UCD Physics 9A Lab Manual

The Staff of the UCD Physics Dep't
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ERRORS AND UNCERTAINTIES

No measurement is exact. A knowledge of the non-exactness of a measurement is important especially when trying to define how "good" or useful a certain result is. For physics experiments we need quantitative words to describe how close a result is to the expected answer and how reliable or reproducible our results are.

The questions following the section are meant to illustrate certain concepts. Your TA may assign some as homework, or work them out as examples. It is recommended you look them over whether or not you work them out.

Since the study of errors and uncertainties is a field in itself (statistics and numerical analysis) we can not answer all questions in this short section. However, for any students wishing to pursue further study we recommend the references:

Hugh D. Young, Statistical Treatment of Experimental Data, McGraw-Hill, 1962.

N.C. Barford, Experimental Measurements: Precision, Error and Truth, Addison-Wesley, 1967.

Philip R. Bevington, Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill, 1969.

John R. Taylor, An Introduction to Error Analysis; The Study of Uncertainties in Physical Measurements, University Science Books, 1982.

William Lichten, Data and Error Analysis in the Introductory Physics Laboratory, Allyn & Bacon, 1988.

OUTLINE OF TOPICS — PAGE

A. DEFINITION OF ERROR

ERROR is defined as the difference between an observation (either directly measured or calculated from measurements) and the true value: Error = Obs.-True. This can be remembered mnemonically as "the true value is the observed value minus the error." (Note the "true" value is not often known since it is the usual reason for doing the experiment! We find then that we will need another concept, the uncertainty, which is an estimate of the error).

B. TYPES OF ERROR

ILLEGITIMATE ERROR is that error introduced by outright blunders. An example would be misreading a ruler or a calculational mistake.

SYSTEMATIC ERROR is a reproducible inaccuracy introduced by imperfect equipment, calibration, or technique. An example would be a shrunken meter stick which would cause measurements to be consistently too high.

RANDOM ERROR is a measure of fluctuation in results during repeated experimentation. An example would be measuring the period of a pendulum three times and getting 3.1, 3.3, and 3.2 seconds.

C. MEASURES OF ERROR

ACCURACY is a measure of how close a measurement comes to the "true" value. In other words, it is dependent on how well we can control or compensate for systematic errors (and eliminate illegitimate errors).

PRECISION is a measure of how "exactly" the result is determined without reference to any "true" value. In other words, it is dependent on how well we can overcome or analyze random errors.

D. DEFINITION OF UNCERTAINTY

Uncertainty is an estimate of the systematic and/or random errors inherent in the measurement or calculation. Often we cannot know what the "true" value is, and therefore cannot say what the actual error is. However, random errors can be estimated from repetition of measurements and systematic errors can be estimated from understanding of the equipment and technique. NOTE the technical words "error" and "uncertainty" are often used interchangeably since the latter is an estimate of the former.

E. NOTATION OF UNCERTAINTY (and ERROR)

ABSOLUTE UNCERTAINTY indicates the magnitude of the uncertainty in the result in the same units as the result. We will use the symbol[1] σ_x for the absolute uncertainty in the quantity x. For example, an uncertainty of $\sigma_L = 2\text{cm}$ in a length L of three meters would be expressed as $L = 3.00 \pm 0.02\text{m}$.

RELATIVE UNCERTAINTY indicates the uncertainty as a fraction of the result. For our previous example the relative uncertainty is

$$\frac{\pm 0.02\,meters}{3.00\,meters} = \pm 0.007.$$

PERCENT UNCERTAINTY is just the relative uncertainty multiplied by 100% to express the fraction as a percentage. We will use the symbol (please refer to previous footnote) e_x to represent the percent uncertainty in the quantity x. Thus in our previous example the percent uncertainty in L would be: $\pm 0.007(100\%) = 0.7\%$. The relationship between absolute and percent uncertainty can thus be expressed:

$$e_x = 100\%\left(\frac{\sigma_x}{x}\right) \qquad (1)$$

$$\sigma_x = x\left(\frac{e_x}{100\%}\right) \qquad (2)$$

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[1] Symbolic notation for uncertainties is not standardized. Some authors use the symbol sigma (σ) to specifically denote standard deviation. Often a capital or lower case delta $(\Delta x\, or\, \delta x)$ is used to denote absolute uncertainty in the quantity *x*. The symbol (*e*) is not standard at all.

