CENCO 33210C Gravitational Torsion Balance

Determining the gravitation constant in accordance with the full deflection method

The gravitation constant G will be determined in accordance with the full deflection method using a torsion balance.

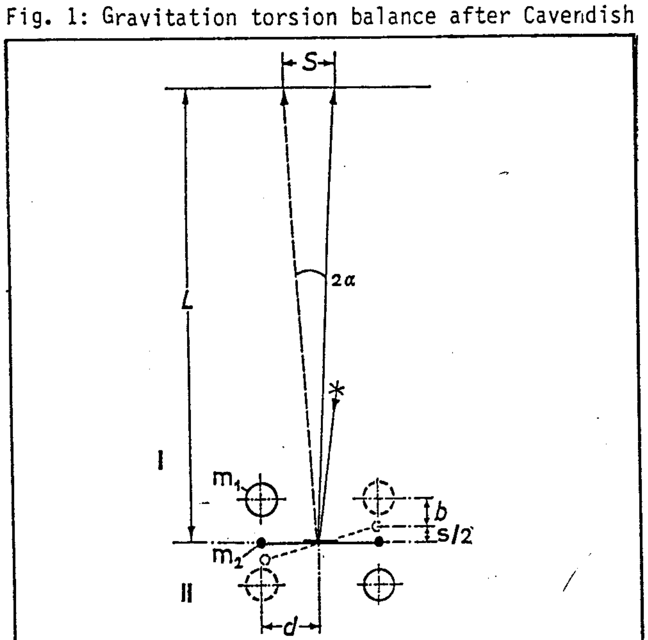
The gravitational force with which two lead balls with masses of m_1 = 1.5 kg and m_2 = 0.015 kg attract each other at a distance of 4.5 cm is less than 10^{-9} N. This force can only be verified with an extremely sensitive torsion balance.

The core of the torsion balance is a light transverse beam suspended horizontally from a thin torsion wire and with a lead ball of mass m_2 = 0.015 kg attached at each end. These balls are attracted by two large lead balls with a mass of m_1 = 1.5 kg. The resultant deflection of the transverse beam from its initial position is determined by means of a light pointer (cf Fig. 1/ Fig 2).

Both the initial and final equilibrium positions of the balance measuring system and the damped intermediate oscillation motion must be observed to determine the gravitation constant. This involves an observation time of approximately 45 minutes.

Carefully check the initial position of the balance, and then start the experiment by moving the large balls to the opposing position, that is to say from position I to position II. After a few oscillations, the measuring system moves from the initial position, designated as one limit position, to the new limit position. The angle between these two limit positions is given by

Fig. 1: Gravitation torsion balance after Cavendish



α. This can be calculated from the dimensions of the setup and the light pointer deflection in accordance with the details given in Fig. 1. The torque M acting on the measuring system in one limit position due to mass attraction is M = 2 Fd, where F is the force of attraction between each pair of balls and d is the axis distance of the small balls in the balance from the torsion band. The equilibrium of this torque is maintained by the twisting of the torsion band by the angle $\frac{\alpha}{2}$.

If the angular writing force of the torsion band is designated as D, this torque $M = D\frac{\alpha}{2}$.

Symbol designations (refer to Fig. 1):
- b: the distance between the center points of the large and small balls,
- s: the path of the small balls in the balance,
- d: distance of the small balls from the axis,
- S: the path of the light pointer on the wall,
- L: the distance of the balance mirror from the wall.

The following applies because of angle doubling due to reflection

$$\frac{s}{2\,d} = \tan\frac{\alpha}{2} \approx \frac{\alpha}{2} \quad ; \quad \frac{S}{2\,L} = \tan\alpha \approx \alpha$$

thus: $\frac{s}{d} = \frac{S}{2\,L} \approx \alpha$

The angular righting force D can only be determined from the oscillation period T of the torsional oscillation of the measuring system:

$$T^2 = 4\pi^2\frac{J}{D}$$

or

$$D = \frac{4\pi^2 J}{T^2}.$$

The moment of inertia J here can be equated with the moment of inertia of the two small balls, $J = 2m_2d^2$, because the suspension with the mirror contributes nothing to the moment of inertia in practice. Thus,

$$D = \frac{8\pi^2 m_2 d^2}{T^2}.$$

If the value $M = 2Fd = 2G\frac{m_1m_2d}{b^2}$ and the values obtained above for D and a are inserted in the equation $M = D\frac{\alpha}{2}$ it follows where $2M = D\alpha$ that

$$4G\frac{m_1m_2}{b^2} = \frac{8\pi^2 dm_2}{T^2}\cdot\frac{S}{2L}.$$