

1: Bubble Chamber

Purpose

The purpose of this experiment is to determine the rest mass of the pion (m_π) and the rest mass of the muon (m_μ).

Introduction

Particle physics (a.k.a. high energy physics) is the division of physics which investigates the behavior of particles involved in “high” energy collisions. (“High” here means energies greater than those found in nuclear reactions, i.e., more than 100 MeV=0.1 GeV. The highest energy particle accelerators available today produce collisions with energies of a few million MeV = TeV.)

The first “new” particles discovered (circa 1940) by particle physicists were the pion (π) and the muon (μ). In spite of roughly similar masses (near 100 MeV, compare: electron mass = .511 MeV and proton mass = 938 MeV), these two particles have quite different properties.

The muon is a relative of the electron (and hence is called a lepton). It comes in particle (μ^-) and anti-particle (μ^+) versions and has spin $\frac{1}{2}$. Unlike the electron, the muon is unstable. It decays into two neutrinos (ν) and an electron (or positron) after a mean life of 2×10^{-6} s:

$$\mu^+ \longrightarrow \bar{\nu} + \nu + e^+ \quad (1.1)$$

$$\mu^- \longrightarrow \nu + \bar{\nu} + e^- \quad (1.2)$$

The pion belongs to the class of particles called mesons. Unlike leptons, mesons interact with protons and neutrons through an additional force called the strong nuclear force (a.k.a., color force). (Particles that can feel this force are called hadrons.) Unlike leptons, mesons are known to be composite particles: each is made of a quark and an antiquark. The pion comes in three versions: π^+ , π^0 , and π^- and has spin 0. All the pions are unstable; the π^+ decays after a mean life of 3×10^{-8} s:

$$\pi^+ \longrightarrow \mu^+ + \nu. \quad (1.3)$$

(The π^0 has a slightly smaller mass and decays much faster than the π^\pm . It is not seen in this experiment.)

Particle Detection

Since the particles studied by particle physics are sub microscopic and decay “quickly”, particle detection is a problem. Most existing particle detectors rely on the fact that as a charged particle moves by an electron (e.g., an electron in an atom of the material through which the charged particle is moving), the electron feels a net impulse. If the charged particle comes close enough to the electron and/or is moving slowly enough (so the interaction is long enough), the impulse on the electron will be sufficient to eject the electron from its atom, producing a free electron and an ion. Thus a charged particle moving through material leaves a trail of ions. This trail can be detected in many ways (e.g., by direct electronic means as in a modern wire chamber or chemically as when the material is a photographic plate or emulsion). In this experiment the ion trail is made visible by vapor bubbles which are seeded by individual ions in boiling material (here liquid hydrogen). The bubbles are large enough to be photographed whereas the ion trail itself is much too narrow.

Relativistic Kinematics

Recall the following from Modern Physics:

$$E = \gamma mc^2 \quad (1.4)$$

$$T \equiv E - mc^2 \quad (1.5)$$

$$pc = \gamma mvc = \gamma mc^2 \beta = E\beta \quad (1.6)$$

$$E^2 - (pc)^2 = [\gamma mc^2]^2 (1 - \beta^2) = [mc^2]^2 \quad (1.7)$$

where:

$$\beta = v/c \quad (1.8)$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad (1.9)$$

and v is the velocity of the particle with rest mass m , momentum p , total energy E and kinetic energy T . Note that E , T , pc , and mc^2 all have the dimensions of energy; it is customary to express each in MeV and even say “the momentum of the particle is 5 MeV” or “the mass of the particle is 938 MeV.” (Of course, technically the momentum of the particle would be 5 MeV/ c and the mass 938 MeV/ c^2 . Basically what we are doing is redefining “momentum” to be pc and “mass” to be mc^2 . Since the “ c ” has disappeared, this re-naming is sometimes called “setting $c = 1$ ”.)

For future reference, note from Equation 1.6 that if $\beta \rightarrow 1$, $E \approx pc$ and from Equation 1.7 that if $m = 0$, $E = pc$. Of course, massless particles (like light) must travel at the speed of light (i.e., $\beta = 1$).

