percent, since the ratio 5/540 reduces to 0.0093 (rounded off) in decimal form.

## Significant Digits

The accuracy of a measurement is often described in terms of the number of significant digits used in expressing it. If the digits of a number resulting from measurement are examined one by one, beginning with the left-hand digit, the first digit that is not 0 is the first significant digit. For example, 2345 has four significant digits and 0.023 has only two significant digits.

The digits 2 and 3 in a measurement such as 0.023 inch signify how many thousandths of an inch comprise the measurement. The 0 s are of no significance in specifying the number of thousandths in the measurement; their presence is required only as "place holders" in placing the decimal point.

A rule that is often used states that the significant digits in a number begin with the first nonzero digit (counting from left to right) and end with the last digit. This implies that 0 can be a significant digit if it is not the first digit in the number. For example, 0.205 inch is a measurement having three significant digits. The 0 between the 2 and the 5 is significant because it is a part of the number specifying how many thousandths are in the measurement.

The rule stated in the foregoing paragraph fails to classify final 0 s on the right. For example, in a number such as 4,700, the number of significant digits might be two, three, or



four. If the 0 s merely locate the decimal point (that is, if they show the number to be approximately forty-seven hundred rather than forty seven), then the number of significant digits is two. However, if the number 4,700 represents a number such as 4,703 rounded off to the nearest hundred, there are three significant digits. The last 0 merely locates the decimal point. If the number 4,700 represents a number such as 4,700.4 rounded off, then the number of significant digits is four.

Unless we know how a particular number was measured, it is sometimes impossible to determine whether right-hand 0 s are the result of rounding off. However, in a practical situation it is normally possible to obtain information concerning the instruments used and the degree of precision of the original data before any rounding was done.

In a number such as 49.30 inches, it is reasonable to assume that the 0 in the hundredths place would not have been recorded at all if it were not significant. In other words, the instrument used for the measurement can be read to the nearest hundredth of an inch. The 0 on the right is thus significant. This conclusion can be reached another way by observing that the 0 in 49.30 is not needed as a place holder in placing the decimal point. Therefore its presence must have some other significance.

The facts concerning significant digits may be summarized as follows:

1. Digits other than 0 are always significant.
2. Zero is significant when it falls between significant digits.

3. Any final 0 to the right of the decimal point is significant.

4. When a 0 is present only as a place holder for locating the decimal point, it is not significant.

5. The following categories comprise the significant digits of any measurement number:

a. The first nonzero left-hand digit is significant.

b. The digit which indicates the precision of the number is significant. This is the digit farthest to the right, except when the right-hand digit is 0. If it is 0, it may be only a place holder when the number is an integer.

c. All digits between significant digits are significant.

Practice problems. Determine the percent of error and the number of significant digits in each of the following measurements:

1. 5.4 feet
2. 0.00042 inch
3. 4.17 sec
4. 147.50 miles

Answers:

1. Percent of error: 0.93%
   Significant digits: 2
2. Percent of error: 1.19%
   Significant digits: 2
3. Percent of error: 0.12%
   Significant digits: 3
4. Percent of error: 0.0034%
   Significant digits: 5



## CALCULATING WITH APPROXIMATE NUMBERS

The concepts of precision and accuracy form the basis for the rules which govern calculation with approximate numbers (numbers resulting from measurement).

### Addition and Subtraction

A sum or difference can never be more precise than the least precise number in the calculation. Therefore, before adding or subtracting approximate numbers, they should be rounded to the same degree of precision. The more precise numbers are all rounded to the precision of the least precise number in the group to be combined. For example, the numbers 2.95, 32.7, and 1.414 would be rounded to tenths before adding as follows:

$$\begin{array}{r} 3.0 \\ 32.7 \\ 1.4 \\ \hline \end{array}$$

### Multiplication and Division

When two numbers are multiplied, the result often has several more digits than either of the original factors. Division also frequently produces more digits in the quotient than the original data possessed, if the division is "carried out" to several decimal places. Results such as these appear to have more significant digits than the original measurements from which they came, giving the false impression of greater accuracy than is justified. In order to correct this situation, the following rule is used:

In order to multiply or divide two approximate numbers having an equal number of significant digits, round the answer to the same number of significant digits as are shown in the original data. If one of the original factors has more significant digits than the other, round the more accurate number before multiplying. It should be rounded to one more significant digit than appears in the less accurate number; the extra digit protects the answer from the effects of multiple rounding. After performing the multiplication or division, round the result to the same number of significant digits as are shown in the less accurate of the original factors.

