PHYSICS 1020

Homework set 11

24 February 1997

[11.1] How fast must Velma move past Mort if Mort is to observe her spaceship's length to be reduced by 50%? If Velma is flying over the United States (about 5000 km wide) at this speed, how wide will she observe the United States to be?
Answer:
The length contraction is given by

$$L = \frac{L_\circ}{\gamma}, \quad \gamma \ = \ \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}$$

In this formula, L is the length of the object as seen by an observer whose speed relative to the object is v, and $L_\circ$ is its "proper length", i.e. the length of the object as measured in its rest frame, i.e. as measured by an observer who is at rest with respect to the object. We are looking for that β for which $\gamma = 2$. Solving the relation between γ and β for β, one obtains:

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

In our case this gives

$$\beta = \sqrt{1 - \frac{1}{2^2}} = \frac{\sqrt{3}}{2} \quad \approx 0.866$$

The speed necessary is therefore $= 0.866c$. (You can read off this number also from Fig.11.6 of the textbook). At this speed, the United States appear (5000/2 km) = 2500 km wide.

[11.2] Mort's swimming pool is 20 m long and 10 m wide. If Velma flies lengthwise over the pool at 60% of lightspeed, how long and how wide will she observe it to be?
Answer:
Using the same relations as in the previous exercise, we find that the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \ = \ \frac{1}{\sqrt{1-0.6^2}} \ = \ \frac{10}{8}.$$

Therefore a meter stick will appear 0.8 m long. Since Velma flies lengthwise over the swimming pool, only its length will appear to be contracted (to 0.8×20 m = 16 m, while its width will appear unchanged. (You can also read off the factor 0.8 from Fig. 11.2).

[11.5] Velma, passing Mort at $0.25c$, launches a rocket forward at a launch speed of 0.5c. According to Galilei's relativity, how fast does the rocket move relative to Mort? How fast according to Einstein? Which answer is correct?

Answer:

According to Galilean relativity, the velocities add linearly, i.e. the total speed relative to Mort $= (0.25 + 0.5)c = 0.75c$.

According to Einstein's relativity, the velocities add according to

$$\beta = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2}.$$

In our case this gives

$$\beta = \frac{0.25 + 0.5}{1 + 0.25 \times 0.5} = \frac{0.75}{1.125} \approx 0.667.$$

The speeds involved are too large for Galilean relativity to be applicable. Einstein's generalization needs to be used.

[11.7] Velma's spaceship has a rest mass of 10000 kg, and she measures its length to be 100 m. She moves past Mort at 0.8c. According to Mort's measurements, what are the mass and the length of her spaceship?

Answer:

The Lorentz factor γ for this speed is:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.8^2}} = \frac{1}{0.6} = \frac{10}{6}$$

To Mort, the mass of Velma's spaceship appears bigger by a factor of γ, while its length (in the direction of motion) appears contracted by a factor $\frac{1}{\gamma}$. Therfore, according to Mort, Velma's spaceship has a mass of 16666.7 kg, and a length of 60 m. (You can also get these numbers from Fig. 11.4).

[11.12] You are in a spaceship moving past Earth at nearly lightspeed and you observe Mort, who is on Earth. You measure his mass, pulse rate, and size. How have they changed?

Answer:

As seen from you, Mort is moving past you at nearly light speed. Therefore his mass appears increased by a large factor, his heartbeat has slowed down, and his size (in the direction of motion) has shrunk by the same factor.

[**11.16**] Making estimates: Show that if all the energy "released" (transformed) when fissioning 1kg of uranium were used to heat water, about 2 billion kg of water could be heated from freezing point up to boiling.
Answer:
The mass loss in fission is about 1%, i.e. in the fission process of 1 kg uranium, 0.01 kg is converted into thermal energy. The corresponding energy is

$$0.01\text{kg} \times (3 \times 10^8 \text{m})^2 = 9 \times 10^{14} \text{ J}$$

To heat 1 kg water from freezing to boiling point requires 100kcal $\approx 4.2 \times 10^5$J. With 9×10^{14} J, one can heat

$$\frac{9 \times 10^{14}}{4.2 \times 10^5} \text{ kg} = 2.14 \times 10^9 \text{kg},$$

i.e. about 2 billion kg of water.

[**11.18**] If you were in a rocketship in space accelerating at $2g$ and you dropped a ball, how would it move as observed by you? What if your acceleration were instead $0.5g$? What if you were not accelerating at all?
Answer:
If the rocketship were accelerating at $2g$, the ball would fall toward the rear (which you would perceive as "down") with an acceleration of $2g$. If the acceleration of the rocketship were $0.5g$, the acceleration of the ball "downwards" (i.e. towards the rear) would be $0.5g$. If the rocketship were not accelerating at all, the dropped ball would stay suspended where you released it.

[**11.19**] Longitudinal (north-south) lines on Earth are "straightest lines" that eventually meet. What about latitudinal (east-west) lines? Are they circles? Are they "straightest" lines? Do they eventually meet?
Answer:
In the (rather good) approximation that the Earth is a sphere, the longitudinal lines on Earth, also called "meridians", are circles. They are "great circles" (i.e the largest circles one can draw on the surface of a sphere, with center = the center of the sphere, and radius = radius of the sphere). Such great circles are "geodesic lines" (i.e the shortest connection between two points) in the two-dimensional space formed by the surface of the sphere.
The latitudinal lines are also circles, but not great circles. They do not meet (intersect), but they are not geodesic ("straightest") lines, apart from one exception, the equator (= the 0 latitude line).

[**11.20**] Is a circle (meaning the perimeter of a flat circular area) a space? How many dimensions does it have? Is it a curved space, or is it flat? give an example of a flat one-dimensional space.
ANswer:
A circle is a curved one-dimensional space. A straight line (infinitely long) is a flat one-dimensional space.