LAB 10b PION-PROTON COLLISION

EXPERIMENTAL GOALS

Under the right conditions, a charged particle moving through liquid hydrogen can leave a trail of tiny bubbles. During the 1960s and 1970s, physicists used photographs of such bubble tracks to study collisions of subatomic particles. (Physicists now use particle detectors that are more easily interfaced directly to computers.) In this lab, you will study the bubble-chamber photograph of a collision between a subatomic particle called a pion and a hydrogen nucleus (proton). Your goal is to check whether this particular collision more closely satisfies the newtonian model or the relativistic model of momentum and energy conservation (the latter will be described below).

LABORATORY SKILLS you will be developing

The main educational purpose of this lab is to introduce the relativistic model of momentum and energy conservation and its application to particle physics. This will give you some concrete experience with the ideas before you encounter the more theoretical presentation near the end of unit *R*. This lab will also give you some practice with handling propagation of uncertainties.

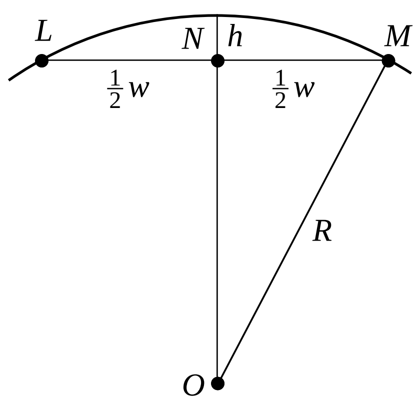
SOME PROCEDURAL SUGGESTIONS AND NOTES

You will each be given bubble-chamber photograph labeled “A-5”. The bubble chamber in this case was a vat filled with liquid hydrogen placed at the business end of a particle accelerator producing a beam of high-energy subatomic particles called *pions*. These pions create the essentially horizontal tracks seen in the photograph. In the center of the photograph you see the consequences of an elastic collision between one of these pions and a hydrogen nucleus (which is simply a proton): the two angled tracks show the paths of the pion and the proton after the collision.

If you look at the tracks, you can see that they are slightly curved. This is because the bubble chamber was placed in a magnetic field. Experimentally, a particle with charge q and mass m moving with speed v perpendicular to a uniform magnetic field $\vec{B}$ follows a circular path of radius

$$R = \frac{mv}{|q|B\sqrt{1-v^2/c^2}} = \frac{mvc}{|q|Bc\sqrt{1-v^2/c^2}} \qquad (1)$$

where c is the speed of light. So knowing R, we can in principle determine the particle’s speed. The magnetic field also bends positive charges and negative charges in opposite directions: in the photograph shown, the magnetic field is oriented so that it bends positive charges into clockwise circles and negative charges into counterclockwise circles. So one can also determine the sign of a particle’s charge by observing which way its track bends.



Each track in the photograph is so gently curved that it would seem to be very difficult to measure its radius of curvature R. However, consider the diagram to the left. Say that we only have a portion of a circular track between points L and M and we would like to know the distance R to the circle’s center O. We can easily measure the chord width w along the straight line between L and M, and the height h of the curve above the chord’s midpoint N. Applying the pythagorean theorem to the right triangle ΔONM in the diagram, we find that

$$R^2 = (\tfrac{1}{2}w)^2 + (R-h)^2 = \tfrac{1}{4}w^2 + R^2 - 2Rh + h^2 \qquad (2)$$

Cancelling R^2 from both sides of this equation yields:

$$0 = \tfrac{1}{4}w^2 - 2Rh + h^2 \;\Rightarrow\; R = \frac{w^2}{8h} + \frac{h}{2} = \frac{w^2}{8h}\left(1 + \frac{4h^2}{w^2}\right) \approx \frac{w^2}{8h} \qquad (3)$$

where the last approximation applies whenever $2h << w$, which will always be strongly true for the tracks we consider. Thus we can easily calculate R by measuring w and h.

Now, particle physicists typically describe a subatomic particle's momentum not in units of kg·m/s (which would yield an awkwardly small number) but rather stating the value of *pc*, where *c* is the speed of light. Note that *pc* has the units of (mass·velocity)·velocity = mass·velocity^2, which are the same as the units of *energy*. Similarly, they describe a particle's mass by stating the value of mc^2, which also has units of energy. To make these numbers even more managable, they also express all quantities with units of energy in **electron volts**, where $1\text{ eV} \equiv 1.602\times10^{-19}$ J. (We the logic behind the name in unit *E*: for right now, just think of it as conveniently small unit of energy.) The value of $m_\pi c^2$ for a pion is 139.6 MeV, while $m_p c^2$ for a proton is 938.3 MeV.

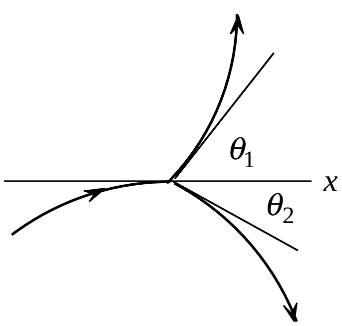
According to the newtonian model, this collision should conserve the system's total vector momentum and its total kinetic energy. Solving equation 1 for *pc* = *mvc*, one gets (after some work)

$$mvc = \frac{|q|BcR}{\sqrt{1+(|q|BcR/mc^2)^2}} \tag{4}$$

In our situation the incoming pions have a negative charge of –*e*, where *e* is the charge of a proton, so $|q| = +e$ for all particles in this experiment. The value of the magnetic field strength *B* in this experiment is such that $|q|BcR$ for the incoming pions is 921 MeV: you can use this to calculate $|q|Bc$. Then by plugging the particle's mass and the radius *R* of its path into the expression above, one can find the *magnitude* of its momentum. One can then determine its kinetic energy *K* using

$$K = \tfrac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{(mvc)^2}{2mc^2} \tag{5}$$

Note that to use equations 4 and 5, you have to determine which outgoing particle is the proton and which is the pion by observing which way the outgoing particle curves.