F. PROPAGATION OF UNCERTAINTIES

Any calculations using quantities which are uncertain will lead to uncertainty in the result. For example if z is a function of x, as $z = f(x)$, then an uncertainty σ_x in the quantity x will give us an uncertainty σ_z in the quantity $z : z \pm \sigma_z = f(x \pm \sigma_x)$. Thus we could find the uncertainty in a calculated quantity from the differences

$$\pm\sigma_z = f(x \pm \sigma_x) - f(x) \qquad (3)$$

For example, if we measure the side of a square to be $x = 3.0 \pm 0.05\text{cm}$ then the area of the square $z = x^2$ is uncertain by the amount $\sigma_z = \pm 0.3\text{cm}^2$ [since $(3.05)^2 = 9.30 = 9.0 + 0.3$ and $(2.95)^2 = 8.70 = 9.0 - 0.3$]. Note the distinction between "+" or "-" sign in Equation (3) is unnecessary for <u>small</u> σ_x (see Question 8).

1. GENERAL FORMULA

a. Functions of one variable

Utilizing the definition of a derivative we find Equation (3) for small uncertainties (see Question 9 and section on Differential Approximations) can be written as:

$$\sigma_z = \sigma_x \frac{\partial f}{\partial x} \qquad (4)$$

In our example above, $\frac{dx^2}{dx} = 2x$ so Equation (4) yields $\sigma_z = \sigma_x(2x) = 0.3\text{cm}^2$ as before.

b. Functions of two or more variables

Where the quantity z is calculated from two (or more) variables x, y, w which are independent,[2]

$$z = f(x,y,w)$$

the uncertainty in z is found from combining the uncertainties in quadrature (square root of the sum of squares):

$$\sigma_z = \sqrt{\left(\sigma_x \frac{\partial f}{\partial x}\right)^2 + \left(\sigma_y \frac{\partial f}{\partial y}\right)^2 + \left(\sigma_w \frac{\partial f}{\partial w}\right)^2} \qquad (5)$$

2. SPECIFIC CASES

General formulas (such as Equation (5)) are often pleasing to the student and TA because they are all-encompassing. However, they can often degenerate into busy-work

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[2]Two quantities x and y are correlated or <u>not</u> independent if, for example, a high value of x is more likely to be observed with a high value of y than with a low value of y. In general where the quantity z is calculated from correlated quantities x and y:

$$z = f(x, y) \qquad (6)$$

the absolute uncertainty σ_z in z is given by:

$$(\sigma_z)^2 = (\sigma_x)^2\left(\frac{\partial f}{\partial x}\right)^2 + (\sigma_y)^2\left(\frac{\partial f}{\partial y}\right)^2 + 2\sigma_{xy}^2\left(\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\right) \qquad (7)$$

where σ_x is the absolute uncertainty in x, σ_y is the absolute uncertainty in y and σ_{xy}^2 is the covariance between x and y.

The covariance is a measure of how x and y are correlated. If it is zero, then we say the two variables are independent and equation (7) reduces to equation (5). For completeness we note that if z and u are two quantities independently calculated from x and y:

$$z=f(x,y);\quad u=g(x,y) \qquad (8)$$

then the covariance of u and z is given by:

$$\sigma_{zu}^2 = \sigma_x^2\left(\frac{\partial f}{\partial x}\frac{\partial g}{\partial x}\right) + \sigma_y^2\left(\frac{\partial f}{\partial y}\frac{\partial g}{\partial y}\right) + \sigma_{xy}^2\left(\frac{\partial f}{\partial x}\frac{\partial g}{\partial y} + \frac{\partial f}{\partial y}\frac{\partial g}{\partial x}\right) \qquad (9)$$

