the parton-parton c.m. scattering angle is $\theta = 90^\circ$, with eigenvalues that may be read off from Eq. (4.5).

The eigenvalues of Γ_S may be written as

$$\begin{aligned}
\lambda_1 &= \frac{1}{3}(X^{1/3} - Y + \Gamma_{11} + 2\Gamma_{22}), \\
\lambda_{2,3} &= \frac{1}{3}\left[-\frac{1}{2}(X^{1/3} - Y) + \Gamma_{11} + 2\Gamma_{22} \pm \frac{1}{2}i\sqrt{3}(X^{1/3} + Y)\right], \qquad (4.6)
\end{aligned}$$

where

$$\begin{aligned}
X &= (\Gamma_{11} - \Gamma_{22})^3 - \frac{9}{16}(N_c^2 - 8)\Gamma_{31}^2(\Gamma_{11} - \Gamma_{22}) \\
&\quad + \frac{3\sqrt{3}}{4}\left[-\frac{(N_c^2+4)^3}{256}\Gamma_{31}^6 - (N_c^2 - 4)\Gamma_{31}^2(\Gamma_{11} - \Gamma_{22})^4\right. \\
&\quad \left. + \frac{1}{8}((N_c^2 - 8)^2 - 12(N_c^2 - 2))\Gamma_{31}^4(\Gamma_{11} - \Gamma_{22})^2\right]^{1/2} \qquad (4.7)
\end{aligned}$$

and

$$Y = -\left[(\Gamma_{11} - \Gamma_{22})^2 + \frac{3}{16}\Gamma_{31}^2(N_c^2 + 4)\right] X^{-1/3}. \qquad (4.8)$$

The eigenvectors of Γ_S are given by

$$e_i = \begin{bmatrix} \frac{\Gamma_{31}}{2(\lambda_i - \Gamma_{11})} \\ \frac{N_c \Gamma_{31}}{4(\lambda_i - \Gamma_{22})} \\ 1 \end{bmatrix}, \qquad (4.9)$$

for each eigenvalue λ_i.

4.2 One- and two-loop expansions of the resummed cross section for $gg \to Q\bar{Q}$

Again, we may expand the resummed cross section for $gg \to Q\bar{Q}$ at one and two loops or higher. In this case the color decomposition is more complicated [21, 31]. The one-loop expansion in the $\overline{\mathrm{MS}}$ scheme is

$$\begin{aligned}
\hat{\sigma}^{\overline{\mathrm{MS}}\,(1)}_{gg\to Q\bar{Q}}(1-z,m^2,s,t_1,u_1) \;&=\; \sigma^B_{gg\to Q\bar{Q}}\frac{\alpha_s}{\pi}\left\{4C_A\left[\frac{\ln(1-z)}{1-z}\right]_+\right.\\
&\left.\quad -2C_A\ln\left(\frac{\mu^2}{s}\right)\left[\frac{1}{1-z}\right]_+\right\}\\
&+\alpha_s^3K_{gg}B_{QED}\left[\frac{1}{1-z}\right]_+\left\{N_c(N_c^2-1)\frac{(t_1^2+u_1^2)}{s^2}\left[\left(-C_F+\frac{C_A}{2}\right)\mathrm{Re}\,L_\beta\right.\right.\\
&\left.+\frac{C_A}{2}\ln\left(\frac{t_1u_1}{m^2s}\right)-C_F\right]+\frac{N_c^2-1}{N_c}(C_F-C_A)\mathrm{Re}\,L_\beta\\
&\left.-(N_c^2-1)\ln\left(\frac{t_1u_1}{m^2s}\right)+C_F\frac{N_c^2-1}{N_c}+\frac{N_c^2}{2}(N_c^2-1)\ln\left(\frac{u_1}{t_1}\right)\frac{(t_1^2-u_1^2)}{s^2}\right\},
\end{aligned} \tag{4.10}$$

where $\sigma^B_{gg\to Q\bar{Q}}$ is the Born cross section,

$$B_{\mathrm{QED}}=\frac{t_1}{u_1}+\frac{u_1}{t_1}+\frac{4m^2s}{t_1u_1}\left(1-\frac{m^2s}{t_1u_1}\right), \tag{4.11}$$

and $K_{gg}=(N^2-1)^{-2}$ is a color average factor.

As we explained in the previous section, we cannot compare our one-loop expansion directly with the approximate NLO results of Ref. [61], but as $\beta\to 0$ our expression becomes identical to the $\beta\to 0$ limit of the results in [61]. Again, we note that even for $\beta>0$ our one-loop expansion is nearly the same as in Ref. [61]. If one uses single-particle inclusive kinematics in the resummation procedure then the agreement with Ref. [61] is exact [37].

The two-loop expansion of the resummed cross section in the $\overline{\mathrm{MS}}$ scheme and with $\mu=Q$ is

$$\begin{aligned}
\hat{\sigma}^{\overline{\mathrm{MS}}\,(2)}_{gg\to Q\bar{Q}}(1-z,m^2,s,t_1,u_1) \;&=\; \sigma^B_{gg\to Q\bar{Q}}\frac{\alpha_s^2}{\pi^2}\left\{8C_A^2\left[\frac{\ln^3(1-z)}{1-z}\right]_+\right.\\
&\left.\quad -\beta_0C_A\left[\frac{\ln^2(1-z)}{1-z}\right]_+\right\}\\
&+\frac{\alpha_s^4}{\pi}K_{gg}B_{\mathrm{QED}}\left[\frac{\ln^2(1-z)}{1-z}\right]_+C_A\,3(N_c^2-1)\left\{\frac{(t_1^2+u_1^2)}{s^2}\right.
\end{aligned}$$

$$\times \left[N_c^2 \ln\left(\frac{t_1 u_1}{m^2 s}\right) - 2N_c\left(C_F - \frac{C_A}{2}\right) \mathrm{Re} L_\beta - 2N_c C_F\right] + 2\frac{C_F}{N_c}$$
$$+ 2\ln\left(\frac{m^4}{t_1\, u_1}\right) + 2\frac{1}{N_c}(C_F - C_A)\,\mathrm{Re}\, L_\beta + N_c^2 \frac{(t_1^2 - u_1^2)}{s^2} \ln\left(\frac{u_1}{t_1}\right)\Bigg\} \quad (4.12)$$

Analogous results may also be obtained in single-particle inclusive kinematics [37].

5 Diagonalization procedure and numerical results

As we saw in the previous two sections, the soft anomalous dimension matrices, Γ_S, are in general not diagonal. They are only diagonal at absolute threshold, $\beta \to 0$, and at a scattering angle $\theta = 90°$. In these cases the exponentiated cross section has a simpler form. Numerical studies of the resummed cross section at $\theta = 90°$ for top quark production at the Fermilab Tevatron and bottom quark production at HERA-B were presented in Ref. [33].

In general, however, we must find new color bases where Γ_S is diagonal so that the resummed cross section can take the simpler form of Eq. (2.35). In this section we outline the required diagonalization procedure and apply it to heavy quark production, giving some numerical results for top quark production at the Tevatron.

5.1 General diagonalization procedure

The Born cross section can be written in an arbitrary color basis as a product of the hard components and the color tensors:

$$\sigma^B = H_{IJ}\, c_J^\dagger\, c_I\,. \quad (5.1)$$

In a diagonal basis, $c'_I = c_K R_{KI}$, the Born cross section can be rewritten as

$$\sigma^B = H_{IJ}\, (R^{-1})^\dagger_{JL} (c')^\dagger_L c'_K (R^{-1})_{KI}\,, \quad (5.2)$$

where R is made from the eigenvectors of the anomalous dimension matrix, and we have used $c_I = c'_K R^{-1}_{KI}$. Then the resummed cross section is given by

$$\sigma^{\rm res} = H_{IJ}\, (R^{-1})^\dagger_{JL} (c')^\dagger_L c'_K (R^{-1})_{KI}\, e^{E_{KL}}\,, \quad (5.3)$$

where the exponent E_{KL} takes contributions at NLL from the eigenvalues λ_K and λ_L, and is given for $\mu = Q$ by Eq. (2.36).

Note that for the original color bases that we have chosen for all the subprocessess, we have $c_I c_J^\dagger = 0$ for $I \neq J$; thus, our expressions simplify. Then the Born cross section can be written as

$$\sigma^B = \sum_I H_{II} |c_I|^2 \,, \tag{5.4}$$

and the resummed cross section is given by

$$\sigma^{\text{res}} = \sum_I H_{II} \, (R^{-1})^\dagger_{IL} (c')^\dagger_L c'_K (R^{-1})_{KI} \, e^{E_{KL}} \,. \tag{5.5}$$

In the next two subsections we give more detailed results for the partonic processes involved in heavy quark production.

5.2 Diagonalization and numerical results for the process $q\bar{q} \to Q\bar{Q}$

The Born cross section for a process with a 2×2 anomalous dimension matrix, Γ_S, such as $q\bar{q} \to Q\bar{Q}$, can be written in a general orthogonal color basis as

$$\sigma^B = H_{11}|c_1|^2 + H_{22}|c_2|^2 \,. \tag{5.6}$$

The eigenvalues and eigenvectors of Γ_S are given by Eqs. (3.10) and (3.11), respectively. Then if $C = (c_1 \quad c_2)$ is the original color basis, the diagonal color basis is

$$C' \equiv (c'_1 \quad c'_2) = CR \,, \tag{5.7}$$

where the columns of R are the eigenvectors of Γ_S:

$$R = [e_1 \; e_2] = \left[\begin{array}{cc} \frac{\Gamma_{12}}{\lambda_1 - \Gamma_{11}} & \frac{\Gamma_{12}}{\lambda_2 - \Gamma_{11}} \\ 1 & 1 \end{array} \right] . \tag{5.8}$$

The diagonalized anomalous dimension matrix is then given by

$$\Gamma_S^{\text{diag}} = R^{-1} \Gamma_S R = \left[\begin{array}{cc} \lambda_1 & 0 \\ 0 & \lambda_2 \end{array} \right] . \tag{5.9}$$

To write down the Born cross section in the diagonal basis we use the inverse of Eq. (5.7), i.e. $C = C'R^{-1}$, where the inverse of the matrix R is

$$R^{-1} = \frac{(\lambda_1 - \Gamma_{11})(\lambda_2 - \Gamma_{11})}{\Gamma_{12}(\lambda_2 - \lambda_1)} \left[\begin{array}{cc} 1 & -\frac{\Gamma_{12}}{\lambda_2 - \Gamma_{11}} \\ -1 & \frac{\Gamma_{12}}{\lambda_1 - \Gamma_{11}} \end{array} \right] . \tag{5.10}$$

We can thus express the old color basis tensors in terms of the new basis.

In addition, we note that the Born cross section for the heavy quark production process $q\bar{q} \to Q\bar{Q}$ is pure octet exchange so that there are further simplifications in the basis $C = (c_{\rm singlet} \quad c_{\rm octet})$. The Born cross section is then

$$\sigma^B_{q\bar{q}\to Q\bar{Q}} = H_{22}|c_2|^2 \,, \tag{5.11}$$

with $c_2 = c_{\rm octet}$. Now, $c_{\rm octet}$ can be written in terms of the diagonal basis as

$$c_{\rm octet} = \frac{(\lambda_1 - \Gamma_{11})c'_1 - (\lambda_2 - \Gamma_{11})c'_2}{(\lambda_1 - \lambda_2)} \,. \tag{5.12}$$

Then using the above equation we can rewrite the Born cross section in terms of $|c'_1|^2, |c'_2|^2, c'_1 c'^*_2$, where

$$|c'_{1,2}|^2 = \frac{N_c^2\,\Gamma_{12}^2}{|\lambda_{1,2} - \Gamma_{11}|^2} + \frac{N_c^2 - 1}{4} \tag{5.13}$$

and

$$c'_1\, c'^*_2 = \frac{N_c^2\,\Gamma_{12}^2}{(\lambda_1 - \Gamma_{11})(\lambda_2 - \Gamma_{11})^*} + \frac{N_c^2 - 1}{4} \,. \tag{5.14}$$

To write the resummed cross section in momentum space a prescription must be chosen to invert the moment space exponentiated cross section. Here we use the simplest method of Ref. [18]; of course the diagonalization procedure is general and any prescription for the inversion can be used.

