The e/m ratio

Objective To measure the electronic charge-to-mass ratio e/m, by injecting electrons into a magnetic field and examining their trajectories. We also estimate the magnitude of the earth's magnetic field.

Introduction This experiment demonstrates how the electronic charge-to-mass ratio can be measured from fairly simple measurements of electron trajectories in a magnetic field. Before we get into the details, consider the following simplified scenario. Let us accelerate an electron using a uniform electrostatic field E maintained over a distance d. The potential difference over this distance with be Ed and the electron will pick up a kinetic energy (KE) of eV. Consequently

$$KE = \frac{1}{2}mv^2 = eV. \tag{1}$$

Now let us inject this electron into a uniform magnetic field B such that the direction of motion is perpendicular to the field lines. We can equate the forces on the electron,

$$\vec{f} = e\vec{v} \times \vec{B} = evB = \frac{mv^2}{r} \tag{2}$$

and combining this with Eqn.(1) we have a compact expressionfor the charge to mass ratio

$$e/m = \frac{2V}{B^2 r^2}. \tag{3}$$

So if we know the magnitude of the B field, the potential difference V and the radius of the path r, we can calculate the charge to mass ratio of the electron. This simple equation is quite informative if we remember that the left hand side is a constant. For example, if we inject electrons into the

magnetic field faster, by increasing the potential V, and we and desire to maintain an orbit with the same radius, then this equation informs us we would have to increase the magnitude of the magnetic field. Alternatively, if we want to increase the radius of the circular orbit we would have to decrease the magnitude of the magnetic field.

The experiment involves injecting electrons into a magnetic field that is generated by Helmholtz coils. The only difference between the experiment that I described above and the one you will perform is that the magnetic field will not be spatially uniform. It is very difficult to generate a spatially uniform magnetic field so we will have to make allowances for the non uniformity in our calculations.

The Apparatus The electron trajectories will be measured in a *vacuum tube* which contains some hydrogen gas. Some of the electrons excite the hydrogen gas molecules and make the electron trajectory visible. The ionized gas emits a faint blue light which can easily be seen with the lights off. The pressure of the hydrogen gas in the tube ($\approx 10^{-2}$mm Hg) is selected so that the space charge produced by the electrons bunches the beam for this range of velocities.

The electrons are generated by heating a filament. This gives some of the electrons in the filament enough energy to overcome the surface barrier and escape from the metal. These electrons are then accelerated by a potential applied between the filament (the cathode) and the metal cover surrounding the filament (the anode). After they have escaped through the nose code of the anode they can be deflected and focused by a grid which is oriented parallel to the beam. We refer to the current in the filament as the *filament current*, the voltage between the filament and the metal cover as the *anode voltage*, and the voltage between the deflector grids as the *grid bias*.

After the electrostatic field, the electrons encounter a magnetic field. The magnetic field is generated by a pair of coils which are placed above and below the tube in the Helmholtz configuration, producing a fairly homogeneous magnetic field. The ideal Helmholtz configuration has the circular coils separated by the radius R. Our coils are not exactly in this configuration, and we will correct for this later. It can be shown that the magnetic field is
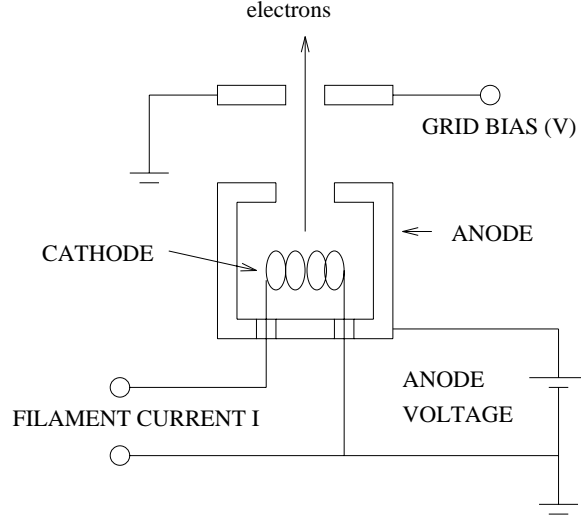
Figure 1: Schematic of the cathode and the electron optics used to accelerate the electrons.

maximum midway between the coils on axis. For an ideal Helmholtz pair, the magnetic field at the centre of the coils is

$$B_o = \frac{8\mu_o n I}{5\sqrt{5}R},$$ (4)

where n is the number of turns. We may simplify this to

$$B_o = KI$$ (5)

where K is a constant for a given pair of coils and B_o is measured in units of Tesla. The Helmholtz pair that you will use have a radius $R = 15.4 \pm 0.5 cm$ and a mean separation of $15.0 \pm 0.5 cm$. They have 130 turns and generate a magnetic induction (KI) defined by $K = (7.73 \pm 0.04) \times 10^{-4} T/A$. Because the radius is not equal to the separation the coils are not an ideal Helmholtz

pair. Therefore the value of K which we have given is slightly different from the ideal value. However, the relationship between B_o and I is still linear.

