CENCO 33210C Gravitational Torsion Balance

Determining the gravitation constant in accordance with the full deflection method

The gravitation constant G will be determined in accordance with the full deflection method using a torsion balance.

The gravitational force with which two lead balls with masses of m_1 = 1.5 kg and m_2 = 0.015 kg attract each other at a distance of 4.5 cm is less than 10^{-9} N. This force can only be verified with an extremely sensitive torsion balance.

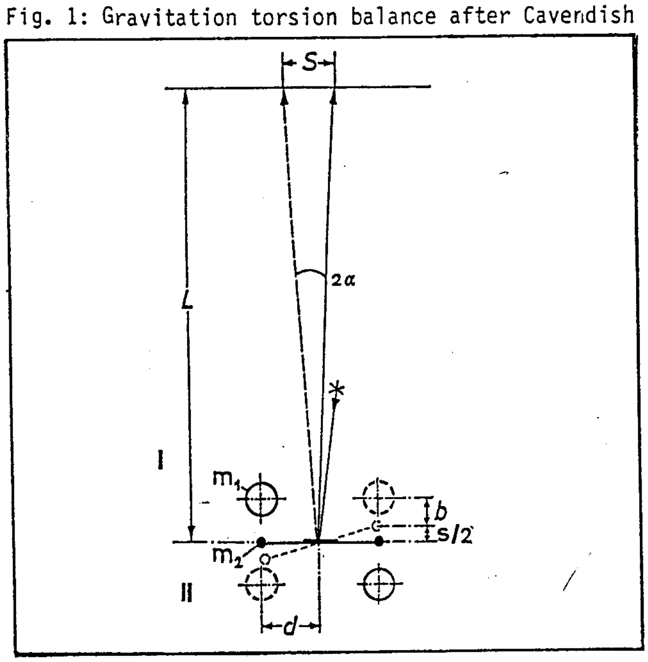
The core of the torsion balance is a light transverse beam suspended horizontally from a thin torsion wire and with a lead ball of mass m_2 = 0.015 kg attached at each end. These balls are attracted by two large lead balls with a mass of m_1 = 1.5 kg. The resultant deflection of the transverse beam from its initial position is determined by means of a light pointer (cf Fig. 1/ Fig 2).

Both the initial and final equilibrium positions of the balance measuring system and the damped intermediate oscillation motion must be observed to determine the gravitation constant. This involves an observation time of approximately 45 minutes.

Carefully check the initial position of the balance, and then start the experiment by moving the large balls to the opposing position, that is to say from position I to position II. After a few oscillations, the measuring system moves from the initial position, designated as one limit position, to the new limit position. The angle between these two limit positions is given by α. This can be calculated from the dimensions of the setup and the light pointer deflection in accordance with the details given in Fig. 1. The torque M acting on the measuring system in one limit position due to mass attraction is M = 2 Fd, where F is the force of attraction between each pair of balls and d is the axis distance of the small balls in the balance from the torsion band. The equilibrium of this torque is maintained by the twisting of the torsion band by the angle $\frac{\alpha}{2}$.

If the angular writing force of the torsion band is designated as D, this torque $M = D\frac{\alpha}{2}$.

Fig. 1: Gravitation torsion balance after Cavendish



Symbol designations (refer to Fig. 1):
- b: the distance between the center points of the large and small balls,
- s: the path of the small balls in the balance,
- d: distance of the small balls from the axis,
- S: the path of the light pointer on the wall,
- L: the distance of the balance mirror from the wall.

The following applies because of angle doubling due to reflection

$$\frac{s}{2d} = \tan\frac{\alpha}{2} \approx \frac{\alpha}{2} \quad ; \quad \frac{S}{2L} = \tan\alpha \approx \alpha$$

thus: $\frac{s}{d} = \frac{S}{2L} \approx \alpha$

The angular righting force D can only be determined from the oscillation period T of the torsional oscillation of the measuring system:

$$T^2 = 4\pi^2\frac{J}{D}$$

or

$$D = \frac{4\pi^2 J}{T^2}.$$

The moment of inertia J here can be equated with the moment of inertia of the two small balls, $J = 2m_2d^2$, because the suspension with the mirror contributes nothing to the moment of inertia in practice. Thus,

$$D = \frac{8\pi^2 m_2 d^2}{T^2}.$$

If the value $M = 2Fd = 2G\frac{m_1m_2d}{b^2}$ and the values obtained above for D and a are inserted in the equation $M = D\frac{\alpha}{2}$ it follows where $2M = D\alpha$ that