The subject of correlation and covariance is a bit sophisticated, and we recommend that the interested student look at the references. See also Questions 15 and 17.

keeping the student from learning some "horse sense" about the dominant sources of uncertainty in a specific case. The three most common types of uncertainty calculations are given below (assuming $\sigma_{xy}^2 = 0$, that is x and y are independent).

a. Addition (Or Subtraction)

Let $z = f(x,y) = x + y$, for which $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = +1$. Equation (5) then is:

$$\boxed{\sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2}} \qquad (10)$$

It can be shown you get the same result for subtraction (see Question 10). Thus, for addition and subtraction the absolute uncertainty of the result is the combination (in quadrature) of the absolute uncertainties in the primary numbers.

b. Multiplication (Or Division)

Let $z = f(x,y) = xy$, for which $\frac{\partial z}{\partial x} = y$, $\frac{\partial z}{\partial y} = x$.. Equation (5) then is:

$$(\sigma_z)^2 = (\sigma_x)^2 y^2 + (\sigma_y)^2 x^2$$

Dividing by $z^2 = x^2 y^2$ gives us a relation between the relative uncertainties:

$$\frac{\sigma_z^2}{z^2} = \frac{\sigma_x^2}{x^2} + \frac{\sigma_y^2}{y^2} \qquad (11)$$

Taking a square root and multiplying both sides by 100%:

$$\boxed{e_z = \sqrt{e_x^2 + e_y^2}} \qquad (12)$$

It can be shown you get the same result for division (see Question 11). Thus, for multiplication and division the percent uncertainty in the result is the combination (in quadrature) of the percent uncertainties in the primary numbers.

c. Powers

Let $z = f(x) = ax^n$ where a and n are constants (have zero uncertainty). Then $\frac{\partial z}{\partial x} = nax^{n-1} = \frac{nz}{x}$, so Equation (4) is:

$$\sigma_z = \sigma_x \left(\frac{nz}{x}\right)$$

and division by z gives us:

$$\frac{\sigma_z}{z} = n\frac{\sigma_x}{x} \quad \text{or} \quad \boxed{e_z = ne_x} \qquad (13)$$

Thus, for a power the percent uncertainty in the result is n times the percent uncertainty in the primary number.

3. APPROXIMATIONS

Note that the combined uncertainty is usually dominated by the largest single uncertainty, which therefore is a fair-to-good approximation of the total combined uncertainty. For example, a 4% distance uncertainty and a 3% time uncertainty give a velocity (distance ÷ time) uncertainty, $\left[(4\%)^2 + (3\%)^2\right]^{\frac{1}{2}} = 5\%$, which is not much larger than the 4% distance uncertainty alone. (A combination 4% + 3% = 7% is incorrect unless the distance and the time measurements were correlated in such a way that over-large distance measurements were accompanied by over-small time measurements, a correlation which contradicts the assumption of uncorrelated independence of the uncertainties.)

Any single uncertainty less than 1/3 the size of others can be considered as negligible in the sense that it would raise the combined uncertainty by a factor closer to 1 than $[1 + 1/9]^{\frac{1}{2}} = 1.05$ and thus would not affect the only significant figure of our uncertainty. Even if two uncertainties were the same size, their combined uncertainty is only larger by a factor of $\sqrt{2} = 1.4$ than either uncertainty alone.

Thus, the largest single uncertainty in the calculated result is usually an acceptable approximation to the routinely required estimated uncertainty.

G. ESTIMATES OF UNCERTAINTIES

Deciding the uncertainties in primary data depends on the circumstances and method of taking data. However, before discussing that, let us agree on the level of uncertainty.

