IO.3. Measurement of the dispersion of glass with a prism spectrometer

1. Purpose: Measure Dispersion Relation of a Glass Prism in the Visible Spectrum.

2. Apparatus: Gaertner Scientific Spectrometer,
Hg and He lamps.

3. Introduction:

The deviation produced by a prism depends on the angle of incidence, i, the refracting angle of the prism, α, and the index of refraction, n. For a given setting of a prism in a spectrometer, a dispersed spectrum is observed, showing a variation of the angle of deviation with wavelength. This is due to a dependence of the refractive index n on the wavelength λ. We may express the dependence by putting:

$$d\vartheta/d\lambda = (d\vartheta/dn) \cdot (dn/d\lambda) \qquad (1)$$

The first factor, $d\vartheta/dn$, depends on the conditions of the experiment; but the second, $dn/d\lambda$ is a function only of the material of the prism, and is called *dispersion* of the material. This is the quantity that we wish to study.

The index of refraction for any wavelength can be determined from the relationship:

$$n = \frac{\sin[(\alpha + \delta)/2]}{\sin(\alpha/2)} \qquad (2)$$

where α is the refracting angle of the prism and δ is the angle of minimum deviation for the particular wavelength used. The value of n can thus be obtained experimentally for any λ at which we have a spectral line available, and an experimental curve of n versus λ can be made. Although it is in general difficult or impossible to derive theoretically a satisfactory formulation of the dependence of n on λ, the experimentally determined curve can be fitted with a fair degree of accuracy (except in the neighborhood of strong absorption bands of the material) by an empirical equation due to Cauchy. This equation is:

$$n = A + B/\lambda^2 \qquad (3)$$

where A and B are constants for a given substance. The dispersion of the material is obtained from equation (3) by differentiating with respect to the wavelength:

$$dn/d\lambda = -2B/\lambda^3 \qquad (4)$$

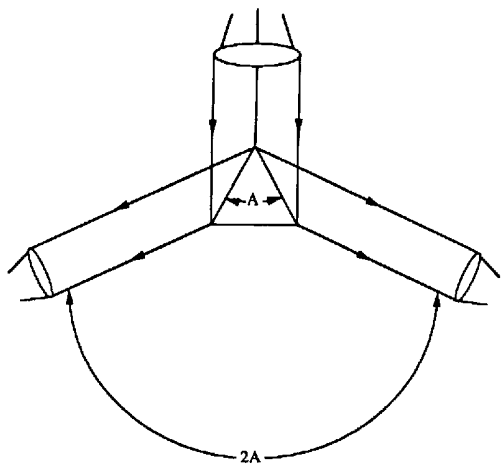
The problem of the experiment is therefore essentially that of determining the constants in Cauchy's equation.

3. Procedure:

3.1 Apparatus:

The spectrometer will probably not be in adjustment, so this must be done first. The adjustment for parallel light should be made by the Gauss eyepiece method.

In order to measure the refracting angle of the prism, place it on the spectrometer table so that the angle α is toward the collimator and so that light from the collimator strikes both refracting faces. An image of the slit reflected in each face can be seen in the telescope (by moving the telescope but not the prism). Measure the angle between the two images of the slit. Show in the write-up that it is equal to 2α. (note that the refracting angle α of the prism is called A in the figure below).



The positions of minimum deviation for each line used should be determined with the prism set to refract in the light first to one side of the straight through beam and then to the other. The difference between these two positions will give 2δ. The position of minimum deviation for the spectral line is found by rotating the prism until the position is found where the line moves away from the direction of the incident light regardless of which way the prism is turned. The position of the prism for minimum deviation is of course different for each different wavelength and therefore must be redetermined for each line. The following table gives the spectral lines from the helium and mercury sources which are to be used in this experiment:

3.2 Spectrum lines to be used:

	Source	λ (nm)	$1/\lambda^2$ (nm^{-2})	$1/\lambda^4$ (nm^{-4})	Description
1)	He	706.5188	2.003303×10^{-6}.......	$4.0133322 \times 10^{-12}$	Farthest red
2)	Hg	623.437	2.5728523.............	6.6195689	Weak but strongest in its region
3)	He	587.5618	2.8966291.............	8.3904602	Bright yellow
4)	Hg	546.0753	3.3534739.............	11.2457874	Bright green
5)	He	501.6675	3.9750374.............	15.8009226	Bright green-middle one of 3 fairly close lines
6)	He	471.3143	4.5017230.............	20.2655096	Blue
7)	He	447.1477	5.0014738.............	25.0147400	Bright blue-violet
8)	Hg	435.8338	5.2645128.............	27.7150948	Bright blue-violet
9)	Hg	404.6571	6.1069683.............	37.2950621	Bright violet -- brighter and shorter of two fairly close lines
10)	He	388.8646	6.6130709............	43.7327069	Farthest violet

3.3 Calculations:

1) Tabulate λ, δ, and n calculated from equation (2) for all lines.

2) Plot a graph with index of refraction as ordinate and wavelength as abscissa.

3) Assuming that Cauchy's equation gives the correct form of the relationship between n and λ, calculate the least square straight line for n vs $1/\lambda^2$, that is, an equation of the form $n = A + B(1/\lambda^2)$. Plot this line on a graph with n as ordinate and $1/\lambda^2$ as abscissa. Show the experimental points on the same graph. Write on the graph your values of A and B.

4) Using equation (3) and your values of A and B calculate n for λ = 200, 500, 800 nm, giving the standard deviation for each.

5) Using equation (4) and your value of B, calculate $dn/d\lambda$ for λ = 200, 500, 800 nm, giving the standard deviation for each

3.4 Notes on the application of the least square method:

a) The abscissae $1/\lambda^2$, may be taken as exact: therefore the transformation of coordinates from λ to $1/\lambda^2$ in order to make the curve a straight line does not change the weighting.

b) Although the individual measurements of angles for different wavelengths may reasonably be assumed to have the same weights, the *n*'s calculated from them will not. In fact, you can verify that if it is assumed that $\sigma(\alpha) = \sigma(\delta) = \sigma$ then

$$\sigma^2 (n) = \frac{1}{4 \sin^2 (\alpha/2)} \left[\frac{\sin^2 (\delta/2)}{\sin^2 (\alpha/2)} + \cos^2((\alpha+\delta)/2) \right] \sigma^2.$$

However, for $\alpha = 60°$ and $\delta = 50°$, $\sigma^2(n) = 1.047\ \sigma^2$, and for $\alpha = 60°$, $\delta = 55°$, $\sigma^2(n) = 1.143\ \sigma^2$. Thus in the range in which the measurements are made, $\sigma^2(n)$ and hence the weighting of n varies by only about 9 percent. We are therefore justified in making the simplifying assumption that the *n*'s are equally weighted if the δ's are, and the δ's will be if the number of measurements of each is the same.

c) The appearance of the "fit" of the least squares curve to the experimental points should be observed critically both as a check on the calculations leading to the line and to notice if the residual show any trends which might suggest an improvement of the original assumptions.

(In fact, Cauchy's equation is known to give a better fit if it is given as

$$n = A + B/\lambda^2 + C/\lambda^4 + ...,$$

and this may very likely show up in your results.)