Practice problems:

1. Find the sum of the sides of a triangle in which the lengths of the three sides are as follows: 2.5 inches, 3.72 inches, and 4.996 inches.
2. Find the product of the length and width of a rectangle which is 2.95 feet long and 0.9046 foot wide.

Answers:

1. 11.2 inches
2. 2.67 square feet

## MICROMETERS AND VERNIERS

Closely associated with the study of decimals is a measuring instrument known as a micrometer. The ordinary micrometer is capable of measuring accurately to one-thousandth of an inch. One-thousandth of an inch is about the thickness of a human hair or a thin sheet of paper. The parts of a micrometer are shown in figure 6-1.



## MICROMETER SCALES

The spindle and the thimble move together. The end of the spindle (hidden from view in figure 6-1) is a screw with 40 threads per inch. Consequently, one complete turn of the thimble moves the spindle one-fortieth of an inch or

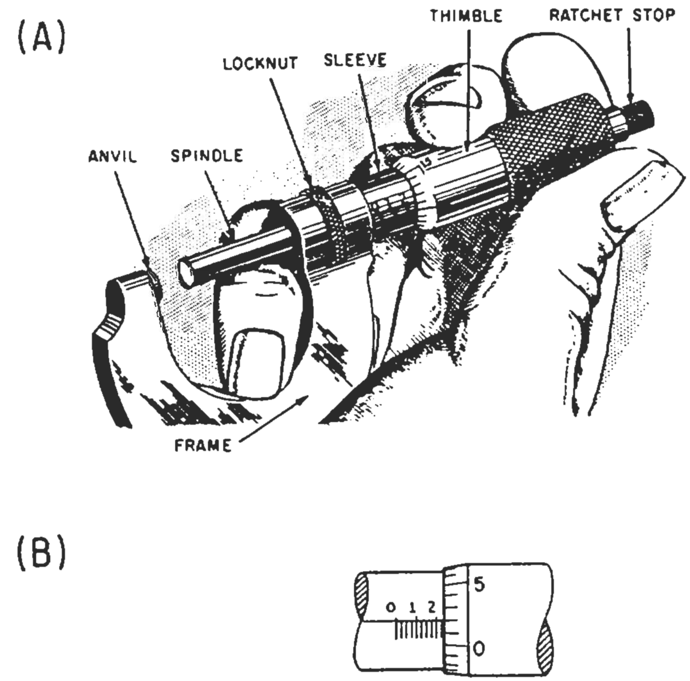


Figure 6-1.—(A) Parts of a micrometer; (B) micrometer scales.

0.025 inch since $\frac{1}{40}$ is equal to 0.025. The sleeve has 40 markings to the inch. Thus each space between the markings on the sleeve is also 0.025 inch. Since 4 such spaces are 0.1 inch (that is, 4 x 0.025), every fourth mark is labeled in tenths of an inch for convenience in reading. Thus, 4 marks equal 0.1 inch, 8 marks equal 0.2 inch, 12 marks equal 0.3 inch, etc.

To enable measurement of a partial turn, the beveled edge of the thimble is divided into 25 equal parts. Thus each marking on the thimble is $\frac{1}{25}$ of a complete turn, or $\frac{1}{25}$ of $\frac{1}{40}$ of an inch. Multiplying $\frac{1}{25}$ times 0.025 inch, we find that each marking on the thimble represents 0.001 inch.