The resummed partonic cross section is then given by [34]

$$\sigma^{\rm res}_{q\bar{q}}(s, m^2) \;\; = \;\; \sum_{i,j=1}^{2} \int_{-1}^{1} d\cos\theta \left[- \int_{s_{\rm cut}}^{s-2ms^{1/2}} ds_4 f_{q\bar{q},ij}(s_4, \theta) \frac{d\overline{\sigma}^{(0)}_{q\bar{q},ij}(s, s_4, \theta)}{ds_4} \right] , \tag{5.15}$$

where $s_4 = s + t_1 + u_1$. The $d\overline{\sigma}^{(0)}_{q\bar{q},ij}(s, s_4, \theta)/ds_4$ are components of the differential of the Born cross section which is given by

$$\begin{aligned}
\frac{d\overline{\sigma}^{(0)}_{q\bar{q}}(s, s_4, \theta)}{ds_4} &= -\pi\alpha_s^2 K_{q\bar{q}} N_c C_F \frac{1}{4s^4}\frac{s - s_4}{\sqrt{(s-s_4)^2 - 4sm^2}} \\
&\times \left[(3(s-s_4)^2 - 8sm^2)(1+\cos^2\theta) + 4sm^2(1-\cos^2\theta)\right] . \quad (5.16)
\end{aligned}$$

The function f is given at NLL by the exponential

$$f_{q\bar{q},ij} = \exp[E_{q\bar{q},ij}] = \exp[E_{q\bar{q}} + E_{q\bar{q}}(\lambda_i, \lambda_j)] \, , \quad (5.17)$$

where in the DIS scheme

$$\begin{aligned}
E^{\rm DIS}_{q\bar{q}} &= \int_{\omega_0}^1 \frac{d\omega'}{\omega'} \Big\{ \int_{\omega'^2 Q^2/\Lambda^2}^{\omega' Q^2/\Lambda^2} \frac{d\xi}{\xi} \Big[\frac{2C_F}{\pi} \Big(\alpha_s(\xi) \\
&\qquad + \frac{1}{2\pi}\alpha_s^2(\xi) K \Big) \Big] - \frac{3}{2}\frac{C_F}{\pi}\alpha_s\Big(\frac{\omega' Q^2}{\Lambda^2}\Big) \Big\} \, , \quad (5.18)
\end{aligned}$$

with $\omega_0 = s_4/(2m^2)$ and Λ the QCD scale parameter. The color-dependent contribution in the exponent is

$$E_{q\bar{q}}(\lambda_i, \lambda_j) = -\int_{\omega_0}^1 \frac{d\omega'}{\omega'} \left\{ \lambda'_i \left[\alpha_s \left(\frac{\omega'^2 Q^2}{\Lambda^2} \right), \theta \right] + \lambda'^*_j \left[\alpha_s \left(\frac{\omega'^2 Q^2}{\Lambda^2} \right), \theta \right] \right\} \, , \quad (5.19)$$

in both mass factorization schemes, where $i = 1, 2$. Here $\lambda' = \lambda - \nu^{(f)}/2$ (see Eq. (2.37)) where the λ's are the eigenvalues of the soft anomalous dimension matrix. The cutoff $s_{\rm cut}$ in Eq. (5.15) regulates the divergence of α_s at low s_4: $s_{\rm cut} > s_{4,\rm min} = 2m^2\Lambda/Q$.

After some algebra, the resummed partonic cross section at NLL accuracy is [34]

$$\begin{aligned}
\sigma^{\rm res}_{q\bar{q}}(s, m^2) &= -\sum_{i,j=1}^{2} \int_{-1}^{1} d\cos\theta \int_{s_{\rm cut}}^{s - 2ms^{1/2}} ds_4 \, \frac{1}{|\lambda_1 - \lambda_2|^2} \frac{d\overline{\sigma}^{(0)}_{q\bar{q}}(s, s_4, \theta)}{ds_4} \\
&\times \left[\left(\frac{4N_c^2}{N_c^2 - 1}\Gamma_{12}^2 + |\lambda_1 - \Gamma_{11}|^2 \right) e^{E_{q\bar{q},11}} + \left(\frac{4N_c^2}{N_c^2 - 1}\Gamma_{12}^2 + |\lambda_2 - \Gamma_{11}|^2 \right) e^{E_{q\bar{q},22}} \right. \\
&\left. - \frac{8N_c^2}{N_c^2 - 1}\Gamma_{12}^2 {\rm Re}\left(e^{E_{q\bar{q},12}} \right) - 2{\rm Re}\left((\lambda_1 - \Gamma_{11})(\lambda_2 - \Gamma_{11})^* e^{E_{q\bar{q},12}} \right) \right] . \quad (5.20)
\end{aligned}$$

The explicit expressions for the quantities in the above equation are long but their derivation is straightforward. Numerical results for the partonic cross sections (resummed and one-loop expansions) are given in [34]. The one-loop expansion of the resummed cross section is a good approximation to the exact NLO result.

The NLL resummed hadronic cross section is given by the convolution of parton distributions $\phi_{i/h}$, for parton i in hadron h, with the partonic cross section

$$\sigma^{\rm res}_{q\overline{q},{\rm had}}(S,m^2) = \sum_{q=u}^{b} \int_{\tau_0}^{1} d\tau \int_{\tau}^{1} \frac{dx}{x} \phi_{q/h_1}(x,\mu^2)\phi_{\overline{q}/h_2}(\frac{\tau}{x},\mu^2)\sigma^{\rm res}_{q\overline{q}}(\tau S,m^2)\,, \tag{5.21}$$

where $\sigma^{\rm res}_{q\overline{q}}(\tau S,m^2)$ is defined in Eq. (5.20) and $\tau_0 = (m+\sqrt{m^2+s_{\rm cut}})^2/S$.

Our numerical results for the $t\overline{t}$ production cross section at the Fermilab Tevatron with $\sqrt{S}=1.8$ TeV are shown in Fig. 5 as functions of the top quark mass. We use the CTEQ 4D DIS parton densities [62, 63]. Since the parton densities are only available at fixed order, the application to a resummed cross section introduces some uncertainty. The NLO exact cross sections, including the factorization scale dependence, are shown in Fig. 5 along with the NLO approximate cross section, i.e. the one-loop expansion of the resummed cross section, calculated with $s_{\rm cut}=0$ and $\mu^2=m^2$. We note the excellent agreement between the NLO exact and approximate cross sections.

As for the NLL resummed cross section, we note that its scale dependence is significantly reduced relative to that of the NLO cross section. In order to match our results to the exact NLO cross section we define the NLL improved cross section

$$\sigma^{\rm imp}_{q\overline{q},{\rm had}} = \sigma^{\rm res}_{q\overline{q},{\rm had}} - \sigma^{\rm NLO,approx}_{q\overline{q},{\rm had}} + \sigma^{\rm NLO,exact}_{q\overline{q},{\rm had}}\,, \tag{5.22}$$

with the same cut applied to the NLO approximate and the NLL resummed cross sections.

In Fig. 5 the hadronic improved cross section is shown for $\mu^2=m^2$ along with the variation with $s_{\rm cut}$. The variation of the improved cross section with change of cutoff is small over the range $30s_{4,{\rm min}} < s_{\rm cut} < 40s_{4,{\rm min}}$. At $m=175$ GeV$/c^2$ and $\sqrt{S}=1.8$ TeV, the value of the improved cross section for $q\overline{q}\rightarrow t\overline{t}$ with $s_{\rm cut}/(2m^2)=0.04$ is 6.0 pb compared to a NLO cross

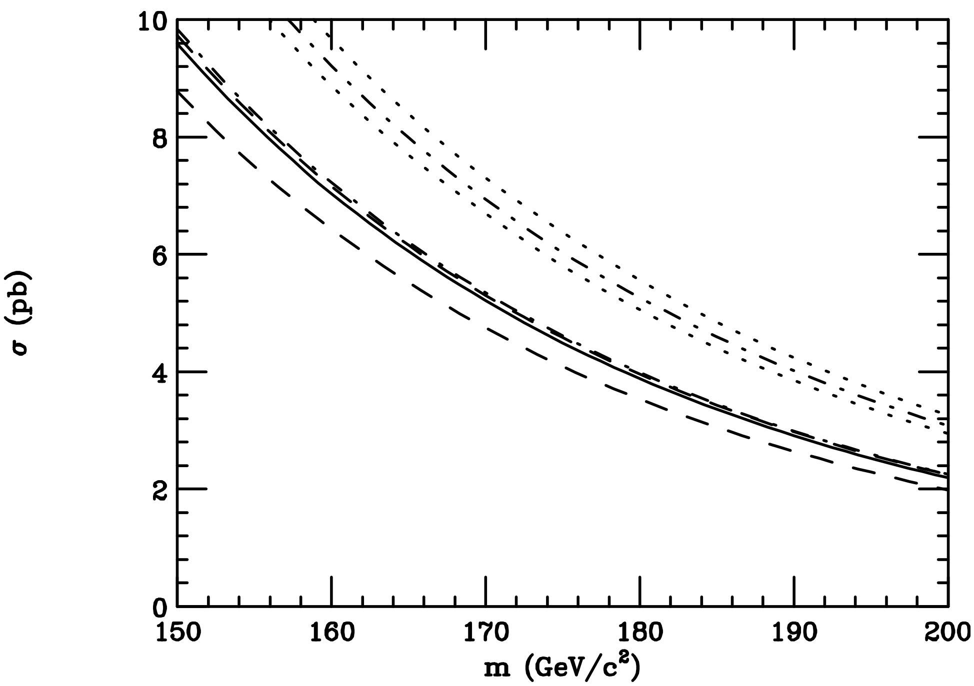


Fig. 5. The NLO exact and approximate and the NLL improved hadronic $t\bar{t}$ production cross sections in the $q\bar{q}$ channel and the DIS scheme are given as functions of top quark mass for $p\bar{p}$ collisions at the Tevatron energy, $\sqrt{S} =$ 1.8 TeV. The NLO exact cross section is given for $\mu^2 = m^2$ (solid curve), $4m^2$ (lower-dashed) and $m^2/4$ (upper-dashed). The NLO approximate cross section with $s_{\rm cut} = 0$ is shown for $\mu^2 = m^2$ (lower dot-dashed). The NLL improved cross section, Eq. (5.22), is given for $s_{\rm cut} = 35 s_{4,\rm min}$ (upper dot-dashed), $30 s_{4,\rm min}$ (upper-dotted) and $40 s_{4,\rm min}$ (lower-dotted).

section of 4.5 pb at $\mu = m$. At the upgraded Tevatron with $\sqrt{S} = 2$ TeV, the corresponding value is 7.8 pb compared to a NLO cross section of 5.9 pb at $\mu = m$. We find that the corrections relative to NLO are larger for larger scales. The gg channel is more complicated and will be discussed in the next subsection. Adding the gg contribution we predict a total cross section of 7 pb at $\sqrt{S} = 1.8$ TeV, in good agreement with experimental values from CDF, $\sigma_{t\bar{t}} = 7.6^{+1.8}_{-1.5}$ pb [64], and D0, $\sigma_{t\bar{t}} = 5.5 \pm 1.8$ pb [65]. Gluon fusion is more important for b-quark production at HERA-B where threshold resummation is also of importance [19, 20].