Momentum Measurements

Classically a charged particle (with mass m and charge q) moving through a magnetic field \vec{B} has an acceleration, \vec{a} , given by

$$m\vec{a} = q\vec{v} \times \vec{B} \quad (1.10)$$

Because of the cross product, the acceleration is perpendicular to both \vec{v} and \vec{B} . Thus there is zero acceleration in the direction of \vec{B} , so v_{\parallel} , the component of velocity parallel to \vec{B} , is constant. On the other hand in the plane perpendicular to \vec{B} , the acceleration and the velocity are perpendicular resulting in centripetal (circular) motion. Thus the particle moves in a circle of radius R even as it travels at constant speed in the direction of \vec{B} . The resulting motion is a helix (corkscrew). Using \perp to denote components in the plane perpendicular to \vec{B} , we have:

$$ma_{\perp} = \frac{mv_{\perp}^2}{R} = qv_{\perp}B \quad (1.11)$$

$$p_{\perp} = mv_{\perp} = qBR \quad (1.12)$$

This last relationship continues to hold for relativistic particles.

SHOW: For a positron, Equation 1.12 means the momentum $p_{\perp}c$ (in MeV) can be calculated as a simple product $3BR$:

$$p_{\perp}c \text{ (in MeV)} = 3BR \quad (1.13)$$

where B is in Tesla and R is in cm.¹

In this experiment, positrons (and electrons) from muon decay circle in an applied magnetic field. You will measure the radii of the positron orbits to determine positron p_{\perp} . Since the rest mass of the muon has been converted to kinetic energy of its decay products, positron p_{\perp} depends on muon mass and measurement of p_{\perp} allows calculation of m_{μ} .

Kinetic Energy Measurement

As stated above, a charged particle moving through a material leaves a trail of ions. The energy needed to form these ions must come from the kinetic energy of the charged particle. Thus, every cm of travel results in a kinetic energy loss. It can be shown (Bethe-Block) that the decrease in kinetic energy depends on the inverse of the velocity squared:

$$\frac{dT}{dx} = - \frac{2.1\rho}{\beta^2} \quad \text{MeV/cm} \quad (1.14)$$

where ρ is the density of the material ($\rho = .07 \text{ g/cm}^3$ for liquid H_2). This is the second example of a “calculator equation”².

A particle with some initial kinetic energy T_0 will travel some definite distance, L , before all of its kinetic energy is lost and it comes to rest. The relationship between T_0 and L can be determined from the energy loss per cm:

$$L = \int_{T_0}^0 \frac{dx}{dT} dT = \int_0^{T_0} \frac{\beta^2}{2.1\rho} dT \quad (1.15)$$

¹This is an example of a “calculator equation” where we seemingly ignore units. That is if $B = 2 \text{ T}$ and $R = 5 \text{ cm}$, this equation says $p_{\perp}c = 3 \times 2 \times 5 = 30 \text{ MeV}$, units seemingly just tacked onto the answer. To derive such an equation, you must demonstrate how the units work out. In particular, $p_{\perp}c$ — which in MKS units in going to naturally come out in Joules — must be converted to the energy unit MeV. The conversion factor is $1 \text{ MeV} = 1.6022 \times 10^{-13} \text{ J}$. Of course, you already know $100 \text{ cm} = 1 \text{ m}$.

²Thus if $\rho = .1 \text{ g/cm}^3$ and $\beta = .5$ we would conclude that dT/dx was $.84 \text{ MeV/cm}$.