## READING THE MICROMETER

It is sometimes convenient when learning to read a micrometer to write down the component parts of the measurement as read on the scales and then to add them. For example, in figure 6-1 (B) there are two major divisions visible (0.2 inch). One minor division is showing clearly (0.025 inch). The marking on the thimble nearest the horizontal or index line of the sleeve is the second marking (0.002 inch). Adding these parts, we have

$$\begin{array}{r} 0.200 \\ 0.025 \\ 0.002 \\ \hline 0.227 \end{array}$$



Thus, the reading is 0.227 inch. As explained previously, this is read verbally as "two hundred twenty-seven thousandths." A more skillful method of reading the scales is to read all digits as thousandths directly and to do any adding mentally. Thus, we read the major division on the scale as "two hundred thousandths" and the minor division is added on mentally. The mental process for the above setting then would be "two hundred twenty-five; two hundred twenty-seven thousandths."

Practice problems:

1. Read each of the micrometer settings shown in figure 6-2.

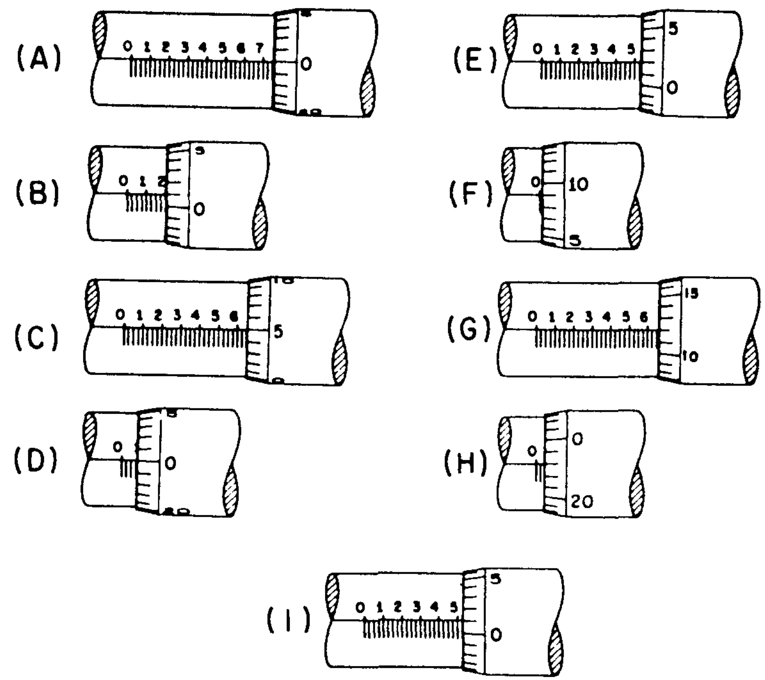


Figure 6-2.—Micrometer settings.

Answers:

1. (A) 0.750 (F) 0.009
   (B) 0.201 (G) 0.662
   (C) 0.655 (H) 0.048
   (D) 0.075 (I) 0.526
   (E) 0.527

## VERNIER

Sometimes the marking on the thimble of the micrometer does not fall directly on the index line of the sleeve. To make possible readings even smaller than thousandths, an ingenious device is introduced in the form of an additional scale. This scale, called a VERNIER, was named after its inventor, Pierre Vernier. The vernier makes possible accurate readings to the ten-thousandth of an inch.

### Principles of the Vernier

Suppose a ruler has markings every tenth of an inch but it is desired to read accurately to hundredths. A separate, freely sliding vernier scale (fig. 6-3) is added to the ruler. It has 10 markings on it that take up the same distance as 9 markings on the ruler scale. Thus, each space on the vernier is $\frac{1}{10}$ of $\frac{9}{10}$ inch, or $\frac{9}{100}$ inch. How much smaller is a space on the vernier than a space on the ruler? The ruler space is $\frac{1}{10}$ inch, or $\frac{10}{100}$ inch, and the vernier