$$4G\frac{m_1m_2}{b^2} = \frac{8\pi^2 dm_2}{T^2}\cdot\frac{S}{2L}.$$

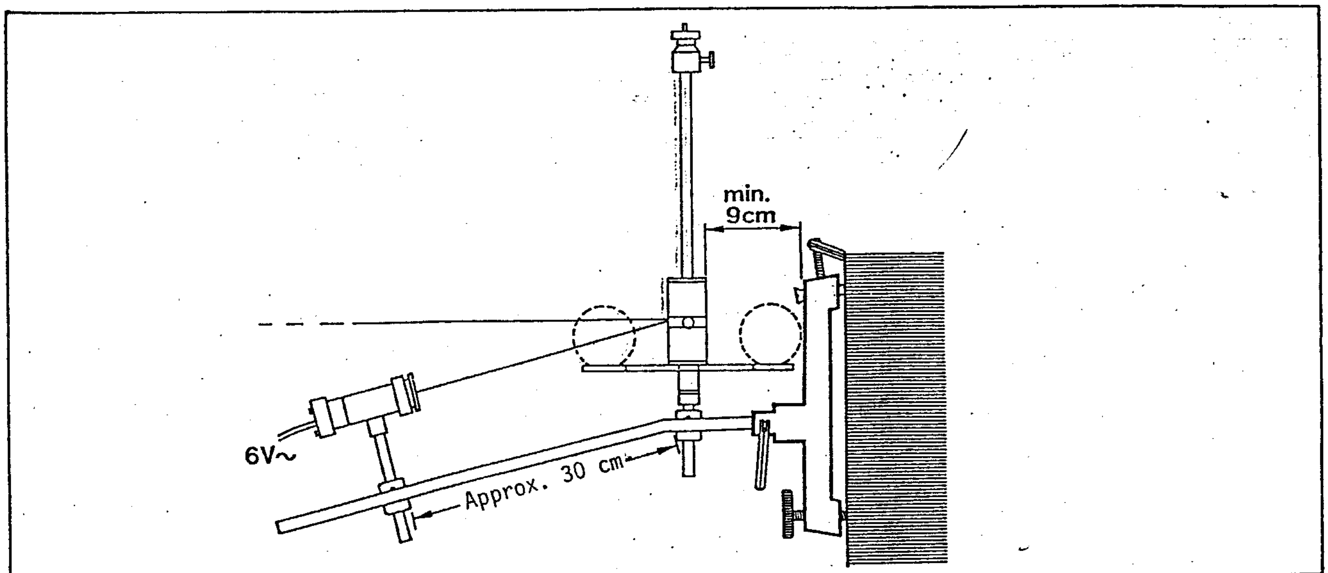


Fig. 2: Gravitation torsion balance with additional equipment set up ready for use

Note:

The following formula applies for the force of attraction between two masses m_1 and m_2:

$$F = G \cdot \frac{m_1 \cdot m_2}{r^2}$$

(r = distance between the masses).

From this, we can calculate

$$G = \frac{\pi^2 b^2 dS}{m_1 T^2 L}.$$

This formula for the gravitation constant only contains measurable variables. The mass of the small balls is not included in the calculation, so it is not necessary to know this.

Apparatus:

1 gravitation torsion balance	332 10
1 steel tape measure	311 77
1 lamp housing	450 60
1 lamp 6 V / 30 W	450 51
1 aspherical condenser	460 20
1 transformer 6 V, 30 W, e.g. transformer 6 V / 30 W	562 73
1 stop-clock	313 05
1 pair of magnets	510 48
2 saddle bases	300 11
1 Leybold multiclamp	301 01
1 stand rod, bent at right angles	300 51
1 stand rod, bent by 15°	300 52
1 large stand base	300 01
1 pair of levelling screws	300 06

Setting up:

Stick the self-adhesive scale included in the scope of delivery of the torsion balance on a steady base or on a wall (refer to the instructions for use 332 10 for the distance of the scale from the torsional balance and for the height h at which it must be set up).