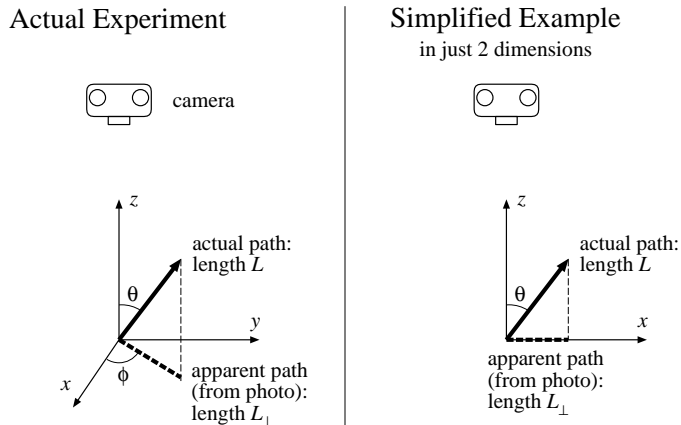


Figure 1.1: This experiment uses photographs of particle paths in a bubble chamber. Two angles ($\theta \in [0, 180^\circ]$, $\phi \in [0, 360^\circ]$) are required to describe the orientation of the path in three dimensional space. The photographic (apparent) path length, L_\perp , is shorter than the actual path length, L , (of course, if $\theta = 90^\circ$, $L_\perp = L$). In general: $L_\perp = L \sin \theta$. The angle ϕ just describes the orientation of the apparent path in the photograph. We can make an easier-to-understand model of perspective effects by just dropping ϕ and considering a two dimensional experiment. In this case to generate all possible orientations $\theta \in [0, 360^\circ]$.

For particles moving much slower than the speed of light, Newton’s mechanics is a good approximation: $T = \frac{1}{2} mv^2 = \frac{1}{2} mc^2 \beta^2$

$$L = \frac{2}{2.1mc^2\rho} \int_0^{T_0} T dT = \frac{T_0^2}{2.1mc^2\rho} \quad (1.16)$$

so $T_0 \propto L^{1/2}$.

In this experiment muons produced by pion decay travel a distance L before coming to rest. You will measure the muon path length to determine muon T_0 . Since the kinetic energy of the muon comes from the rest mass of the decaying pion, the mass of the pion can be calculated from muon T_0 .

Perspective Effects

In real particle physics experiments, decay events are reconstructed in three dimensions. However in this experiment you will measure *apparent* muon path lengths from photographs. Because of perspective effects, typically the true path length (L) is longer than the apparent (photographic) path length (L_\perp), as the particle will generally be moving towards or away from the camera in addition to sideways. In this experiment we need to “undo” the perspective effect and determine L from the measurements of L_\perp .

There are several ways this could be done. Perhaps the easiest would be to pick out the longest L_\perp , and argue that it is longest only because it is the most perpendicular, i.e.,

$$\max(\{L_\perp\}) \approx L \quad (1.17)$$

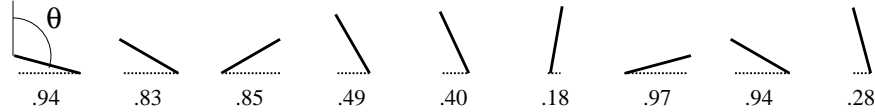


Figure 1.2: Nine randomly-oriented, fixed-length segments are placed on a plane and the corresponding horizontal lengths L_{\perp} (dotted lines) are measured (results displayed below the segment). The resulting data set $\{.94, .83, .85, .49, .40, .18, .97, .94, .28\}$ of L_{\perp} can be analyzed to yield the full segment length L . (The angle $\theta \in [0, 360^{\circ}]$ describes the orientation, but it is not measured in this “experiment”: only L_{\perp} is measured.)

Essentially this is a bad idea because it makes use of only one collected data point (the maximum L_{\perp}). For example, it is likely you will make at least one misidentification or mismeasurement in your 60+ measurements. If the longest L_{\perp} happens to be a bad point, the whole experiment is wrong. Additionally since L is the net result of interactions with randomly placed electrons, L is not actually exactly constant. (That is, Equation 1.14 is true only “on average”.) Paths that happen to avoid electrons are a bit longer. The L - T_0 relationship is based on average slowdown; it should not be applied to one special path length.

One way of using all the data is to note that randomly oriented, fixed-length paths will produce a definite average L_{\perp} related to L . So by measuring the average L_{\perp} (which we will denote with angle brackets: $\langle L_{\perp} \rangle$), you can calculate the actual L .