The size of the uncertainty depends upon the meaning attributed to it. It is often incorrectly assumed that the uncertainty should be an "error limit" in the sense of being large enough that the correct value is certain to be within the limits. Taken to its logical extreme, this would lead to error limits much too large to contain any information about the actual uncertainties or fluctuations in the measurements. On the other extreme, we also want to avoid uncertainties so small that they would be unlikely to contain the correct value. We will adopt the usual criterion of a "68% confidence level" uncertainty, representing an estimation that the error limits have a 68% probability of containing the correct value (and conversely, a 32% probability of not containing it)[3]

1. INSTRUMENTAL ACCURACY

All measuring instruments have an inherent "limiting accuracy" due to imperfections in manufacture. A simple way to estimate uncertainty is to consider the smallest divisions that can be read unambiguously and take half their size as the limiting instrumental accuracy. For example a meter stick with 1 mm divisions has an implied inherent uncertainty of ±0.5 mm. The student is expected routinely to record at least this uncertainty for each measurement or instrument.

2. EXPERIMENTAL PRECISION

Even if our measuring instrument had perfect accuracy, different measurements of the same quantity, say *x*, would yield different results $x_1, x_2, x_3, ...$ due to uncontrollable fluctuations in that quantity. "Uncontrollable" means that, even if we knew exactly the mean value of the quantity (x_0) and the exact RMS size of the fluctuations (s_0) and their reason (such as air molecules hitting a mass and changing its location), we could not predict the measurements $x_1, x_2, ...$ We might, however, be able to predict the distribution of an infinite number of such measurements. For example, if the fluctuations are indeed random, statistical theory says the measurements will have a Gaussian distribution $e^{-(x-x_0)^2/2s_0^2}$ as shown. This means (among other things) that the most probable result of a measurement is the mean (x_0) and that 68% of the fluctuations are smaller than the RMS deviation $(x_0 - s_0 < x < x_0 + s_0)$.

What we want to do is use a finite number of measurements, $x_1, x_2, x_3, ..., x_N$ to obtain an estimate (s) for the exact size (s_0) of the fluctuations, an estimate $(\bar{x})$ for the exact mean (x_0) and the uncertainty $(\sigma_{\bar{x}})$ of this estimated mean. The best values to use for these estimates are the standard deviation (s) and the average $(\bar{x})$ and the precision $(\sigma_{\bar{x}} = s/\sqrt{N})$, which we will now discuss.

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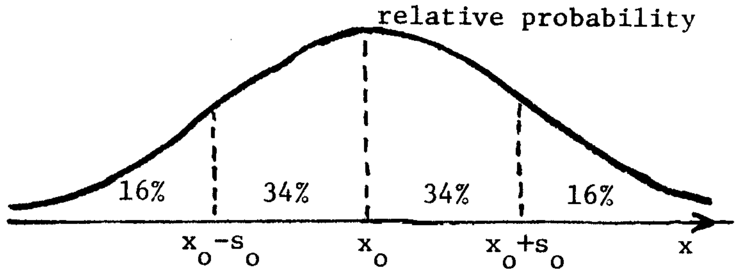
[3] This corresponds to a "standard deviation" in statistical theory-68% of the deviations from the mean of a Gaussian distribution are smaller than the RMS deviation (square Root of the Mean of the Squares of the deviations).

Another criterion used is the "Probable Error" representing an estimation that the error limits have a 50% probability of containing the correct value.



a. Average and Standard Deviation

Let $x_1, x_2, x_3, ..., x_N$ be N readings of a physical quantity. The <u>average</u> $\bar{x}$ of this set of readings is:

$$\bar{x} = \frac{x_1 + x_2 + x_3 ... + x_N}{N} \equiv \frac{\Sigma x}{N}, \quad (14a)$$

and the <u>deviation</u> of each of the readings from the average is

$$\delta_i \equiv x_i - \bar{x}. \quad (14b)$$

The <u>standard deviation,</u> s, of this set of readings is defined as the square root of

$$s^2 \equiv \frac{\Sigma(\delta_i)^2}{N-1} = \frac{\Sigma(x - \bar{x})^2}{N-1} = \frac{\Sigma\left(x^2 - 2x\bar{x} + \bar{x}^2\right)}{N-1} = \frac{\Sigma\left(x^2\right) - N\bar{x}^2}{N-1} \quad (15)$$