The magnitude of the field midway between the coils is of limited usefulness since the electron will be traversing an orbit in a plane parallel to the plane of the coils a distance r from the centre of the coil. Luckily the field is radially symmetric and we can allow for the fact that the field will be smaller as we move out from the centre of the coil. To estimate the field at a distance r from the centre of the coil use the following data which was supplied with the coils. The normalized distance from the centre of the coil r/R is listed with the normalized field B/B_o, where B_o is the field at the centre.

r/R	B/B$_o$
0.0	1.0000
0.1	0.99996
0.2	0.99928
0.3	0.99621
0.4	0.98728
0.5	0.96663
0.6	0.92525
0.7	0.85121
0.8	0.73324
0.9	0.56991
1.0	0.38007

Power supplies There is a power supply manufactured by Leybold. The switch S2 is ON/OFF. The switch S1 is to remain in the up position. It is a range selector and we will be using the upper 0-25 V and 0-300 V ranges. Beware, to increase the anode voltage, you rotate the knob clockwise. To increase the grid bias, you rotate the knob counter clockwise. Don't as me why ! Before switching on, turn both to zero. After a few minutes, the filament will warm up and emit a yellow glow. Once it is hot, increase the anode voltage until the beam becomes visible. Change grid bias slightly to obtain best beam definition.
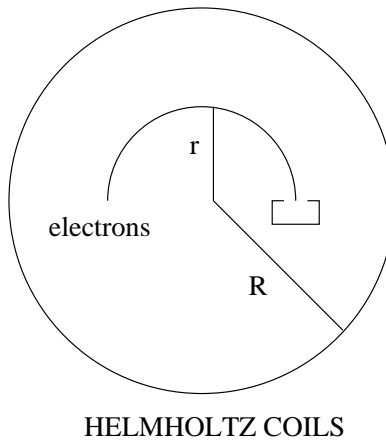
HELMHOLTZ COILS

Figure 2: Plan view of the electron trajectory, which is approximately a circle with radius r, and the Helmholtz coils, which have radius R.
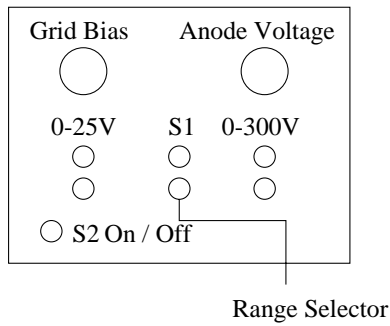

Figure 3: Schematic of the Leybold power supply.

The d.c. current for the Helmholtz field coils is adjusted by using the large rheostat located on the bench. By increasing the current in the coils you should be able to see the electron beam bend.

The anode voltage controls both the beam intensity and the diameter of the electron orbit. For optimum sharpness the anode voltage should be between 150 and 250 V. The diameter of the beam circle is measured using the illuminated scale. Adjust the position of the scale so that images formed by the two surfaces of the plate glass appear parallel to and on each side of the plane of the electron beam, and along a diameter of the circular path of the beam.

Procedure

- 1. Set the anode voltage (V) to 150 V and increase the current in the Helmholtz coil until the diameter of the electron orbit (D) is 8 cm. Turn the tube gently in its wooden clamps about its long axis so that the beam forms a closed circle rather than a spiral. From the magnitude of the current in the Helmholtz coil calculate the field required to bend the electron into a circular orbit of diameter $D = 8cm$ with an anode voltage of 150 V.

- 2. Increase the anode voltage to 250 V in 20 V increments, measuring the current required to maintain the electrons in a circular orbit with $D = 8cm$.

- 3. Repeat 1 and 2 for $D = 10cm$.

- 4. Turn the tube through 180 degrees and reverse the direction of the current in the coils using the reversing switch. This bends the beam into a circle with the direction of electron motion reversed.

- 5. Repeat steps 1-3 with the polarity of the current reversed.

- 6. When you flip the orientation of the tube you must still apply the same total B field (B_{total}) in both cases, if the radius of the orbit is the same. However, you will find that the currents you apply to maintain the electrons in orbits of the same radius are different. The larger we

will call I_{big} and the smaller we will call I_{small}. Obviously, in the first case the earth's magnetic field B_{earth} is opposing the field generated by the coil and in the second case the earth's field is adding to the field generated by the coil. Considering both cases in turn, we have

$$B_{total} = B_{big} - B_{earth} =$$
$$KI_{big} - B_{earth} \tag{6}$$

and

$$B_{total} = B_{small} + B_{earth} =$$
$$KI_{small} + B_{earth} \tag{7}$$

These can be added to give

$$B_{total} = \frac{K}{2}(I_{small} + I_{big}), \tag{8}$$

and subtracted to give

$$B_{earth} = \frac{K}{2}(I_{big} - I_{small}). \tag{9}$$

Use B_{total} to calculate e/m for each measurement pair. Average these to produce an estimate of e/m.

- 7. Calculate the component of the earth's magnetic field normal to the plain of the Helmholtz pair by averaging all estimates of B_{earth}. Subtract 1% from your anode voltage measurements to correct for the voltage drop in a protection resistor included in the leads. Errors in the digital meters should be taken as ± 1 in the last digit.

After the experiment Do not disconnect the leads, but ensure that the power supply and the coil current are switched off. The reversing switch should be in the central position.

Analysis What is the discrepancy between your experimental determination of e/m and the accepted value? Does this lie within your estimated uncertainty?

$$\frac{e}{m} = 1.7588 \times 10^{11} C/kg. \tag{10}$$

Estimate the magnitude of the earth's magnetic field (actually the component normal to the coil). For the formal report discuss whether you would expect the orbit with $D = 8cm$ or the orbit with $D = 10cm$ to give you a more accurate estimate of the e/m ratio.

Draw a picture of the magnetic flux lines generated by the earth.