5.3 Diagonalization for the process $gg \to Q\bar{Q}$

The Born cross section for a process with a 3×3 anomalous dimension matrix, Γ_S, such as $gg \to Q\bar{Q}$, can be written in a general orthogonal color basis as

$$\sigma^B = H_{11}|c_1|^2 + H_{22}|c_2|^2 + H_{33}|c_3|^2 \, . \tag{5.23}$$

The eigenvalues and eigenvectors of Γ_S are given by Eqs. (4.6) and (4.9), respectively. Then if $C = (c_1 \quad c_2 \quad c_3)$ is the original color basis, the diagonal color basis is

$$C' \equiv (c'_1 \quad c'_2 \quad c'_3) = CR \, , \tag{5.24}$$

where

$$R = [e_1 \; e_2 \; e_3] \equiv \begin{bmatrix} e_1^1 & e_2^1 & e_3^1 \\ e_1^2 & e_2^2 & e_3^2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\Gamma_{31}}{2(\lambda_1-\Gamma_{11})} & \frac{\Gamma_{31}}{2(\lambda_2-\Gamma_{11})} & \frac{\Gamma_{31}}{2(\lambda_3-\Gamma_{11})} \\ \frac{N_c\Gamma_{31}}{4(\lambda_1-\Gamma_{22})} & \frac{N_c\Gamma_{31}}{4(\lambda_2-\Gamma_{22})} & \frac{N_c\Gamma_{31}}{4(\lambda_3-\Gamma_{22})} \\ 1 & 1 & 1 \end{bmatrix} , \tag{5.25}$$

where, to simplify the expressions below, we denote by e_i^j the jth element of the ith eigenvector. The diagonalized anomalous dimension matrix is $\Gamma_S^{\rm diag} = R^{-1}\Gamma_S R$.

To write down the Born cross section in the diagonal basis we use the inverse of Eq. (5.24), i.e. $C = C'R^{-1}$, where the inverse of the matrix R is

$$R^{-1} = D^{-1} \begin{bmatrix} e_2^2 - e_3^2 & e_3^1 - e_2^1 & e_2^1 e_3^2 - e_3^1 e_2^2 \\ e_3^2 - e_1^2 & e_1^1 - e_3^1 & e_1^2 e_3^1 - e_1^1 e_3^2 \\ e_1^2 - e_2^2 & e_2^1 - e_1^1 & e_1^1 e_2^2 - e_1^2 e_2^1 \end{bmatrix} \tag{5.26}$$

with

$$D = (e_1^1 - e_3^1)e_2^2 - (e_1^1 - e_2^1)e_3^2 - (e_2^1 - e_3^1)e_1^2 \,. \tag{5.27}$$

Then, the old basis may be expressed in terms of the new basis color tensors, and we can rewrite the Born cross section in terms of the diagonal basis as

$$\sigma^B = \sum_{K,L=1}^{3} \sigma^B_{KL} \,, \tag{5.28}$$

where σ^B_{KL} is the component of the Born cross section proportional to $c'_K c'^*_L$. The explicit expressions for these components are, again, long but they are straightforward to derive. Then the resummed cross section is given by

$$\sigma^{\rm res} = \sum_{K,L=1}^{3} \sigma^B_{KL} \, e^{E_{KL}} \,, \tag{5.29}$$

where the exponent E_{KL} takes contributions at NLL from the eigenvalues λ_K and λ_L.

6 Threshold resummation for dijet production

In this section we review the resummation formalism relevant to the hadronic production of a pair of jets. We will follow the same methods as for heavy quark production but will encounter additional complications due to the presence of final state jets.

6.1 Factorized dijet cross section

We study dijet production in hadronic processes

$$h_a(p_a) + h_b(p_b) \to J_1(p_1, \delta_1) + J_2(p_2, \delta_2) + X(k) \,, \tag{6.1}$$

at fixed rapidity interval,

$$\Delta y = \frac{1}{2} \, \ln \left(\frac{p_1^+ \, p_2^-}{p_1^- \, p_2^+} \right) \,, \tag{6.2}$$

with total rapidity,

$$y_{JJ} = \frac{1}{2}\ \ln\left(\frac{p_1^+ + p_2^+}{p_1^- + p_2^-}\right)\,. \tag{6.3}$$

The jets are defined by cone angles δ_1 and δ_2. The introduction of cones removes all the final-state collinear singularities from the partonic cross section, which is then infrared safe, once the initial-state collinear singularities have been factored into universal parton distribution functions. We shall assume that the cones are small enough so that contributions proportional to $\delta_i \ll 1$ may be neglected, but large enough so that $\alpha_s(Q)\ln(1/\delta_i) \ll 1$, where Q is any of the hard scales of the cross section, such as the momentum transfer [66, 67].

To construct the dijet cross sections, we define a large invariant, M_{JJ}, which is held fixed. A natural choice is the dijet invariant mass,

$$M_{JJ}^2 = (p_1 + p_2)^2\,, \tag{6.4}$$

which is the analog of Q^2 for heavy quark production, but other choices are possible, for example the scalar product of the two jet momenta

$$M_{JJ}^2 = 2p_1 \cdot p_2\,. \tag{6.5}$$

In both cases, large M_{JJ} at fixed Δy implies a large momentum transfer in the partonic subprocess. The nature of the resummed cross section depends critically on this choice. As we shall see, the leading behavior of the resummed cross section is the same as for Drell-Yan or heavy quark production for the first choice for M_{JJ}, Eq. (6.4), while it is different for the second choice, Eq. (6.5).

The dijet cross section is given in factorized form by

$$\begin{aligned}\frac{d\sigma_{h_a h_b \to J_1 J_2}(S, M_{JJ}, y, \Delta y, \delta_1, \delta_2)}{dM_{JJ}^2\, dy_{JJ}\, d\Delta y} \quad &= \quad \sum_{f_a, f_b = q, \bar{q}, g} \int \frac{dx_a}{x_a}\, \frac{dx_b}{x_b}\ \phi_{f_a/h_a}(x_a, \mu^2) \\ &\times \phi_{f_b/h_b}(x_b, \mu^2)\ H_{f_a f_b}\left(\frac{M_{JJ}^2}{x_a x_b S}, y, \Delta y, \frac{M_{JJ}}{\mu}, \alpha_s(\mu^2), \delta_1, \delta_2\right)\,,\end{aligned} \tag{6.6}$$

where again H is the hard scattering and the ϕ's are parton distribution functions. The threshold for the partonic subprocess is given in terms of the variable z,

$$z = \frac{M_{JJ}^2}{x_a x_b S} = \frac{M_{JJ}^2}{s}\,, \tag{6.7}$$

where, as before, $S = (p_a + p_b)^2$ and $s = x_a x_b S$. At $z_{\max} = 1$ (partonic threshold) there is just enough partonic energy to produce the observed final state, with no additional radiation. The lower limit of z is

$$z_{\min} \equiv \tau = \frac{M_{JJ}^2}{S}\,. \tag{6.8}$$

We may rewrite the cross section, introducing an explicit integration over z, as

$$\begin{aligned}\frac{d\sigma_{h_a h_b \to J_1 J_2}(S, M_{JJ}, y, \Delta y, \delta_1, \delta_2)}{dM_{JJ}^2\, dy_{JJ}\, d\Delta y} &= \sum_{f_a, f_b = q, \bar{q}, g} \int_\tau^1 dz \int \frac{dx_a}{x_a}\frac{dx_b}{x_b} \\ &\times \phi_{f_a/h_a}(x_a, \mu^2)\, \phi_{f_b/h_b}(x_b, \mu^2)\, \delta\left(z - \frac{M_{JJ}^2}{s}\right)\, \delta\left(y_{JJ} - \frac{1}{2}\ln\frac{x_a}{x_b}\right) \\ &\times \sum_{f_1, f_2 = q, \bar{q}, g} \hat{\sigma}_{f_a f_b \to f_1 f_2}\left(1 - z, \frac{M_{JJ}}{\mu}, \Delta y, \alpha_s(\mu^2), \delta_1, \delta_2\right)\,, \end{aligned} \tag{6.9}$$

where again we have used the observation [47] that we can treat the total rapidity of the dijets as a constant, equal to its value at threshold. Thus we now have a simplified hard scattering function, $\hat{\sigma}_{f_a f_b \to f_1 f_2}$.

To calculate $\hat{\sigma}_{f_a f_b \to f_1 f_2}$ at any order in perturbation theory, we construct the partonic cross section for the process $f_a + f_b \to f_1 + f_2$, as above,

$$\begin{aligned}\frac{d\sigma_{f_a f_b \to J_1 J_2}(S, M_{JJ}, \Delta y, \delta_1, \delta_2)}{dM_{JJ}^2\, d\Delta y} &= \int_\tau^1 dz \int \frac{dx_a}{x_a}\frac{dx_b}{x_b}\, \phi_{f_a/f_a}(x_a, \mu^2) \\ &\times \phi_{f_a/f_a}(x_a, \mu^2)\, \phi_{f_b/f_b}(x_b, \mu^2)\, \delta\left(z - \frac{M_{JJ}^2}{s}\right) \\ &\times \sum_{f_1, f_2 = q, \bar{q}, g} \hat{\sigma}_{f_a f_b \to f_1 f_2}\left(1 - z, \frac{M_{JJ}}{\mu}, \Delta y, \alpha_s(\mu^2), \delta_1, \delta_2\right)\,, \end{aligned} \tag{6.10}$$

where we have integrated over the total rapidity. We then factorize the initial-state collinear divergences into the light-cone distribution functions $\phi_{f/f}$, expanded to the same order in α_s, and thus obtain the perturbative expansion for the infrared-safe hard scattering function, $\hat{\sigma}$. As for heavy

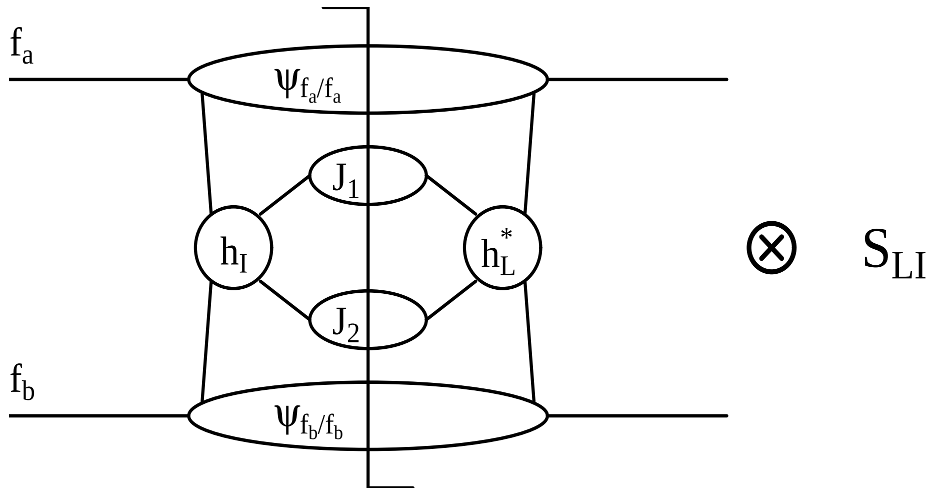


Fig. 6. Refactorization for dijet production. The soft gluon function is as in Fig. 1(b).

quark production, again we note that the leading power as $z \to 1$ comes entirely from flavor diagonal parton distributions. Moreover, we may sum, at leading power, over the flavors of the final-state partons f_1, f_2.

By taking a Mellin transform of the rapidity-integrated partonic cross section (6.10) with respect to τ, the above convolution becomes a product

$$\int_0^1 d\tau\; \tau^{N-1}\; \frac{d\sigma_{f_a f_b \to J_1 J_2}(S, M_{JJ}, \Delta y, \delta_1, \delta_2)}{dM_{JJ}^2\; d\Delta y} \;=\; \sum_{\mathrm{f}} \tilde{\phi}_{f_a/f_a}(N, \mu^2, \epsilon) \times \tilde{\phi}_{f_b/f_b}(N, \mu^2, \epsilon)\; \tilde{\sigma}_{\mathrm{f}}(N, M_{JJ}/\mu, \alpha_s(\mu^2), \delta_1, \delta_2)\,, \qquad (6.11)$$

where f denotes the partonic processes $f_a + f_b \to f_1 + f_2$, and with $\tilde{\sigma}_{\mathrm{f}}(N) = \int_0^1 dz\; z^{N-1} \hat{\sigma}_{\mathrm{f}}(z)$, and $\tilde{\phi}(N) = \int_0^1 dx\; x^{N-1} \phi(x)$, as before.