The root-mean-square, RMS, deviation is the same except for dividing by N instead of N-1. [These calculations are done on many calculators simply by inputting each number ($x_1, x_2, ..$) using a special accumulation button (most are marked <u>Σ +</u> or <u>M+</u>) which accumulates the sum of the numbers ($\sum x$) the sum of their squares ($\sum x^2$), and the number of numbers entered ($N = \sum 1$). Another button (marked $\bar{x}$) will then give the average of the numbers entered and another (marked σ_{n-1} or s) will give their standard deviation. (Some also have buttons marked σ_n which give the RMS deviation.) Even if your calculator does not have such functions, you can accumulate $\sum x$ and $\sum x^2$ and use the last forms of (14a) and (15) to quickly find $\bar{x}$ and s. [Warning: Do not round off before subtracting the nearly-equal numbers in $\left(\Sigma\left(x^2\right) - N\bar{x}^2.\right)$]

For example, if N = 6 readings were found to be 18.1, 18.2, 18.3, 18.5, 18.6, and 18.7, the average would be $\bar{x} = 18.4$, the deviations would be -0.3, -0.2, -0.1, +0.1, +0.2, +0.3, and the standard deviation would be $s = \sqrt{0.28/5} = 0.24$. We see that the deviations add up to zero (a check that we have done the arithmetic correctly) and that 4/6 (~68%) of the deviations are smaller than the standard deviation and 2/6 are larger, as expected.

We may take the standard deviation $s = \sigma_x$ as the random uncertainty of any <u>single</u> measurement (see references). Thus, in our example, the 3rd reading is 18.3 ± 0.24.

b. Uncertainty in the Average

As we increase the number of measurements, the individual readings will continue to fluctuate by about the same amount, s, but the average will change by less and less. It may be shown (see Question 21) that the random uncertainty in such an average may be taken to be the square root of

$$\sigma_{\bar{x}}^2 = \frac{\Sigma(\delta_i)^2}{N(N-1)} = \frac{s^2}{N}, \quad (16)$$

which we note is smaller, by a factor of

$\sqrt{N}$, than the random uncertainty in the individual readings. Therefore, we can increase our precision (but not our accuracy, see Question 22), by repeating the measurement many times.

In our example, we would state our resulting average as

$$\bar{x} = 18.4 \pm 0.1$$

since $\sigma_{\bar{x}} = \sqrt{0.28/30} = 0.24/\sqrt{6} = 0.097$. In order to improve the precision by an order of magnitude (factor of ten) to ± 0.01, one would have to increase the number of measurements by two orders of magnitude (a factor of $10^2 = 100$) to N = 600.

H. SIGNIFICANT FIGURES

It is generally proper to assume that all digits needed to write down a number are significant. Therefore, numbers reported with higher precision than appropriate are misleading and incorrect, so the student must learn how many figures are actually significant. The basic rule is that no completely uncertain digits should be used.[4]

We have already discussed how to estimate the uncertainty in every number used in lab, so it is relatively easy to round off such a number to the first decimal place of the uncertainty. For example, the overly precise number 11.378965 $\pm$ 0.2 should be rounded off to 11.4 ± 0.2.

The uncertainty itself is uncertain and therefore should not be reported with undeservedly high precision. In this lab, although we may take pains with uncertainties, they are not expected to be known to better than one significant figure (except perhaps when the first figure is 1) and should not be reported with higher precision in your final results. For example, an uncertainty you have calculated to be ± 0.356 should be written as ± 0.4 in a final result, but you have the option of rounding ± 0.146 to either ± 0.1 or ± 0.15.

The correct numbers of significant figures must be used for all data recorded and for all final results. For convenience, intermediate calculations will be allowed to be written down with one or two extra (insignificant) figures, but students should avoid absurdities such as ten-figure numbers for experiments that are only good to 1%.