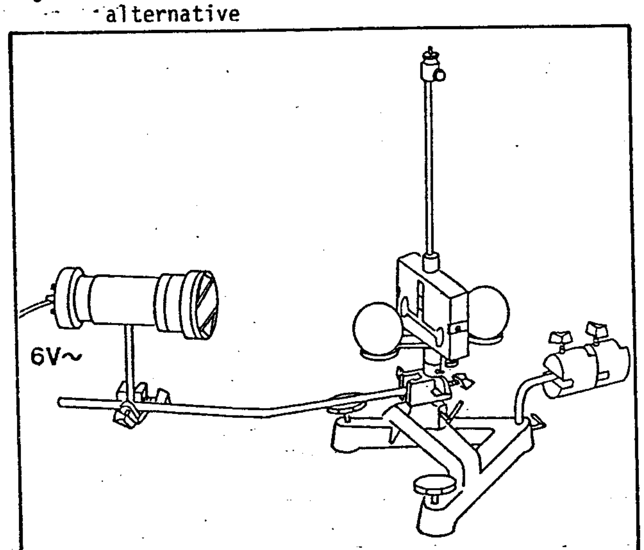
Set up the torsion balance in accordance with the instructions for use 332 10, at first without the large lead balls.

An alternative way of setting up the balance is to fit it to a stand base, and then place the latter on a flat, rough (concrete or stone) surface. Two saddle bases can be used here as counterweights for the illuminating equipment. The setup must remain completely still during measurement.

Setting up the light pointer:

Equip the condenser with the diaphragm holder and insert the 1 mm slit. Place the lamp in the correct position for the experiment (refer to Fig. 2 or Fig. 3), and focus the lamp filament on the torsion balance mirror by moving the lamp insert. The light filament must be set up vertically. Finally, focus the slit on the scale by moving the whole stand setup along the angled stand rod. Always make readings on the scale from the same shadow edge of the slit. To do this, focus the shadow edge so that it is possible to make readings to within 0.5 mm (also refer to the instructions for use 332 10 for how to set up the light pointer).

Fig. 3: Gravitation torsion balance: setup alternative



Adjust the zero point of the light pointer in accordance with the torsion balance instructions for use.

After adjusting the zero point, place the lead balls in position and move to one limit position (position I or II, Fig. 1). You must not touch the housing with your fingers or with the lead balls under any circumstances. Allow the apparatus to stand for at least two hours without touching it.

Before starting measurement, observe the stability of the zero point for at least 10 minutes and record this.

Carrying out the experiment:

Measurement must always be carried out by at least two people; one of these should exclusively follow the light pointer on the scale, and the other should displace the lead balls, call out the measurement times ("READY - NOW") and record the measurements made.

At the time t = 0, move the lead balls quickly from one limit position to the other, but ensuring that the housing is not touched by either your fingers or the lead balls. At the same time, start the stop-clock.

Read off the pointer position every 5 seconds and note this. After one complete oscillation period, increase the measurement interval to 10 seconds, and after two further periods to 30 seconds. The total measurement time must be at least 3 oscillation periods.

Then measure the distance L between the light pointer mirror and the scale using a tape measure.

Measurement example:

Technical data of the torsion balance need not be checked and are as follows:

m = 1.5 kg
d = 0.05 m (see Fig. 1)
b = 0.0465 m

Note:

b = 0.0465 m as the distance between the center points of the large and small balls is an approximate value. The distance is smaller in the initial and final equilibrium position if the small balls are exactly in the center of the metal housing of the torsion balance for the selected zero point.

Initial position x_0 of the pointer before the start of measurement:

x_0 = 60.4 cm
L = 437 cm

Fig. 4: Measurement example - oscillation of the gravitation balance about the final equilibrium position.

Evaluation and results:

From the graph, we can see that one oscillation period T = 245 seconds.

The equilibrium position x_∞ of the light pointer after oscillation has died out is determined from three successive amplitudes, e.g. $x_1 \ldots x_3$ in accordance with the algorithm (approximate value)

$$x_\infty \approx \frac{x_1}{4} + \frac{x_2}{2} + \frac{x_3}{4} = 58{,}1 \text{ cm}$$