We can refactorize the cross section, as shown in Fig. 6, into hard components H_{IL}, which describe the truly short-distance hard-scattering, center-of-mass distributions ψ, associated with gluons collinear to the incoming partons, a soft gluon function S_{LI} associated with soft gluons, and jet functions J_i, associated with gluons collinear to the outgoing jets. As before, I and L are color indices that describe the color structure of the hard scattering. The refactorized cross section may then be written as

$$\int_0^1 d\tau\; \tau^{N-1}\; \frac{d\sigma_{f_a f_b \to J_1 J_2}(S, M_{JJ}, \Delta y, \delta_1, \delta_2)}{dM_{JJ}^2\; d\Delta y}$$

$$
\begin{aligned}
&= \sum_{\mathrm{f}} \sum_{IL} H_{IL}^{(\mathrm{f})} \left(\frac{M_{JJ}}{\mu}, \Delta y, \alpha_s(\mu^2) \right) \; \tilde{S}_{LI}^{(\mathrm{f})} \Big(\frac{M_{JJ}}{N\mu}, \Delta y, \alpha_s(\mu^2) \Big) \\
&\quad \times \tilde{\psi}_{f_a/f_a} \left(N, \frac{M_{JJ}}{\mu}, \alpha_s(\mu^2), \epsilon \right) \; \tilde{\psi}_{f_b/f_b} \left(N, \frac{M_{JJ}}{\mu}, \alpha_s(\mu^2), \epsilon \right) \\
&\quad \times \tilde{J}_{(f_1)} \left(N, \frac{M_{JJ}}{\mu}, \alpha_s(\mu^2), \delta_1 \right) \; \tilde{J}_{(f_2)} \left(N, \frac{M_{JJ}}{\mu}, \alpha_s(\mu^2), \delta_2 \right) \\
&\qquad + \mathcal{O}(1/N) \, .
\end{aligned} \tag{6.12}
$$

This refactorization is similar to the heavy quark case except that now we have to include in addition outgoing jet functions in order to absorb the final state collinear singularities (the mass of the heavy quarks eliminates final state collinear singularities in heavy quark production). Expressions for the hard components H_{IL} and the center-of-mass distributions ψ were given in Section 2 in the context of heavy quark production.

The soft function again represents the coupling of soft gluons to the partons in the hard scattering; this coupling, as we saw in Section 2.2, is described by eikonal lines. The construction of the eikonal cross section for dijet production is similar to the heavy quark case, apart from the outgoing jets. The collinear dynamics of the eikonal final-state jets are summarized by the matrix elements [38]

$$
\begin{aligned}
j_{\mathrm{OUT}}^{(f_i)} \left(\frac{w_i M_{JJ}}{\mu}, \alpha_s(\mu^2), \delta_i \right) \;&=\; \sum_{\xi} \, \delta \left(w_i - w(\xi, \delta_i) \right) \\
&\times \langle 0 | \, \mathrm{Tr} \Big\{ \, \bar{T} [\Phi_{\beta_i}^{(f_i)\dagger}(\infty, 0; 0)] | \xi \rangle \langle \xi | T [\Phi_{\beta_i}^{(f_i)}(\infty, 0; 0)] \, \Big\} \, | 0 \rangle \, ,
\end{aligned} \tag{6.13}
$$

where the ordered exponentials Φ_β are defined in Eq. (2.13), and with $i = 1, 2$ and ξ a set of intermediate states which contribute to the weight w_i.

We then construct moments of the soft function by dividing the moments of the eikonal cross section (defined in analogy to Eq. (2.15), see Ref. [38]) by the product of moments of the eikonal jets, Eqs. (2.17) and (6.13),

$$
\begin{aligned}
\tilde{S}_{LI}^{(\mathrm{f})} \left(\frac{M_{JJ}}{N\mu}, \Delta y, \alpha_s(\mu^2) \right) \;&=\; \frac{\tilde{\sigma}_{LI}^{(\mathrm{f,eik})} \left(\frac{M_{JJ}}{N\mu}, \Delta y, \alpha_s(\mu^2), \epsilon \right)}{\tilde{j}_{\mathrm{IN}}^{(f_a)} \left(\frac{M_{JJ}}{N\mu}, \alpha_s(\mu^2), \epsilon \right) \; \tilde{j}_{\mathrm{IN}}^{(f_b)} \left(\frac{M_{JJ}}{N\mu}, \alpha_s(\mu^2), \epsilon \right)} \\
&\times \frac{1}{\tilde{j}_{\mathrm{OUT}}^{(f_1)} \left(\frac{M_{JJ}}{N\mu}, \alpha_s(\mu^2), \delta_1 \right) \; \tilde{j}_{\mathrm{OUT}}^{(f_2)} \left(\frac{M_{JJ}}{N\mu}, \alpha_s(\mu^2), \delta_2 \right)} \, .
\end{aligned} \tag{6.14}
$$

6.2 Resummed dijet cross section

Comparing Eqs. (6.11) and (6.12), the refactorized expression for the Mellin transform of the hard scattering function is

$$\begin{aligned}\tilde{\sigma}_{\mathrm{f}}(N) &= \left[\frac{\tilde{\psi}_{f_a/f_a}(N, M_{JJ}/\mu, \epsilon)\, \tilde{\psi}_{f_b/f_b}(N, M_{JJ}/\mu, \epsilon)}{\tilde{\phi}_{f_a/f_a}(N, \mu^2, \epsilon)\, \tilde{\phi}_{f_b/f_b}(N, \mu^2, \epsilon)}\right] \\ &\quad \times \sum_{IL} H^{(\mathrm{f})}_{IL}\left(\frac{M_{JJ}}{\mu}, \Delta y, \alpha_s(\mu^2)\right)\, \tilde{S}^{(\mathrm{f})}_{LI}\left(\frac{M_{JJ}}{N\mu}, \Delta y, \alpha_s(\mu^2)\right) \\ &\quad \times \tilde{J}_{(f_1)}\left(N, \frac{M_{JJ}}{\mu}, \alpha_s(\mu^2), \delta_1\right)\, \tilde{J}_{(f_2)}\left(N, \frac{M_{JJ}}{\mu}, \alpha_s(\mu^2), \delta_2\right) . \end{aligned} \tag{6.15}$$

We discussed resummation for the ratio ψ/ϕ and the soft gluon function in the context of heavy quark production in Section 2. Thus, all we need to write down the resummed dijet cross section are expressions for the resummation of the final state jets.

The moments of the final-state jet with $M^2_{JJ} = 2p_1 \cdot p_2$ are given by [38]

$$\tilde{J}_{(f_i)}\left(N, \frac{M_{JJ}}{\mu}, \alpha_s(\mu^2), \delta_i\right) = \exp\left[E'_{(f_i)}(N, M_{JJ})\right] , \tag{6.16}$$

with

$$\begin{aligned} E'_{(f)}(N, M_{JJ}) &= \int_0^1 dz \frac{z^{N-1}-1}{1-z} \left\{ \int_{(1-z)^2}^{(1-z)} \frac{d\lambda}{\lambda} A^{(f)}\left[\alpha_s(\lambda M^2_{JJ})\right] \right. \\ &\qquad \left. + B'_{(f)}\left[\alpha_s((1-z)M^2_{JJ})\right] \right\} , \end{aligned} \tag{6.17}$$

where the function $A^{(f)}$ is the same as in Eq. (2.21) and the lowest-order term in $B'_{(f)}$ may be read off from the one-loop jet function. The results include a gauge dependence, which cancels against a corresponding dependence in the soft anomalous dimension matrix. The leading logarithms for final-state jets with $M^2_{JJ} = 2p_1 \cdot p_2$ are negative and give a suppression to the cross section, in contrast to the initial-state leading-log contributions.

The moments of the final-state jet with $M^2_{JJ} = (p_1 + p_2)^2$ are given by Eq. (6.16) with [38]

$$E'_{(f_i)}(N, M_{JJ}) = \int_{\mu}^{M_{JJ}/N} \frac{d\mu'}{\mu'}\; C'_{(f_i)}(\alpha_s(\mu'^2)) , \tag{6.18}$$

where the first term in the series for $C'_{(f_i)}(\alpha_s)$ may be read off from a one-loop calculation. The leading logarithmic behavior of the cross section in this case is not affected by the final state jets, so we always have an enhancement of the cross section at leading logarithm, as is the case for Drell-Yan and heavy quark cross sections.

Using Eqs. (6.15), (2.27), (2.31), and (6.16), we can write the resummed dijet cross section in moment space as

$$\begin{aligned}
\tilde{\sigma}_{\mathrm{f}}(N) \quad = \quad & R^2_{(f)} \; \exp\Bigg\{ \sum_{i=a,b} \Big[E^{(f_i)}(N, M_{JJ}) \\
& \qquad - 2\int_{\mu}^{M_{JJ}} \frac{d\mu'}{\mu'} \; \Big[\gamma_{f_i}(\alpha_s(\mu'^2)) - \gamma_{f_i f_i}(N, \alpha_s(\mu'^2))\Big]\Big]\Bigg\} \\
& \times \; \exp\Bigg\{ \sum_{j=1,2} E'_{(f_j)}(N, M_{JJ}) \Bigg\} \\
& \times \; \mathrm{Tr}\Bigg\{ H^{(\mathrm{f})}\left(\frac{M_{JJ}}{\mu}, \Delta y, \alpha_s(\mu^2)\right) \\
& \times \; \bar{P}\exp\left[\int_{\mu}^{M_{JJ}/N} \frac{d\mu'}{\mu'} \, \Gamma_S^{(\mathrm{f})\dagger}\left(\alpha_s(\mu'^2)\right)\right] \; \tilde{S}^{(\mathrm{f})}\left(1, \Delta y, \alpha_s\left(M_{JJ}^2/N^2\right)\right) \\
& \times \; P\exp\left[\int_{\mu}^{M_{JJ}/N} \frac{d\mu'}{\mu'} \, \Gamma_S^{(\mathrm{f})}\left(\alpha_s(\mu'^2)\right)\right]\Bigg\} \; , \qquad (6.19)
\end{aligned}$$

where $E^{(f_i)}$ is given by Eq. (2.20), and $E'_{(f_j)}$ is given by Eq. (6.17) or Eq. (6.18). This expression is similar to Eq. (2.33) for heavy quark production except for the addition of the exponents for the final-state jets.

We give explicit expressions for the soft anomalous dimensions for the partonic processes relevant to dijet production in the next three sections.

7 Soft anomalous dimension matrices for processes involving quarks

In this section we present results for the soft anomalous dimension matrices for the processes $q\bar{q} \to q\bar{q}$, $qq \to qq$, and $\bar{q}\bar{q} \to \bar{q}\bar{q}$. The matrices for all these processes are 2×2. Hence the general diagonalization procedure for these processes follows along the same lines as we discussed at the beginning of Section 5.2.

Since the results are somewhat lengthy, we first introduce some notation which will simplify the expressions for Γ_S.

7.1 Notation

We consider partonic processes $f_a\,(p_a, r_a) + f_b\,(p_b, r_b) \to f_1\,(p_1, r_1) + f_2\,(p_2, r_2)$, where the r_i are color labels, and the p_i are momenta. To facilitate the presentation of the results for Γ_S we introduce the notation

$$T \equiv \ln\left(\frac{-t}{s}\right) + \pi i\,, \qquad U \equiv \ln\left(\frac{-u}{s}\right) + \pi i\,, \tag{7.1}$$

where

$$s = (p_a + p_b)^2\;, \quad t = (p_a - p_1)^2\;, \quad u = (p_a - p_2)^2\;, \tag{7.2}$$

are the usual Mandelstam invariants. Concerning the choice of physical channel s, t, or u, for the definition of the color basis, we note that we can choose any channel. For the processes $q\bar{q} \to q\bar{q}$, $qq \to qq$ and $\bar{q}\bar{q} \to \bar{q}\bar{q}$, $qg \to qg$ and $\bar{q}g \to \bar{q}g$, as well as the process $gg \to gg$ we will use t-channel bases, which seem to be the natural choice when analyzing forward scattering [68]. The processes $q\bar{q} \to gg$ and $gg \to q\bar{q}$ are better described in terms of s-channel color structures, so we will give results for them in s-channel color bases.