Ambiguous Trailing Zeros

It should be understood that zeros are generally considered as significant figures except for zeros in front of a number (called leading zeros) used to indicate the decimal point. For example,

4.0670 has five significant figures (intermediate and trailing zeros are both significant) while 0.00986 has three significant figures (leading zeros are not significant figures).

A trailing zero leads to ambiguities only if there is no decimal point. For example, it is unclear if the number 800 has one, two, or even three significant figures. To avoid this ambiguity, it is better to write such numbers in exponential notation. Thus 8.0×10^2 has two significant figures while 8.00×10^2 has three significant figures, etc.

It is interesting to note that, if we were only concerned with the orders of magnitude of uncertainties we could replace explicit uncertainty calculations with corresponding consideration of the proper use of significant figures. Thus, the absolute uncertainty is in the last decimal place written, and the relative uncertainty is known by the number of significant figures (N significant figures is 10^{-N} relative uncertainty or $e = 10^{2-N}\%$).

[4] Outside of our lab, it is often (but not always) presumed that all digits given are certain. Then a number such as 10.6 has an implied error limit of ± 0.05. In our example, 11.4 $\pm$ 0.2 would then be written as 11 and might be considered to have two (instead of three) significant figures. A compromise form of these slightly different presumptions would say that our example has three significant figures but would insist on printing the uncertain digit in smaller type, as $11._4$. In our lab, these ambiguities are settled by including one uncertain digit but insisting on its uncertainty being given.

These are related to the following simple rules of thumb for significant figures corresponding to uncertainty formulas (10) and (12):

RULES OF THUMB

1. In addition and/or subtraction, the result is known only to the decimal place given by the least accurately known term. It can have fewer significant figures than either term if the operation involves subtracting two nearly equal numbers.

Example: If we add two numbers, N_1= 102.36 and N_2= 12, the result is 114, not 114.36.

Example: If we take the difference between two lengths, L_1 = 1021.6 cm L_2 = 1023.5 cm, the result is 1.9 cm. Even though each term has five significant figures, the result has only two.

2. In multiplication and/or division, the result is known only to as many significant figures as occurs in the least accurately known factor.

Example: If we multiply 327.6 by 0.28, we obtain 92, not 91.728. Such use of significant figures can aid a more intuitive understanding and facilitates the calculation of more precise uncertainties.

QUESTIONS (The TA may assign some of these as homework or work them out as examples to illustrate concepts).

Section A and B

Question 1. Al Caggie measured the acceleration of gravity to be 975 cm/sec^2 as compared to the true value of 980 cm/sec^2. What is his error?

Question 2. Kermit the Frog measures the length of his legs with a steel meter stick and finds that the average of his measurements yields a result of 0.582 meters for the length. He subsequently learns that the meter stick was calibrated at 25˚ C and expands at the rate of 0.5 mm/˚ C, i.e., at 27˚ C it would be 1 mm longer. Kermit made his measurement at 20˚ C. What type of error is involved here? What is the “true” length of his legs?

Question 3. Five engineering students independently measured the length of the blackboard in a lecture hall with a 12′′ ruler. Their results ranged from 32′10′′ to 33′3′′. What type of error is involved here? (Hint - it is not the wrong major).

Question 4. Fannie Farkel discovers after finishing her calculations that her calculator has been giving a random fictitious number every time she pushed the square root button. Thus her final answer is wrong. What type of error is involved here?

Section C, D, and E

Question 5. Given the mass of an object is 32 ± 8 gm, what is the percent uncertainty?

Question 6. Can you say what percent 0.02 cm is of zero cm? Why or why not?

Question 7. Often you will need to express the difference of a quantity (x_1) from another (x_2) as a percent (of either x_1 or x_2) Given x_1= 980 and x_2= 975 from Question 1 above, what is the percent difference in terms of x_1? In terms of x_2? In terms of the average of x_1 and x_2? Is the distinction between these three possibilities important in this case?