space is $\frac{9}{100}$ inch. The vernier space is smaller by the difference between these two numbers, as follows:

$$\frac{10}{100} - \frac{9}{100} = \frac{1}{100}$$

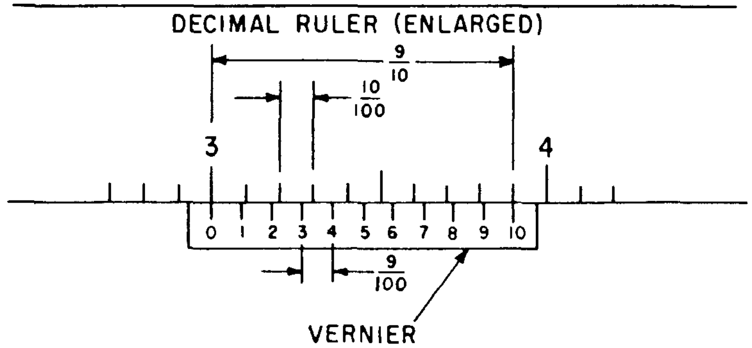


Figure 6-3.—Vernier scale.

Each vernier space is $\frac{1}{100}$ inch smaller than a ruler space.

As an example of the use of the vernier scale, suppose that we are measuring the steel bar shown in figure 6-4. The end of the bar almost reaches the 3-inch mark on the ruler, and we estimate that it is about halfway between 2.9 inches and 3.0 inches.

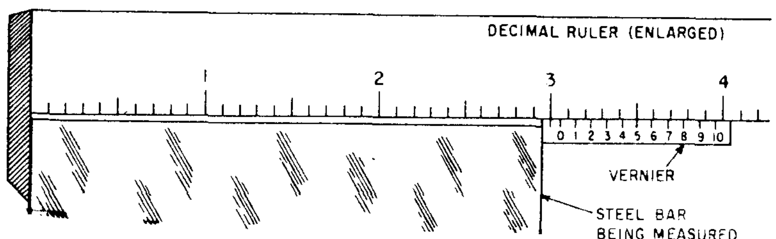


Figure 6-4.—Measuring with a vernier.

The 0 on the vernier scale is spaced the distance of exactly one ruler mark (in this case, one tenth of an inch) from the left hand end of the vernier. Therefore the 0 is at a position between ruler marks which is comparable to the position of the end of the bar. In other words, the 0 on the vernier is about halfway between two adjacent marks on the ruler, just as the end of the bar is about halfway between two adjacent marks. The 1 on the vernier scale is a little closer to alinement with an adjacent ruler mark; in fact, it is one hundredth of an inch closer to alinement than the 0. This is because each space on the vernier is one hundredth of an inch shorter than each space on the ruler.

Each successive mark on the vernier scale is one hundredth of an inch closer to alinement than the preceding mark, until finally alinement is achieved at the 5 mark. This means that the 0 on the vernier must be five hundredths of an inch from the nearest ruler mark, since five increments, each one hundredth of an inch in size, were used before a mark was found in alinement.

We conclude that the end of the bar is five hundredths of an inch from the 2.9 mark on the ruler, since its position between marks is exactly comparable to that of the 0 on the vernier scale. Thus the value of our measurement is 2.95 inches.

The foregoing example could be followed through for any distance between markings. Suppose the 0 mark fell seven tenths of the distance between ruler markings. It would take



seven vernier markings, a loss of one-hundredth of an inch each time, to bring the marks in line at 7 on the vernier.

The vernier principle may be used to get fine linear readings, angular readings, etc. The principle is always the same. The vernier has one more marking than the number of markings on an equal space of the conventional scale of the measuring instrument. For example, the vernier caliper (fig. 6-5) has 25 markings on the vernier for 24 on the caliper scale. The caliper is marked off to read to fortieths (0.025) of an inch, and the vernier extends the accuracy to a thousandth of an inch.