Since the full cross section is gauge independent, i.e. independent of the choice of the axial gauge-fixing vector n^μ, the gauge dependence in the product of the hard and soft functions, $H^{(\mathrm{f})}\,S^{(\mathrm{f})}$, must cancel the gauge dependence of the incoming and outgoing jets, ψ and $J_{(f_i)}$. Now, the jets are incoherent relative to the hard and soft functions, so the gauge dependence of the anomalous dimension matrices $\Gamma_S^{(\mathrm{f})}$ must be proportional to the identity matrix. Then we can rewrite the anomalous dimension matrix as

$$(\Gamma_S^{(\mathrm{f})})_{KL} = (\Gamma_{S'}^{(\mathrm{f})})_{KL} + \delta_{KL}\frac{\alpha_s}{\pi}\sum_{i=a,b,1,2} C_{(f_i)}\,\frac{1}{2}\,(-\ln\nu_i - \ln 2 + 1 - \pi i)\,, \tag{7.3}$$

with $C_{f_i} = C_F\;(C_A)$ for a quark (gluon), and with $\nu_i \equiv (v_i\cdot n)^2/|n|^2$, as in Eq. (3.9). Here the dimensionless and lightlike velocity vectors v_i^μ are defined by

$$p_i^\mu = \frac{M_{JJ}}{\sqrt{2}}v_i^\mu\,, \qquad i = a, b, 1, 2\,, \tag{7.4}$$

and satisfy $v_i^2 = 0$.

For the subprocesses involved in dijet production we will present the explicit expressions for $\Gamma_{S'}^{(\mathrm{f})}$. The full anomalous dimension matrix can then be retrieved from Eq. (7.3).

We now give the results for the anomalous dimension matrices $\Gamma_{S'}^{(\mathrm{f})}$ for partonic processes involving quarks.

7.2 Soft anomalous dimension for $q\bar{q} \to q\bar{q}$

First, we present the 2×2 soft anomalous dimension matrix for the process

$$q\,(p_a, r_a) + \bar{q}\,(p_b, r_b) \to q\,(p_1, r_1) + \bar{q}\,(p_2, r_2)\ , \tag{7.5}$$

in the t-channel singlet-octet color basis

$$\begin{aligned} c_1 &= \delta_{r_a r_1} \delta_{r_b r_2}\ , \\ c_2 &= (T_F^c)_{r_1 r_a} (T_F^c)_{r_b r_2} = -\frac{1}{2N_c} \delta_{r_a r_1} \delta_{r_b r_2} + \frac{1}{2} \delta_{r_a r_b} \delta_{r_1 r_2}. \end{aligned} \tag{7.6}$$

We find the soft anomalous dimension matrix [55, 39]

$$\Gamma_{S'} = \frac{\alpha_s}{\pi} \left[\begin{array}{cc} 2C_F T & -\frac{C_F}{N_c} U \\ -2U & -\frac{1}{N_c}(T - 2U) \end{array} \right]. \tag{7.7}$$

The dependence on the logarithmic ratio T is diagonal in this t-channel color basis. In the forward region of the partonic scattering ($T \to -\infty$), where Γ_S becomes diagonal, color singlet exchange is exponentially enhanced relatively to color octet [68]. The general forms of the eigenvalues and eigenvectors of Γ_S are given by Eqs. (3.10) and (3.11); explicit expressions are given in Ref. [39].

Finally we note that this result for Γ_S is consistent with the massless limit of the anomalous dimension matrix for the heavy quark production process $q\bar{q} \to Q\bar{Q}$ [21, 31].

7.3 Soft anomalous dimension for $qq \to qq$ and $\bar{q}\bar{q} \to \bar{q}\bar{q}$

Next, we consider the process

$$q\,(p_a, r_a) + q\,(p_b, r_b) \to q\,(p_1, r_1) + q\,(p_2, r_2)\ , \tag{7.8}$$

in the t-channel singlet-octet color basis

$$\begin{aligned} c_1 &= (T_F^c)_{r_1 r_a}(T_F^c)_{r_2 r_b} = -\frac{1}{2N_c}\delta_{r_a r_1}\delta_{r_b r_2} + \frac{1}{2}\delta_{r_a r_2}\delta_{r_b r_1}\,, \\ c_2 &= \delta_{r_a r_1}\delta_{r_b r_2}. \end{aligned} \tag{7.9}$$

The anomalous dimension matrix is given by [55, 39]

$$\Gamma_{S'} = \frac{\alpha_s}{\pi}\left[\begin{array}{cc} -\frac{1}{N_c}(T+U) + 2C_F U & 2U \\ \frac{C_F}{N_c}U & 2C_F T \end{array}\right]. \tag{7.10}$$

Again, the dependence on the logarithmic ratio T is diagonal in this t-channel color basis, and the color singlet dominates the octet in the forward region of the partonic scattering ($T \to -\infty$), where Γ_S becomes diagonal.

Note that the same anomalous dimension matrix applies to the process

$$\bar{q}\,(p_1, r_1) + \bar{q}\,(p_2, r_2) \to \bar{q}\,(p_a, r_a) + \bar{q}\,(p_b, r_b)\ . \tag{7.11}$$

Again, the general forms of the eigenvalues and eigenvectors of Γ_S are given by Eqs. (3.10) and (3.11). Explicit expressions for the eigenvalues and eigenvectors are given in Ref. [39].

8 Soft anomalous dimension matrices for processes involving quarks and gluons

Here we present the soft anomalous dimension matrices for the processes $q\bar{q} \to gg$, $gg \to q\bar{q}$, $qg \to qg$, and $\bar{q}g \to \bar{q}g$. The matrices for all these processes are 3×3. Moreover, we shall see that they are of the same general form as the anomalous dimension matrix for the heavy quark production process $gg \to Q\bar{Q}$. Hence the general diagonalization procedure for these processes follows the same lines as we discussed in Section 5.3 in the context of heavy quark production in the channel $gg \to Q\bar{Q}$.

8.1 Soft anomalous dimension for $q\bar{q} \to gg$ and $gg \to q\bar{q}$

First, we present the soft anomalous dimension matrix for the process

$$q\,(p_a, r_a) + \bar{q}\,(p_b, r_b) \to g\,(p_1, r_1) + g\,(p_2, r_2)\ , \tag{8.1}$$

in the s-channel color basis

$$c_1 = \delta_{r_a r_b} \delta_{r_1 r_2}\,, \quad c_2 = d^{r_1 r_2 c} (T_F^c)_{r_b r_a}\,, \quad c_3 = i f^{r_1 r_2 c} (T_F^c)_{r_b r_a}\,. \tag{8.2}$$

We find [39]

$$\Gamma_{S'} = \frac{\alpha_s}{\pi} \left[\begin{array}{ccc} 0 & 0 & U-T \\ 0 & \frac{C_A}{2}\,(T+U) & \frac{C_A}{2}\,(U-T) \\ 2\,(U-T) & \frac{N_c^2-4}{2N_c}\,(U-T) & \frac{C_A}{2}\,(T+U) \end{array} \right]. \tag{8.3}$$

The same anomalous dimension describes also the time-reversed process [21, 31]

$$g\,(p_1, r_1) + g\,(p_2, r_2) \to \bar{q}\,(p_a, r_a) + q\,(p_b, r_b)\,. \tag{8.4}$$

We note that the anomalous dimension matrix, Eq. (8.3), is of the general form of Eq. (4.4). Then the general forms of its eigenvalues and eigenvectors are given by Eqs. (4.6) and (4.9); explicit expressions are given in Ref. [39].

Finally we note that this result for Γ_S is consistent with the massless limit of the anomalous dimension matrix for the heavy quark production process $gg \to Q\bar{Q}$ [21, 31].

8.2 Soft anomalous dimension for $qg \to qg$ and $\bar{q}g \to \bar{q}g$

Next, we consider the "Compton" process

$$q\,(p_a, r_a) + g\,(p_b, r_b) \to q\,(p_1, r_1) + g\,(p_2, r_2)\;. \tag{8.5}$$

The soft anomalous dimension matrix in the t-channel color basis

$$c_1 = \delta_{r_a r_1} \delta_{r_b r_2}\,, \quad c_2 = d^{r_b r_2 c} (T_F^c)_{r_1 r_a}\,, \quad c_3 = i f^{r_b r_2 c} (T_F^c)_{r_1 r_a}\,, \tag{8.6}$$

is given by [39]

$$\Gamma_{S'} = \frac{\alpha_s}{\pi} \left[\begin{array}{ccc} (C_F + C_A)\,T & 0 & U \\ 0 & C_F T + \frac{C_A}{2} U & \frac{C_A}{2} U \\ 2U & \frac{N_c^2-4}{2N_c} U & C_F T + \frac{C_A}{2} U \end{array} \right], \tag{8.7}$$

which also applies to the process

$$\bar{q}\,(p_1, r_1) + g\,(p_2, r_2) \to \bar{q}\,(p_a, r_a) + g\,(p_b, r_b)\ . \tag{8.8}$$

The dependence on the logarithmic ratio T is diagonal in this t-channel color basis, and the t-channel color singlet dominates in the forward region ($T \to -\infty$) where Γ_S becomes diagonal. Again, we note that the anomalous dimension matrix, Eq. (8.7), is of the general form of Eq. (4.4), so the general forms of its eigenvalues and eigenvectors are given by Eqs. (4.6) and (4.9). Explicit expressions for the eigenvalues and eigenvectors are given in Ref. [39].

9 Soft anomalous dimension matrix and diagonalization for $gg \to gg$

9.1 Soft anomalous dimension matrix for $gg \to gg$

The final, and by far more complicated, process that we consider is

$$g\,(p_a, r_a) + g\,(p_b, r_b) \to g\,(p_1, r_1) + g\,(p_2, r_2)\ . \tag{9.1}$$

The choice of a color basis, in which a four-gluon diagram can be expanded [69, 70], is more difficult in this case. In Ref. [39] an initial overcomplete color basis of nine elements was found convenient for the calculations, resulting in a 9×9 anomalous dimension matrix. This was then reduced in a complete color basis to an 8×8 matrix.

A complete color basis for the process $gg \to gg$ is given by the eight color structures [39]

$$\begin{aligned}
c_1 &= \frac{i}{4}\left[f^{r_a r_b l} d^{r_1 r_2 l} - d^{r_a r_b l} f^{r_1 r_2 l}\right]\,, \\
c_2 &= \frac{i}{4}\left[f^{r_a r_b l} d^{r_1 r_2 l} + d^{r_a r_b l} f^{r_1 r_2 l}\right]\,, \\
c_3 &= \frac{i}{4}\left[f^{r_a r_1 l} d^{r_b r_2 l} + d^{r_a r_1 l} f^{r_b r_2 l}\right]\,, \\
c_4 &= P_1(r_a, r_b; r_1, r_2) = \frac{1}{8}\delta_{r_a r_1}\delta_{r_b r_2}\,, \\
c_5 &= P_{8_S}(r_a, r_b; r_1, r_2) = \frac{3}{5} d^{r_a r_1 c} d^{r_b r_2 c}\,,
\end{aligned}$$

$$\begin{aligned}
c_6 &= P_{8_A}(r_a, r_b; r_1, r_2) = \frac{1}{3} f^{r_a r_1 c} f^{r_b r_2 c}\,, \\
c_7 &= P_{10+\overline{10}}(r_a, r_b; r_1, r_2) = \frac{1}{2}(\delta_{r_a r_b}\delta_{r_1 r_2} - \delta_{r_a r_2}\delta_{r_b r_1}) - \frac{1}{3} f^{r_a r_1 c} f^{r_b r_2 c}\,, \\
c_8 &= P_{27}(r_a, r_b; r_1, r_2) = \frac{1}{2}(\delta_{r_a r_b}\delta_{r_1 r_2} + \delta_{r_a r_2}\delta_{r_b r_1}) - \frac{1}{8}\delta_{r_a r_1}\delta_{r_b r_2} \\
& -\frac{3}{5} d^{r_a r_1 c} d^{r_b r_2 c}\,, \qquad (9.2)
\end{aligned}$$

where the P's are t-channel projectors of irreducible representations of $SU(3)$ [71], and we have used explicitly $N_c = 3$.