Of course, equation 4 only gives you the *magnitude* of the particle's momentum: it is its momentum *components* that are conserved. Now, if the components of $\vec{p} = m\vec{v}$ are conserved, then the components of $m\vec{v}c$ will also conserved. For a given particle, you can calculate $mv_x c = (mvc)\cos\theta$ and $mv_y c = \pm(mvc)\sin\theta$ if you measure the angle θ of the particle's path *just after the collision* makes with some *x* axis (which we might conveniently choose to be the original pion's direction of motion *just before* the collision). We can measure this angle by drawing a line tangent to the particle's path just as it emerges from the collision, as shown to the right (the path curvatures are exaggerated), and measuring its angle relative to the *x* axis using a protractor. (Note that these tangent lines will *not* be at the same angle as the chord lines you draw to compute *R*.)

In summary, to check conservation of newtonian momentum and energy,

1. Identify which outgoing particle is which by the way it curves.
2. Draw straight-line chords stretching from end to end along each particle's path using a very fine pencil, and measure *w* and *h* (try to measure *h* to ±0.1 mm). Note that the incoming pions all have almost exactly the same momentum in this experiment by design, so you can measure the curvature of *any* incoming pion's path and confidently assume that the path of the pion actually involved in the collision has the same curvature.
3. Use *w* and *h* (and their uncertainties) to determine *R* (and its uncertainty).
4. Carefully draw tangent lines to the particle's paths just before or after the collision and measure the angles θ_1 and θ_2 for the outgoing particles. (Remember that the uncertainty in these angles should include an estimate of how well you can draw those tangent lines.)
5. Use *R* and the particle's mass *m* to calculate mvc (and its uncertainty) for each particle.
6. Use these quantities to calculate the newtonian kinetic energy and components of the total momentum before and after the collision. Are these conserved within uncertainties?

Do steps 1 through 4 *by yourself*, and then compare results with your partners to come up with common estimates of curvature radii and angles (and their uncertainties), and then proceed as a team with the rest of the analysis.

As we will see in unit *R*, the theory of relativity predicts that the quantities conserved in such a collision are the components of the system's total **4-momentum**. The 4-momentum of an individual particle is a four-dimensional vector whose components are

$$\begin{bmatrix} E \\ P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} \sqrt{(mc^2)^2 + P^2} \\ mv_x c \,/\, \sqrt{1 - v^2/c^2} \\ mv_y c \,/\, \sqrt{1 - v^2/c^2} \\ mv_z c \,/\, \sqrt{1 - v^2/c^2} \end{bmatrix} \tag{6}$$

$$\text{where} \quad P \equiv \sqrt{P_x^2 + P_y^2 + P_z^2} = \frac{mvc}{\sqrt{1 - v^2/c^2}} \tag{7}$$

We call P the magnitude of the particle's *relativistic* momentum and E the particle's *relativistic* energy. Note that a particle has relativistic energy $E = mc^2$ when it is at rest (does this formula ring a bell?). Note also that in all these expressions, the particle's mass m is a speed-independent constant (no matter what you may have heard about mass in relativity).

This formula looks complicated, but it is actually very simple to use in this case. If you compare equation 7 with equation 1, you will see that the magnitude P of a particle's relativistic momentum is related in a very simple way to the radius of curvature R of its track:

$$R = \frac{mvc}{|q|Bc\sqrt{1 - v^2/c^2}} = \frac{P}{|q|Bc} \quad \Rightarrow \quad P = |q|BcR \tag{8}$$

Since we know the value of $|q|Bc = eBc$ from before, we can easily calculate P if we know R. Then, if the particle's track makes an angle of θ with the $+x$ direction, its 4-momentum is simply

$$\begin{bmatrix} E \\ P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} \sqrt{(mc^2)^2 + P^2} \\ P\cos\theta \\ \pm P\sin\theta \\ 0 \end{bmatrix} \tag{9}$$

(where the sign in the next-to-last component depends on whether the particle's path is above or below the x axis). Since you already determined R for each particle when you did the newtonian analysis, it is an easy matter to compute these 4-momentum components for each particle. Add up the 4-momentum column vectors for all particles before and after the collision and compare to see if 4-momentum is conserved. (Treat these vectors just like ordinary 3-component column vectors that happen to have an additional component.)

Your job is to determine whether the newtonian conservation laws or conservation of 4-momentum better model the collision shown in the photograph. To do this successfully, you will have to pay very careful attention to uncertainties. Work carefully to make the best uncertainty estimates you can, and think carefully about how you will handle propagation of uncertainties. It will also help if you are *very* careful when you construct your lines and make your measurements.

(Note that *PropUnc* expects angles in radians. The program predefines the constant "pi", so you can type something like "cos(theta*pi/180)" if theta is an angle in degrees.)