Section F

Question 8. Using Equation (3) repeat the example $z = x^2$ where the uncertainty in x = 3.0 cm is now $\sigma_x = \pm 0.5 cm$. Does it make a difference whether you use the “+” or “-” sign in Equation (3)?

Question 9. Derive Eqn. 4 from Eqn. 3 for small uncertainties recalling the definition of a derivative.

Question 10. Given $z = f(x,y) = x\text{-}y$; prove that Equation (10) is still valid.

Question 11. Given $z = f(x,y) = x/y;$ prove that Equation (12) is still valid.

Question 12. Given that the percent uncertainty in x is 3% and the percent uncertainty in y is 5%, what is the percent uncertainty in $z = \sqrt{(x/y)}$?

Question 13. Given $z = \ln(x)$, derive an expression for both the absolute and percent uncertainty of z in terms of the absolute (or percent) uncertainty in x. (Note ln is the "natural" logarithm, i.e., log to the base e). Given $x = 1.0 \pm 0.05$ what is the uncertainty in $\ln x$?

Question 14. Given $z = R \sin \theta$, derive an expression for the percent uncertainty in z in terms of the percent uncertainty in R and the absolute uncertainty in θ.

Question 15. Compare the following three statements:

(a) If $z = 2x$ the absolute uncertainty in z is $\sigma_z = 2\sigma_x$.

(b) If $z = x + y$ and the absolute uncertainty in x and y are the same, then $\sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2} = \sigma_x\sqrt{2}$.

(c) If $y = x$ in (b) then clearly $z = x + x = 2x$ as in (a), so $\sigma_z = 2\sigma_x$, which contradicts the (b) result $\sigma_z = \sigma_x\sqrt{2}$ Which one is wrong and why?

Question 16. At one second intervals a cart's position is measured along a meter stick (in cm):

X_0	X_1	X_2	X_3	X_4	X_5
0.1	5.3	10.9	15.8	21.5	25.9

Five observers (A thru E) calculated the velocity according to the following methods:

A took how far the cart went in 5 seconds, divided distance by time to get

$$V_A = \frac{x_5 - x_0}{5 \text{ sec}} = 5.16 \text{cm/ sec}$$

B took how far the cart went from X_0 and divided by the time to each point. He then averaged the five velocities to get his "best" velocity:

$$\frac{X_1 - X_0}{1} = 5.2; \quad \frac{X_2 - X_0}{2} = 5.4;$$

$$\frac{X_3 - X_0}{3} = 5.23; \quad \frac{X_4 - X_0}{4} = 5.35;$$

$$\frac{X_5 - X_0}{5} = 5.16;$$

Average is $V_B = 5.27$ cm/sec

C calculated how far the cart went each second and took an average of these numbers to get the best velocity. $(X_1 - X_0) = 5.2$; $(X_2 - X_1) = 5.6$; $(X_3 - X_2) = 4.9$; $(X_4 - X_3) = 5.7$; $(X_5 - X_4) = 4.4$; Average is $V_C = 5.16$ cm/sec.

D calculated the distance traveled in each 3 second interval, and averaged the resulting three numbers to get his best velocity:

$$\frac{X_3 - X_0}{3} = 5.27; \quad \frac{X_4 - X_1}{3} = 5.40;$$

$$\frac{X_5 - X_2}{3} = 5.00;$$

Average is $V_D = 5.22$ cm/sec.

E took every other velocity from observer C and averaged: $(X_1 - X_0) = 5.2$; $(X_3 - X_2) = 4.9$; $(X_5 - X_4) = 4.4$

Average is $V_E = 4.83$ cm/sec.

16a. Why did "C" get the same result as "A" even though he did a more complicated calculation? Show mathematically they really did the same calculation. What lesson is to be learned here?