The soft anomalous dimension matrix in this basis is [39]

$$\Gamma_{S'} = \left[\begin{array}{cc} \Gamma_{3\times 3} & 0_{3\times 5} \\ 0_{5\times 3} & \Gamma_{5\times 5} \end{array} \right], \qquad (9.3)$$

with

$$\Gamma_{3\times 3} = \frac{\alpha_s}{\pi} \left[\begin{array}{ccc} 3T & 0 & 0 \\ 0 & 3U & 0 \\ 0 & 0 & 3\,(T+U) \end{array} \right] \qquad (9.4)$$

and

$$\Gamma_{5\times 5} = \frac{\alpha_s}{\pi} \left[\begin{array}{ccccc} 6T & 0 & -6U & 0 & 0 \\ 0 & 3T + \frac{3U}{2} & -\frac{3U}{2} & -3U & 0 \\ -\frac{3U}{4} & -\frac{3U}{2} & 3T + \frac{3U}{2} & 0 & -\frac{9U}{4} \\ 0 & -\frac{6U}{5} & 0 & 3U & -\frac{9U}{5} \\ 0 & 0 & -\frac{2U}{3} & -\frac{4U}{3} & -2T + 4U \end{array} \right]. \qquad (9.5)$$

We note that the dependence on T is diagonal and in the forward region of the partonic scattering, $T \to -\infty$, where Γ_S becomes diagonal, color singlet exchange dominates. This has been a general trend for Γ_S for all the processes we analyzed in t-channel bases; suppression increases with the dimension of the exchanged color representation.

The eigenvalues of the anomalous dimension matrix, Eq. (9.3), are

$$\begin{aligned}
\lambda_1 &= \lambda_4 = 3\frac{\alpha_s}{\pi}T, \quad \lambda_2 = \lambda_5 = 3\frac{\alpha_s}{\pi}U, \quad \lambda_3 = \lambda_6 = 3\frac{\alpha_s}{\pi}(T+U), \\
\lambda_{7,8} &= 2\frac{\alpha_s}{\pi}\left[T + U \mp 2\sqrt{T^2 - TU + U^2}\right]. \qquad (9.6)
\end{aligned}$$

The eigenvectors have the general form

$$e_i = \begin{bmatrix} e_i^{(3)} \\ 0^{(5)} \end{bmatrix}, \; i = 1,2,3\,, \quad e_i = \begin{bmatrix} 0^{(3)} \\ e_i^{(5)} \end{bmatrix}, \; i = 4\ldots 8\,, \tag{9.7}$$

where the superscripts refer to the dimension. The three-dimensional vectors $e_i^{(3)}$ are defined by

$$e_i^{(3)} = \begin{bmatrix} \delta_{i1} \\ \delta_{i2} \\ \delta_{i3} \end{bmatrix}, \; i = 1,2,3\,, \tag{9.8}$$

while $0^{(5)}$ and $0^{(3)}$ are the five- and three-dimensional null vectors. The five-dimensional vectors $e_4^{(5)}$, $e_5^{(5)}$ and $e_6^{(5)}$ are given by [39]

$$e_4^{(5)} = \begin{bmatrix} -15 \\ 6 - \frac{15}{2}\frac{T}{U} \\ -\frac{15}{2}\frac{T}{U} \\ 3 \\ 1 \end{bmatrix}, \quad e_5^{(5)} = \begin{bmatrix} 0 \\ -\frac{3}{2} \\ 0 \\ \frac{3}{4} - \frac{3}{2}\frac{T}{U} \\ 1 \end{bmatrix}, \quad e_6^{(5)} = \begin{bmatrix} -15 \\ -\frac{3}{2} + \frac{15}{2}\frac{T}{U} \\ \frac{15}{2} - \frac{15}{2}\frac{T}{U} \\ -3 \\ 1 \end{bmatrix}. \tag{9.9}$$

The expressions for $e_7^{(5)}$ and $e_8^{(5)}$ are long but can be given succinctly by [39]

$$e_i^{(5)} = \begin{bmatrix} b_1(\lambda_i') \\ b_2(\lambda_i') \\ b_3(\lambda_i') \\ b_4(\lambda_i') \\ 1 \end{bmatrix}, \; i = 7,8\,, \tag{9.10}$$

where

$$\lambda_i' = \frac{\pi}{\alpha_s}\lambda_i\,, \tag{9.11}$$

and where the b_i's are given by

$$\begin{aligned} b_1(\lambda_i') = \frac{3}{U^2 K'}[&80T^4 + 103U^4 - 280UT^3 - 300TU^3 + 404T^2U^2 \\ &+ (40T^3 - 16U^3 - 60T^2U + 52TU^2)\lambda_i']\,, \end{aligned}$$

$$\begin{aligned}
b_2(\lambda'_i) &= \frac{3}{2K'}[20T^2 - 50UT + 44U^2 + (10T - 5U)\lambda'_i]\,,\\
b_3(\lambda'_i) &= -\frac{3}{2UK'}[40T^3 - 64U^3 - 120T^2U + 130TU^2\\
&\qquad + (20T^2 + 13U^2 - 20TU)\lambda'_i]\,,\\
b_4(\lambda'_i) &= \frac{3U}{K'}(2T + 5U - 2\lambda'_i)\,,
\end{aligned} \tag{9.12}$$

with

$$K' = 20T^2 - 20UT + 21U^2\,. \tag{9.13}$$

With these results for the eigenvalues and eigenvectors we have all the required elements for the diagonalization procedure.

9.2 Diagonalization procedure for $gg \to gg$

We now give a discussion of the diagonalization procedure for the process $gg \to gg$. Since the anomalous dimension matrix is 8×8 its diagonalization is quite more complicated than for the other processes that we have considered.

The Born cross section for $gg \to gg$ can be decomposed in the original basis, Eq. (9.2), as

$$\begin{aligned}
\sigma^B &= H_{11}|c_1|^2 + H_{22}|c_2|^2 + H_{33}|c_3|^2 + H_{44}|c_4|^2\\
&\qquad + H_{55}|c_5|^2 + H_{66}|c_6|^2 + H_{77}|c_7|^2 + H_{88}|c_8|^2\,.
\end{aligned} \tag{9.14}$$

Then if $C = (c_1\ c_2\ c_3\ c_4\ c_5\ c_6\ c_7\ c_8)$ is the original color basis, the diagonal color basis is

$$C' \equiv (c'_1\ c'_2\ c'_3\ c'_4\ c'_5\ c'_6\ c'_7\ c'_8) = CR\,, \tag{9.15}$$

where the columns of the matrix R are the eigenvectors of $\Gamma_{S'}$:

$$R = [e_1\ e_2\ e_3\ e_4\ e_5\ e_6\ e_7\ e_8] = \begin{bmatrix} 1_{3\times 3} & 0_{3\times 5} \\ 0_{5\times 3} & R_{5\times 5} \end{bmatrix}. \tag{9.16}$$

Here

$$R_{5\times 5} = \begin{bmatrix}
-15 & 0 & -15 & b_1(\lambda'_7) & b_1(\lambda'_8) \\
6 - \frac{15}{2}\frac{T}{U} & \frac{-3}{2} & \frac{-3}{2} + \frac{15}{2}\frac{T}{U} & b_2(\lambda'_7) & b_2(\lambda'_8) \\
-\frac{15}{2}\frac{T}{U} & 0 & \frac{15}{2} - \frac{15}{2}\frac{T}{U} & b_3(\lambda'_7) & b_3(\lambda'_8) \\
3 & \frac{3}{4} - \frac{3}{2}\frac{T}{U} & -3 & b_4(\lambda'_7) & b_4(\lambda'_8) \\
1 & 1 & 1 & 1 & 1
\end{bmatrix}, \tag{9.17}$$

and the diagonalized anomalous dimension matrix is $\Gamma_{S'}^{\rm diag} = R^{-1}\Gamma_{S'}R$. To decompose the Born cross section in the diagonal basis we use the relation $C = C'R^{-1}$. The inverse of the matrix R is

$$R^{-1} = \left[\begin{array}{cc} 1_{3\times3} & 0_{3\times5} \\ 0_{5\times3} & R^{-1}_{5\times5} \end{array}\right] \tag{9.18}$$

where

$$R^{-1}_{5\times5} = \left[\begin{array}{ccccc} \frac{-U^2}{12K_1} & \frac{4U^2-5TU}{15K_1} & -\frac{UT}{3K_1} & \frac{U^2}{3K_1} & \frac{3U^2}{20K_1} \\ 0 & -\frac{16U^2}{15K_2} & 0 & \frac{4U^2-8TU}{3K_2} & \frac{12U^2}{5K_2} \\ -\frac{U^2}{12K_3} & \frac{-U^2+5TU}{15K_3} & \frac{U^2-TU}{3K_3} & -\frac{U^2}{3K_3} & \frac{3U^2}{20K_3} \\ d_1(\lambda_7') & d_2(\lambda_7') & d_3(\lambda_7') & d_4(\lambda_7') & d_5(\lambda_7') \\ -d_1(\lambda_8') & -d_2(\lambda_8') & -d_3(\lambda_8') & -d_4(\lambda_8') & -d_5(\lambda_8') \end{array}\right]. \tag{9.19}$$

Here we have simplified the expression for the inverse of R by introducing the variables d, which are given as functions of the eigenvalues λ_i, $i = 7, 8$, by:

$$\begin{aligned} d_1(\lambda_i') &= U^2\Big[10T^3 + 15T^2U - 19TU^2 + 24U^3 \\ &\qquad - \lambda_i'(10T^2 - 10TU + 9U^2)\Big]/(96K_0K_4)\,, \\ d_2(\lambda_i') &= U^2\Big[180T^5 - 360T^4U + 509T^3U^2 - 339T^2U^3 + 166TU^4 - 6U^5 \\ &\quad - \frac{\lambda_i'}{2}(60T^4 - 120T^3U + 163T^2U^2 - 103TU^3 + 48U^4)\Big]/(12K_0K_5)\,, \\ d_3(\lambda_i') &= U\Big[30T^4 - 45T^3U + 59T^2U^2 - 32TU^3 + 18U^4 \\ &\qquad - \frac{\lambda_i'}{2}(10T^3 - 15T^2U + 13TU^2 - 4U^3)\Big]/(12K_0K_4)\,, \\ d_4(\lambda_i') &= 5U\Big[240T^6 - 600T^5U + 972T^4U^2 - 888T^3U^3 + 583T^2U^4 \\ &\qquad - 205TU^5 + 48U^6 - \lambda_i\Big(40T^5 - 100T^4U + 152T^3U^2 \\ &\qquad - 128T^2U^3 + 70TU^4 - 17U^5\Big)\Big]/(24K_0K_5)\,, \\ d_5(\lambda_i') &= \Big[2400T^7 - 7200T^6U + 13440T^5U^2 - 15180T^4U^3 + 12306T^3U^4 \\ &\quad - 6321T^2U^5 + 2169TU^6 - 264U^7 - \lambda_i\Big(400T^6 - 1200T^5U + 2140T^4U^2 \\ &\quad - 2280T^3U^3 + 1666T^2U^4 - 726TU^5 + 181U^6\Big)\Big]/(32K_0K_5)\,, \end{aligned} \tag{9.20}$$

where, again for brevity, we have introduced the notation

$$\begin{aligned}
K_0 &= \sqrt{T^2 - TU + U^2}, \quad K_1 = 4U^2 - 4UT + 5T^2, \\
K_2 &= 4T^2 - 4UT + 5U^2, \quad K_3 = 5U^2 - 6UT + 5T^2, \\
K_4 &= 20U^4 - 44TU^3 + 69T^2U^2 - 50UT^3 + 25T^4, \\
K_5 &= 100U^6 - 300U^5T + 601U^4T^2 - 702U^3T^3 + 601T^4U^2 \\
&\quad -300T^5U + 100T^6. \qquad (9.21)
\end{aligned}$$

With these results we can now express the original basis in terms of the new diagonal basis color tensors. The Born cross section can then be rewritten in the diagonal basis as

$$\sigma^B = \sum_{K,L=1}^{8} \sigma^B_{KL}\,, \qquad (9.22)$$

where σ^B_{KL} is the component of the Born cross section proportional to $c'_K c'^*_L$. While the calculation of these components is straightforward, the explicit expressions are quite long.