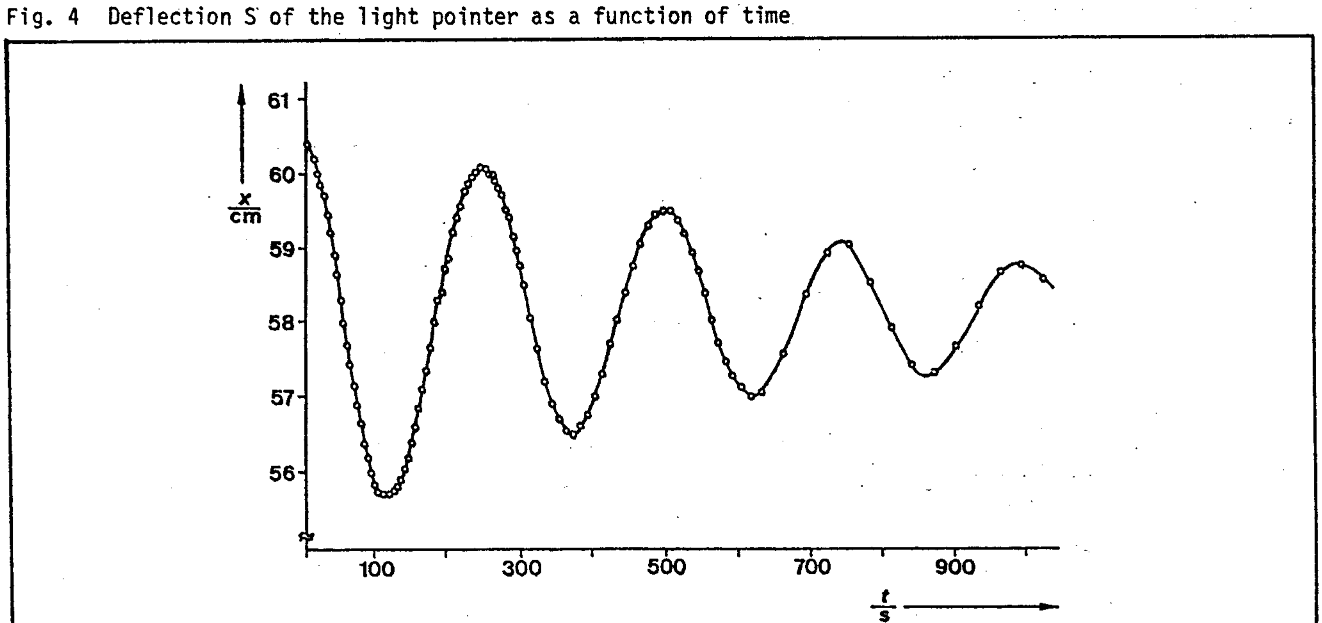
It is possible to check this by means of the amplitudes $x_2 \ldots x_4$ and $x_3 \ldots x_5$.

In accordance with this, the pointer deflection S is equal to
$S = x_0 - x_\infty = 2{,}3$ cm

By inserting this in equation (1) we obtain the gravitation constant:

$$G = \frac{\pi^2 \cdot (0{,}045)^2 \cdot 0{,}05 \cdot 0{,}023}{1{,}5 \cdot (245)^2 \cdot 4{,}37} = 6{,}237 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$$

Fig. 4 Deflection S of the light pointer as a function of time



The determined value is subject to the following systematic error: the small ball is also attracted by the more distant second large ball, albeit with a much lower force (refer to Fig. 5).

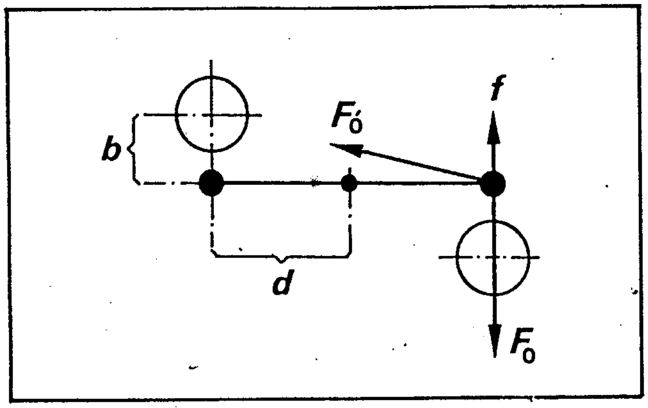


Fig. 5: Calculating the correction factor

In accordance with the gravitation law, this force is given by

$$F_0' = 2G \frac{m_1 m_2}{b^2 + 4d^2}$$

and has a component f opposed to the force F_0 to be measured

$$f = 2G \frac{m_1 m_2}{b^2 + 4d^2} \frac{b}{\sqrt{b^2 + 4d^2}} = \beta F_0,$$

if β is the fraction by which the observed force F_0 is reduced.

$$\beta = \frac{b^3}{(b^2 + 4d^2)\sqrt{b^2 + 4d^2}}.$$

β = 0.075 where d = 0.05 and b = 0.0465 m. The value for G obtained without this correction must therefore be increased by 7.5 %.

For our measurement example, we thus obtain

$G = 6{,}698 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$

instead of the theoretical value

$G = 6{,}705 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$.