It will be easier to explain this method if we drop a dimension and start by considering randomly oriented, fixed-length segments in two dimensions. Figure 1.2 shows³ nine randomly oriented segments in a plane with the corresponding measured L_{\perp} . The different measured L_{\perp} are a result of differing orientations of a fixed-length segment:

$$L_{\perp} = L |\sin \theta| \quad (1.18)$$

From a sample of N measurements of the horizontal distance L_{\perp} (i.e., a data set of measured L_{\perp} : $\{x_i\}$ for $i = 1, 2, \dots, N$, with corresponding orientations $\{\theta_i\}$ with $\theta_i \in [0, 2\pi]$), the average L_{\perp} could be calculated

$$\langle L_{\perp} \rangle = \frac{1}{N} \sum_{i=1}^N x_i = \frac{L}{N} \sum_{i=1}^N |\sin \theta_i| \quad (1.19)$$

The θ_i should be approximately evenly distributed with an average separation of $\Delta\theta = 2\pi/N$ (because there are N angles distributed throughout $[0, 2\pi]$). Thus, using a Riemann sum approximation for an integral:

$$\langle L_{\perp} \rangle = \frac{L}{N} \sum_{i=1}^N |\sin \theta_i| = L \sum_{i=1}^N |\sin \theta_i| \left(\frac{\Delta\theta}{2\pi} \right) \quad (1.20)$$

$$\approx \frac{L}{2\pi} \int_0^{2\pi} |\sin \theta| d\theta = L \frac{\int_0^{2\pi} |\sin \theta| d\theta}{\int_0^{2\pi} d\theta} \quad (1.21)$$

³Note that if we applied Equation 1.17, we would conclude $L = .97$ with no estimate for the uncertainty in this result (i.e., δL).

The above integral is easily evaluated:

$$\int_0^{2\pi} |\sin \theta| d\theta = 2 \int_0^\pi \sin \theta d\theta = 2 [-\cos \theta]_0^\pi = 4 \quad (1.22)$$

Thus we have the desired relationship between $\langle L_\perp \rangle$ and L :

$$\langle L_\perp \rangle = L \frac{2}{\pi} \quad (1.23)$$

With the example data set we have: $\langle L_\perp \rangle = 0.653$ with standard deviation $\sigma_{L_\perp} = 0.314$. Using the standard deviation of the mean we have:

$$0.65 \pm \frac{.314}{\sqrt{10}} = 0.65 \pm .10 = L \frac{2}{\pi} \quad (1.24)$$

$$1.03 \pm .16 = L \quad (1.25)$$

Note that our argument for finding averages is quite general, so if random values of x are uniformly selected from the interval $[a, b]$, the average value of any function of x , $f(x)$, can be calculated from:

$$\langle f(x) \rangle = \frac{\int_a^b f(x) dx}{\int_a^b dx} \quad (1.26)$$

For the actual experiment, the path orientations have a uniform distribution in space. That is, if all the paths originated from the same point, the path ends would uniformly populate the surface of a sphere of radius L . The element of surface area of a sphere of radius L is:

$$L^2 d\Omega = L^2 \sin \theta d\theta d\phi \quad (1.27)$$

where Ω is called the solid angle and plays an analogous role to radian measure in a plane:

$$\text{plane angle in radians} = \frac{\text{arc length}}{R} \quad (1.28)$$

$$\text{solid angle in steradians} = \frac{\text{sphere surface area}}{R^2} \quad (1.29)$$

Thus the relationship between $\langle L_\perp \rangle$ and L in three dimensions is:

$$\langle L_\perp \rangle = L \langle \sin \theta \rangle = L \frac{\int \sin \theta d\Omega}{\int d\Omega} = L \frac{\pi}{4} \quad (1.30)$$

SHOW this result! Note: $d\Omega = \sin \theta d\theta d\phi$ and the range of the double integral is $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$