The resummed cross section is then given by

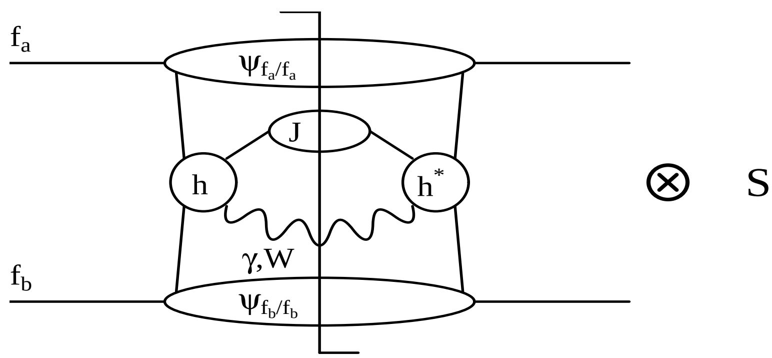
$$\sigma^{\rm res} = \sum_{K,L=1}^{8} \sigma^B_{KL}\, e^{E_{KL}}\,, \qquad (9.23)$$

where E_{KL} takes contributions at NLL from the eigenvalues λ_K and λ_L.

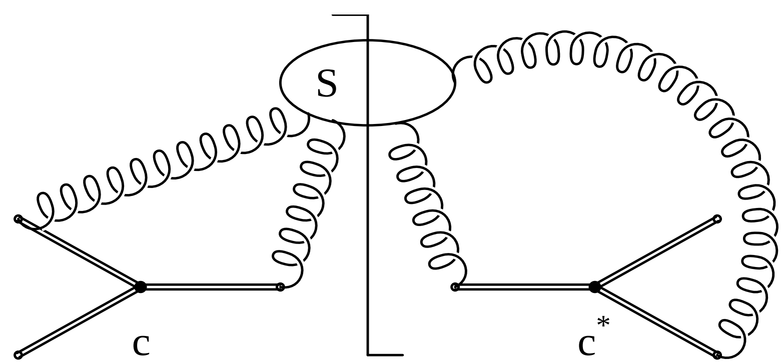
10 Threshold resummation for direct photon and W boson production

10.1 Factorization and resummed cross sections

In this section we discuss resummation for direct photon and electroweak boson production. Threshold resummation may be important for these processes at large transverse momentum. Here we formulate the resummation in single-particle inclusive kinematics [41]. Resummation will follow, as we saw before, from the factorization properties of the cross section.



(a)



(b)

Fig. 7. (a) Factorization for direct photon or W + jet production near partonic threshold. (b) The soft-gluon function S, in which the vertices c link ordered exponentials.

We consider hadronic processes of the form

$$h_A(p_A) + h_B(p_B) \to \gamma, W(Q) + X\,. \tag{10.1}$$

The partonic subprocesses contributing to direct photon or W + jet production are

$$q(p_a) + g(p_b) \longrightarrow q(p_J) + \gamma\,, W(Q) \tag{10.2}$$

and

$$q(p_a) + \bar{q}(p_b) \longrightarrow g(p_J) + \gamma\,, W(Q)\,. \tag{10.3}$$

We define Mandelstam invariants

$$s = (p_a + p_b)^2\,, \quad t = (p_a - Q)^2\,, \quad u = (p_b - Q)^2\,, \tag{10.4}$$

which satisfy $s + t + u = Q^2$ at threshold, with $Q^2 = M_W^2$ for W production and $Q^2 = 0$ for direct photon production.

The factorized form of the cross section for direct photon or W + jet production is a convolution of the parton distribution functions ϕ with the hard scattering function, $\hat{\sigma}$:

$$\begin{aligned} E_Q \frac{d\sigma_{h_A h_B \to \gamma, W}}{d^3Q} &= \sum_{ab} \int dx_a dx_b \, \phi_{f_a/h_A}(x_a, \mu^2)\, \phi_{f_b/h_B}(x_b, \mu^2) \\ &\qquad \times \hat{\sigma}(s_2, t, u, Q^2, \alpha_s(\mu^2))\,, \end{aligned} \tag{10.5}$$

where we define $s_2 = s + t + u - Q^2$, with $Q = p_W$ for W production and $Q = p_\gamma$ for direct photon production. The threshold region is given by $s_2 = 0$.

The cross section can be refactorized into distributions ψ, defined in direct analogy with the c.m. distributions that we presented in Section 2, hard components $H = h^* h$, and a soft gluon function S. This refactorization, similar to the ones for heavy quark and dijet production, is shown in Fig. 7. There are some differences in the details of the formalism here as compared to the results in the previous sections because the resummation is now carried out in single-particle inclusive kinematics. More details are given in Ref. [41].

The color structure of the hard scattering for direct photon or W boson production is much simpler than for heavy quark or dijet production. The color basis here consists of only one tensor; hence Γ_S is simply a 1×1 matrix, and no color traces or path ordering appear in the resummed expressions.

The resummation of the N-dependence of each of the functions in the refactorized cross section leads to the expression in the $\overline{\mathrm{MS}}$ factorization scheme in single-particle inclusive kinematics:

$$\begin{aligned}
\tilde{\sigma}(N) \;=\; & \exp\Bigg\{\sum_{i=a,b}\Big[E^{(f_i)}(N_i, p_i\cdot\zeta) \\
& \qquad - 2\int_{\mu}^{2p_i\cdot\zeta}\frac{d\mu'}{\mu'}\;\Big[\gamma_{f_i}(\alpha_s(\mu'^2)) - \gamma_{f_if_i}(N_i,\alpha_s(\mu'^2))\Big]\Big]\Bigg\} \\
& \times\; \exp\left\{E'_{(f_J)}(N, p_J\cdot n)\right\}\; H\left(t,u,Q^2,\alpha_s(\mu^2)\right) \\
& \times\; S\left(1,\beta_i,\zeta,n,\alpha_s(S/N^2)\right)\;\exp\left[\int_{\mu}^{\sqrt{S}/N}\frac{d\mu'}{\mu'}\,2\,\mathrm{Re}\Gamma_S\left(\alpha_s(\mu'^2)\right)\right]\Bigg\}\;,
\end{aligned} \tag{10.6}$$

where $\zeta^\mu = p_3^\mu/\sqrt{S}$ [41] and β_i are the particle velocities. The first exponent $E^{(f_i)}$ has the same form as in Eq. (2.20) with $N_a = N(-u/s)$ and $N_b = N(-t/s)$. The exponent $E'_{(f_J)}$ has the same form as in Eq. (6.17), with $A^{(f)}$ given by (2.21) and $B'_{(f)}$ given for quarks by [41]

$$B'_{(q)} = \frac{\alpha_s}{\pi}C_F\left[-\frac{7}{4} + \ln(2\nu_q)\right] \tag{10.7}$$

and for gluons by

$$B'_{(g)} = \frac{\alpha_s}{\pi}C_A\left[\frac{n_f}{6C_A} - \frac{11}{12} - 1 + \ln(2\nu_g)\right]\;. \tag{10.8}$$

10.2 Soft anomalous dimensions and expansions of the resummed cross sections

Here we give the anomalous dimensions for direct photon and W + jet production. The calculation follows the same lines as for heavy quark and dijet production. The main difference here is that the color basis consists of only one tensor, $c = T_F$, and there are three eikonal lines connecting at the color vertex. The one-loop eikonal vertex corrections for the partonic subprocesses in direct photon and W + jet production are shown in Fig. 8.

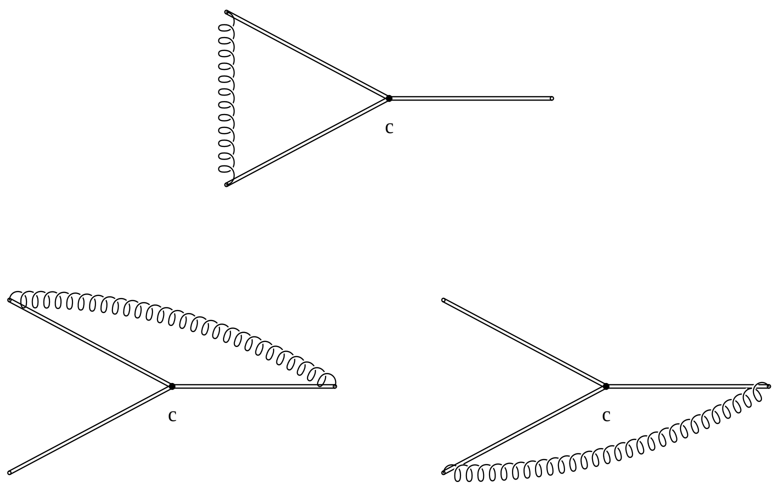


Fig. 8. One-loop eikonal vertex corrections for partonic subprocesses in direct photon and W + jet production.

The anomalous dimension for the process $q(p_a) + g(p_b) \rightarrow q(p_J) + \gamma(Q)$ or the process $q(p_a) + g(p_b) \rightarrow q(p_J) + W(Q)$ is given by

$$\begin{aligned}\Gamma_S &= \frac{\alpha_s}{2\pi}\left\{C_F\left[2\ln\left(\frac{-u}{s}\right) - \ln(4\nu_{q_a}\nu_{q_J}) + 2\right]\right.\\ &\qquad \left. + C_A\left[\ln\left(\frac{t}{u}\right) - \ln(2\nu_g) + 1 - \pi i\right]\right\}\,. \qquad (10.9)\end{aligned}$$

The one-loop expansion of the resummed cross section for direct-photon production is given in Ref. [41]. The authors have found agreement with the exact NLO results in Ref. [6]. Numerical results are given in Ref. [72]. We may also expand the resummed cross section for W + jet production. The $\overline{\rm MS}$ one-loop expansion at NLL of s_2/Q^2 in single-particle inclusive kinematics is

$$\begin{aligned}\hat{\sigma}^{\overline{\rm MS}\,(1)}_{qg\rightarrow qW}(s_2,s,t,u,Q^2) &= \sigma^B_{qg\rightarrow qW}\frac{\alpha_s}{\pi}\left\{(C_F+2C_A)\left[\frac{\ln(s_2/Q^2)}{s_2}\right]_+\right.\\ &\left. -(C_F+C_A)\ln\left(\frac{\mu^2}{Q^2}\right)\left[\frac{1}{s_2}\right]_+ + \left[-\frac{3}{4}C_F + C_A\ln\left(\frac{sQ^2}{tu}\right)\right]\left[\frac{1}{s_2}\right]_+\right\}, \qquad (10.10)\end{aligned}$$

where $\sigma^B_{qg\rightarrow qW}$ denotes the Born cross section for this partonic channel. This result agrees with the exact NLO expressions in Ref. [7].

The anomalous dimension for the process $q(p_a)+\bar{q}(p_b) \rightarrow g(p_J)+\gamma(Q)$ or the process $q(p_a)+\bar{q}(p_b) \rightarrow g(p_J)+W(Q)$ is given by

$$\begin{aligned}
\Gamma_S &= \frac{\alpha_s}{2\pi}\left\{C_F\left[-\ln(4\nu_q\nu_{\bar{q}})+2-2\pi i\right]\right. \\
&\qquad \left. + C_A\left[\ln\left(\frac{tu}{s^2}\right)-\ln(2\nu_g)+1+\pi i\right]\right\}. \qquad (10.11)
\end{aligned}$$

Again, the one-loop expansion for direct-photon production is given in Ref. [41] and agreement has been found with the exact NLO results in [6]. The $\overline{\mathrm{MS}}$ one-loop expansion at NLL of s_2/Q^2 for W + jet production in this channel and in single-particle inclusive kinematics is

$$\begin{aligned}
\hat{\sigma}^{\overline{\mathrm{MS}}\,(1)}_{q\bar{q}\rightarrow gW}(s_2,s,t,u,Q^2) &= \sigma^B_{q\bar{q}\rightarrow gW}\frac{\alpha_s}{\pi}\left\{(4C_F-C_A)\left[\frac{\ln(s_2/Q^2)}{s_2}\right]_+\right. \\
& - 2C_F\ln\left(\frac{\mu^2}{Q^2}\right)\left[\frac{1}{s_2}\right]_+ + \left[2C_F\ln\left(\frac{sQ^2}{tu}\right)\right. \\
& \left.\left. + C_A\left(\frac{n_f}{6C_A}-\frac{11}{12}-\ln\left(\frac{sQ^2}{tu}\right)\right)\right]\left[\frac{1}{s_2}\right]_+\right\}, \qquad (10.12)
\end{aligned}$$

which again agrees with the exact NLO results in Ref. [7].