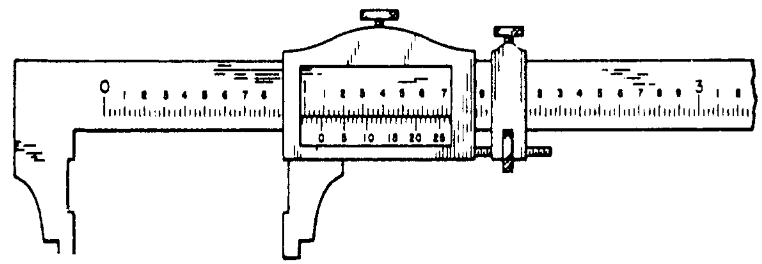

Figure 6-5.—A vernier caliper.

## Vernier Micrometer

By adding a vernier to the micrometer, it is possible to read accurately to one ten-thousandth of an inch. The vernier markings are on the sleeve of the micrometer and are parallel to the thimble markings. There are 10 divisions on the vernier that occupy the same space as 9 divisions on the thimble. Since a thimble space is one thousandth of an inch, a vernier space is

$\frac{1}{10}$ of $\frac{9}{1000}$ inch, or $\frac{9}{10000}$ inch. It is $\frac{1}{10000}$ inch less than a thimble space. Thus, as in the preceding explanation of verniers, it is possible to read the nearest ten-thousandth of an inch by reading the vernier digit whose marking coincides with a thimble marking.

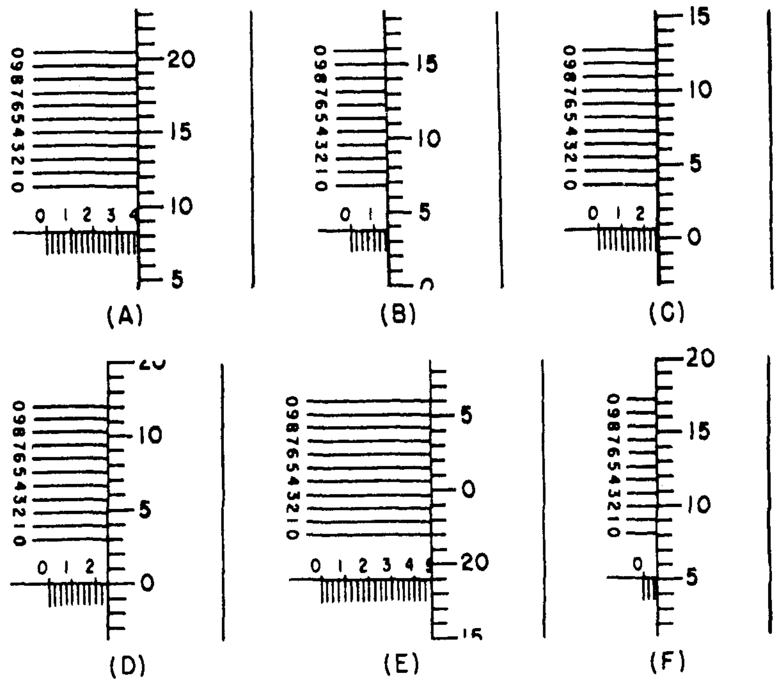


Figure 6-6.—Vernier micrometer settings.

In figure 6-6 (A), the last major division showing fully on the sleeve index is 3. The third minor division is the last mark clearly showing (0.075). The thimble division nearest and below the index is the 8 (0.008). The vernier marking that matches a thimble marking is the fourth (0.0004). Adding them all together, we have,



$$
\begin{array}{r}
0.3000 \\
0.0750 \\
0.0080 \\
\underline{0.0004} \\
0.3834
\end{array}
$$

The reading is 0.3834 inch. With practice these readings can be made directly from the micrometer, without writing the partial readings.

Practice problems:

1. Read the micrometer settings in figure 6-6.

Answers:

1. (A) See the foregoing example.

   (B) 0.1539

   (C) 0.2507

   (D) 0.2500

   (E) 0.4690

   (F) 0.0552