16b. Using the general error propagation formula (Equation 4) calculate an expression for the absolute uncertainty in the mean velocity for each observer. To simplify the algebra, take the absolute uncertainty in each position to be the same:

$$\sigma = \sigma_0 = \sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5$$

Hence for $V = f(X_0, X_1, X_2, X_3, X_4, X_5)$ Eqn. (5) reduces to

$$(\sigma_V)^2 = (\sigma)^2 \left[\left(\frac{\partial f}{\partial X_0}\right)^2 + \left(\frac{\partial f}{\partial X_1}\right)^2 + \left(\frac{\partial f}{\partial X_2}\right)^2 + \left(\frac{\partial f}{\partial X_3}\right)^2 + \left(\frac{\partial f}{\partial X_4}\right)^2 + \left(\frac{\partial f}{\partial X_5}\right)^2 \right]$$

Express your results for σ_V as a number times σ (different numbers for A, B, C, D, E in general).

16c. Based on your results from 16b, which observer gives the most precise result? Can you say which observer gives the most accurate result? Why or why not?

16d. Compare E's result to D's. Which is better? Why is one observer's technique better than the others?

16e. What is the weak point in B's method? (Clue - why does he get an average which is higher than everyone else’s?)

<u>Question 17</u>. Suppose we convert a set of Cartesian coordinates x and y into polar coordinates:

$$R = \sqrt{x^2 + y^2} \quad \theta = \arctan(y/x).$$

Given that the uncertainties in x and y are independent (i.e., $\sigma_{xy}^2 = 0$) are the uncertain-ties in R and θ independent? (Hint: Calculate $\sigma_{R\theta}^2$ using Eqn. 9).

<u>Section G</u>

<u>Question 18</u>. What is the instrumental uncertainty for a balance which has divisions of 0.1 gm?

<u>Question 19.</u> If I average four numbers, each (independently) with absolute uncertainty of ± 0.8, what will be the absolute uncertainty of the average of the four numbers?

<u>Question 20.</u> Suppose we are trying to determine whether a measurement “p” is consistent with a calculated value “q” of the same quantity, where q has negligible uncertainty and p has uncertainty σ_p. If they differ by $p - q = (1.6)\sigma_p$, which of the following is true?

a. Surely $p \neq q$ (measurement and calculation are not consistent).

b. Not sure, but probably $p \neq q$.

c. Not sure, but probably $p = q$.

d. Surely $p = q$ (measurement and calculation are consistent).

Reanswer for $p - q = (0.6)\,\sigma_p$.

<u>Question 21.</u> Derive Equation (16) from the propagation of uncertainties formula (Eqn. 5) and the definition of an average (Eqn. 14).

<u>Question 22.</u> Suppose we have a watch (not unlike our digital laboratory timers) which only measured to the nearest second. What is its limiting accuracy?

Using this timer we measure the period of a pendulum three times and get 11 seconds each time. What is the precision of our measurement?

Unknown to us, another student was timing the period of the pendulum at the same time but with a timer that measured to the tenth of a second. He got 11.4 seconds for the three measurements which is consistent with our timer since ours reads only to the nearest second. Who has the best measurement of the period? Why?

If we made a million measurements and got 11 seconds each time would our answer be any better? Why?

Section H

Question 23. The latitude of the observatory at Davis is given as $38^\circ 32' 28'' N$. (Recall $1^\circ = 60', 1' = 60''$).

a. What is the implied uncertainty in the latitude?

b. Given that the radius of the earth is 6.371×10^6 meters, what is the uncertainty in the position of the observatory in meters? Is the uncertainty larger than the size of the campus?

c. Write the latitude in decimal degrees with the correct number of significant figures.

d. Suppose the next digit was measured (e.g., $28.3''$) what would be the implied uncertainty? Would you now be able to tell which side of Hutchison Hall (approximately 15 meters wide) the observatory is on?

Question 24. The number 0.0070001 rounded to two significant figures is what?

Question 25. The number 3758 rounded to three significant figures is what? To two significant figures? to one significant figure?

Question 26. How many significant figures has: 900.000?, 0.005070 ?, 13.80?

Question 27. Give the answers with the correct number of significant figures:

a. $19.293 - 12.3 = ?$
b. $19.293 \div 12.3 = ?$
c. $9.278 - 2.9\pi = ?$