One may of course expand the resummed cross sections to two loops or higher for both direct photon [72] and W boson [43] production. For example, the $\overline{\mathrm{MS}}$ two-loop expansion at NLL of $1-w=s_2/(s+t)$ for direct photon production in the channel $q\bar{q}\rightarrow g\gamma$ is

$$\begin{aligned}
\hat{\sigma}^{\overline{\mathrm{MS}}\,(2)}_{q\bar{q}\rightarrow g\gamma}(1-w,s,v) &= \sigma^B_{q\bar{q}\rightarrow g\gamma}\frac{\alpha_s^2}{\pi^2}\left\{\left(8C_F^2-4C_FC_A+\frac{C_A^2}{2}\right)\left[\frac{\ln^3(1-w)}{1-w}\right]_+\right. \\
& + \left[-12C_F^2\left(\ln\left(\frac{1-v}{v}\right)+\ln\left(\frac{\mu^2}{s}\right)\right)+C_A^2\left(-\frac{3}{2}\ln(1-v)-\frac{n_f}{4C_A}+\frac{11}{8}\right)\right. \\
& + C_FC_A\left(9\ln(1-v)-3\ln v+\frac{n_f}{C_A}-\frac{11}{2}+3\ln\left(\frac{\mu^2}{s}\right)\right) \\
& \left.\left. - \beta_0\left(C_F-\frac{3}{8}C_A\right)\right]\left[\frac{\ln^2(1-w)}{1-w}\right]_+\right\}, \qquad (10.13)
\end{aligned}$$

where $v=1+t/s$.

11 Conclusion

We have reviewed the resummation of threshold logarithms for heavy quark, dijet, direct photon, and W boson production in hadronic collisions. Resummation follows from the factorization properties of the cross sections and is influenced by the color exchange in the hard scattering. We have presented full results for the soft anomalous dimension matrices for all the relevant partonic subprocesses. The one-loop expansions of the resummed cross sections agree with exact NLO calculations and one can expand the cross sections to two-loops or higher orders. We have discussed the general diagonalization procedure that can be implemented in the calculation of the resummed cross sections. Numerical results have been presented for top quark production at the Fermilab Tevatron. The resummation formalism is quite general and can be applied as well to the calculation of transverse momentum or other differential distributions for a variety of processes, such as heavy quark production.

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A Evaluation of one-loop eikonal vertex corrections

Here we give some calculational details for the one-loop corrections of Figs. 2 and 8.

In our calculations we use Feynman eikonal rules as discussed in Sections 3 and 4, and a general axial gauge gluon propagator,

$$D^{\mu\nu}(k) = \frac{-i}{k^2 + i\epsilon} N^{\mu\nu}(k), \quad N^{\mu\nu}(k) = g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{n \cdot k} + n^2 \frac{k^\mu k^\nu}{(n \cdot k)^2}, \quad \text{(A.1)}$$

with n^μ the axial gauge-fixing vector.

We denote the kinematic part of the one-loop vertex correction to c_I, with the virtual gluon linking lines v_i and v_j, as $\omega_{ij}(\delta_i v_i, \delta_j v_j, \Delta_i, \Delta_j)$. The δ's and Δ's are defined as in Sections 3 and 4 except that here we use $\Delta = -i$ $(+i)$ for a gluon located below (above) the eikonal line in order to present the results for both quarks and gluons in a uniform fashion.

The expression for ω_{ij} is then

$$\begin{aligned}\omega_{ij}(\delta_i v_i, \delta_j v_j, \Delta_i, \Delta_j) &= g_s^2 \int \frac{d^n q}{(2\pi)^n} \frac{-i}{q^2 + i\epsilon} \left\{ \frac{\Delta_i\, \Delta_j\; v_i \cdot v_j}{(\delta_i v_i \cdot q + i\epsilon)(\delta_j v_j \cdot q + i\epsilon)} \right. \\ &\left. - \frac{\Delta_i v_i \cdot n}{(\delta_i v_i \cdot q + i\epsilon)} \frac{P}{(n \cdot q)} - \frac{\Delta_j v_j \cdot n}{(\delta_j v_j \cdot q + i\epsilon)} \frac{P}{(n \cdot q)} + n^2 \frac{P}{(n \cdot q)^2} \right\}, \qquad \text{(A.2)}\end{aligned}$$

with $g_s^2 = 4\pi\alpha_s$, and where P stands for principal value,

$$\frac{P}{(q \cdot n)^\beta} = \frac{1}{2} \left(\frac{1}{(q \cdot n + i\epsilon)^\beta} + (-1)^\beta \frac{1}{(-q \cdot n + i\epsilon)^\beta} \right). \qquad \text{(A.3)}$$

We may rewrite (A.2) as

$$\begin{aligned}\omega_{ij}(\delta_i v_i, \delta_j v_j, \Delta_i, \Delta_j) &= \mathcal{S}_{ij} \left[I_1(\delta_i v_i, \delta_j v_j) - \frac{1}{2} I_2(\delta_i v_i, n) - \frac{1}{2} I_2(\delta_i v_i, -n) \right. \\ &\left. - \frac{1}{2} I_3(\delta_j v_j, n) - \frac{1}{2} I_3(\delta_j v_j, -n) + I_4(n^2) \right], \qquad \text{(A.4)}\end{aligned}$$

where $\mathcal{S}_{ij}$ is an overall sign

$$\mathcal{S}_{ij} = \Delta_i\, \Delta_j\, \delta_i\, \delta_j. \qquad \text{(A.5)}$$

We now evaluate the ultraviolet poles of the integrals. For the integrals when both v_i and v_j refer to massive quarks we have (with $\epsilon = 4 - n$) [21, 31]

$$\begin{aligned} I_1^{\text{UV pole}} &= \frac{\alpha_s}{\pi} \frac{1}{\epsilon} L_\beta \,, \\ I_2^{\text{UV pole}} &= -\frac{\alpha_s}{\pi} \frac{1}{\epsilon} L_i \,, \\ I_3^{\text{UV pole}} &= -\frac{\alpha_s}{\pi} \frac{1}{\epsilon} L_j \,, \\ I_4^{\text{UV pole}} &= -\frac{\alpha_s}{\pi} \frac{1}{\epsilon} \,, \end{aligned} \qquad \text{(A.6)}$$

where the L_β is the velocity-dependent eikonal function

$$L_\beta = \frac{1-2m^2/s}{\beta}\left(\ln\frac{1-\beta}{1+\beta}+\pi i\right), \tag{A.7}$$

with $\beta = \sqrt{1-4m^2/s}$. The L_i and L_j are rather complicated functions of the gauge vector n. Their contributions are cancelled by the inclusion of self energies. Their explicit expressions are:

$$L_i = \frac{1}{2}\left[L_i(+n)+L_i(-n)\right], \tag{A.8}$$

where

$$\begin{aligned} L_i(\pm n) &= \frac{1}{2}\frac{|v_i\cdot n|}{\sqrt{(v_i\cdot n)^2-2m^2n^2/s}} \\ &\times\left[\ln\left(\frac{\delta(\pm n)\,2m^2/s-|v_i\cdot n|-\sqrt{(v_i\cdot n)^2-2m^2n^2/s}}{\delta(\pm n)\,2m^2/s-|v_i\cdot n|+\sqrt{(v_i\cdot n)^2-2m^2n^2/s}}\right)\right. \\ &\left.+\ln\left(\frac{\delta(\pm n)\,n^2-|v_i\cdot n|-\sqrt{(v_i\cdot n)^2-2m^2n^2/s}}{\delta(\pm n)\,n^2-|v_i\cdot n|+\sqrt{(v_i\cdot n)^2-2m^2n^2/s}}\right)\right] \end{aligned} \tag{A.9}$$

with $\delta(n)\equiv |v_i\cdot n|/(v_i\cdot n)$. Then, using Eq. (A.4), we get

$$\omega_{ij}(\delta_i v_i,\delta_j v_j,\Delta_i,\Delta_j) = \mathcal{S}_{ij}\,\frac{\alpha_s}{\pi\epsilon}\left[L_\beta+L_i+L_j-1\right]. \tag{A.10}$$

The contribution of the heavy quark self energy graphs in Fig. 2(b) to the anomalous dimension matrices for heavy quark production in either partonic channel is $(\alpha_s/\pi)C_F(L_1+L_2-2)\delta_{IJ}$, which cancels the gauge-dependent contribution from the ω_{12} in Eq. (A.10).

When v_i refers to a massive quark and v_j to a massless quark we have [21, 31]

$$\begin{aligned} I_1^{\text{UV pole}} &= \frac{\alpha_s}{2\pi}\left\{\frac{2}{\epsilon^2}-\frac{1}{\epsilon}\left[\gamma+\ln\left(\frac{v_{ij}^2 s}{2m^2}\right)-\ln(4\pi)\right]\right\}, \\ I_2^{\text{UV pole}} &= -\frac{\alpha_s}{\pi}\frac{1}{\epsilon}L_i\,, \\ I_3^{\text{UV pole}} &= \frac{\alpha_s}{2\pi}\left\{\frac{2}{\epsilon^2}-\frac{1}{\epsilon}\left[\gamma+\ln\nu_j-\ln(4\pi)\right]\right\}, \\ I_4^{\text{UV pole}} &= -\frac{\alpha_s}{\pi}\frac{1}{\epsilon}\,, \end{aligned} \tag{A.11}$$

where

$$\nu_a = \frac{(v_a \cdot n)^2}{|n|^2}\,, \tag{A.12}$$

and $v_{ij} = v_i \cdot v_j$. Note that the double poles cancel in the sum over the I's and we get

$$\omega_{ij}(\delta_i v_i, \delta_j v_j, \Delta_i, \Delta_j) = \mathcal{S}_{ij}\,\frac{\alpha_s}{\pi\epsilon}\left[-\frac{1}{2}\ln\left(\frac{v_{ij}^2 s}{2m^2}\right) + L_i + \frac{1}{2}\ln\nu_j - 1\right]. \tag{A.13}$$

Finally, when both v_i and v_j refer to massless quarks we have [55, 21, 31]

$$\begin{aligned}
I_1^{\text{UV pole}} &= \frac{\alpha_s}{\pi}\left\{\frac{2}{\epsilon^2} - \frac{1}{\epsilon}\left[\gamma + \ln\left(\delta_i\delta_j\,\frac{v_{ij}}{2}\right) - \ln(4\pi)\right]\right\}\,,\\
I_2^{\text{UV pole}} &= \frac{\alpha_s}{2\pi}\left\{\frac{2}{\epsilon^2} - \frac{1}{\epsilon}\left[\gamma + \ln\nu_i - \ln(4\pi)\right]\right\}\,,\\
I_3^{\text{UV pole}} &= \frac{\alpha_s}{2\pi}\left\{\frac{2}{\epsilon^2} - \frac{1}{\epsilon}\left[\gamma + \ln\nu_j - \ln(4\pi)\right]\right\}\,,\\
I_4^{\text{UV pole}} &= -\frac{\alpha_s}{\pi}\frac{1}{\epsilon}\,.
\end{aligned} \tag{A.14}$$

Again, note that the double poles cancel in the sum over the I's and we get

$$\omega_{ij}(\delta_i v_i, \delta_j v_j, \Delta_i, \Delta_j) = \mathcal{S}_{ij}\,\frac{\alpha_s}{\pi\epsilon}\left[-\ln\left(\delta_i\,\delta_j\,\frac{v_{ij}}{2}\right) + \frac{1}{2}\ln(\nu_i\nu_j) - 1\right]. \tag{A.15}$$

In order to obtain contributions to the different entries of the matrix of renormalization constants, the above expression has still to be multiplied by the color decomposition of its corresponding diagram into the basis color structures [21, 31, 55].

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