Comment: The above discussion has been phrased in terms of position vectors, but it applies as well to any vector. In particular, you will be measuring the perpendicular component of momentum, p_\perp , and need to deduce the actual momentum, p . Exactly as above, if the particles have the same speed with direction uniformly distributed in space:

$$\langle p_\perp \rangle = p \frac{\pi}{4} \quad (1.31)$$

If the particles actually have differing speeds we can still conclude:

$$\langle p_\perp \rangle = \langle p \rangle \frac{\pi}{4} \quad (1.32)$$

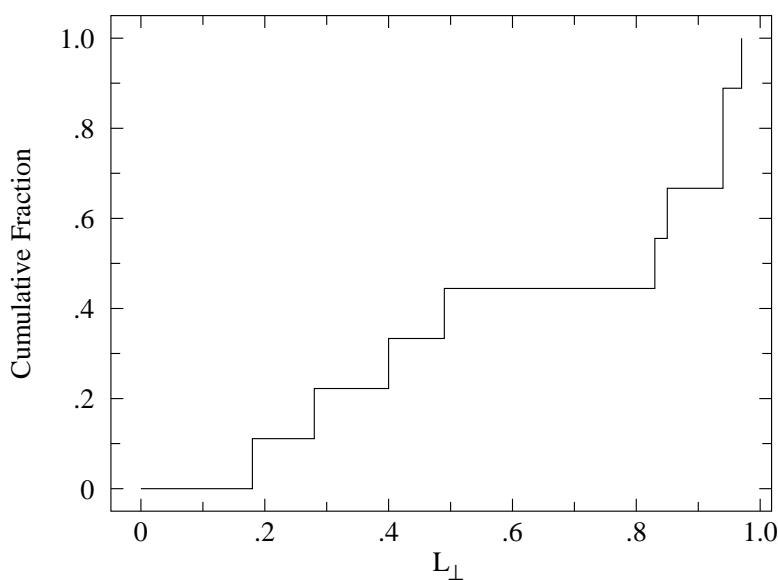


Figure 1.3: The distribution of data set: $\{.94, .83, .85, .49, .40, .18, .97, .94, .28\}$ of Figure 1.2 (nine randomly oriented segments) displayed as a cumulative fraction.

Displaying Distributions

As discussed above, when finding L it is best to use the entire data set. Although the $\langle L_{\perp} \rangle$ method uses all the data, it quickly reduces the whole data set to one number. Is there some way of graphically displaying and using all the data? In particular, is there some way of checking to see if the data have the expected distribution (i.e., the right proportion of long and short L_{\perp} s)?

Perhaps the easiest way to understand the idea of a distribution is to consider the idea of the cumulative fraction function for some data set: $\{x_i\}$, for $i = 1, 2, \dots, N$. The cumulative fraction function⁴, $c(x)$, reports the fraction of the data set $\{x_i\}$ that is less than or equal to x . Obviously if $a < \min(\{x_i\})$, $c(a) = 0$; if $b \geq \max(\{x_i\})$, $c(b) = 1$; and if $c(x) = .5$ then x is the median (i.e., middle) data point. In the 2d example data set, $c(.60) = 4/9$, because four of the nine data points are smaller than .6. Similarly $c(.84) = 5/9$. Every time x increases past one of the x_i , $c(x)$ has a jump. See Figure 1.3 for a plot of this function.

The function $c(x)$ depends on the data set, so if the experiment is repeated generating a new data set $\{x_i\}$, a new function $c(x)$ is also generated. The new $c(x)$ should be slightly different, but generally similar to the old $c(x)$. If the data set is sufficiently large, the new and old $c(x)$ will be quite similar and both $c(x)$ would approximate the function $\hat{c}(x)$, the cumulative fraction function that would be generated from an infinite-sized data set⁵. How can $\hat{c}(x)$ be best approximated from one finite-sized data set $\{x_i\}$? To answer this question it will be convenient to consider the data set $\{x_i\}$ already sorted so x_1 is the minimum and

⁴The cumulative fraction function is also known as the empirical distribution function, and is closely related to percentiles and order statistics.

⁵The usual name for $\hat{c}(x)$ is the distribution function.

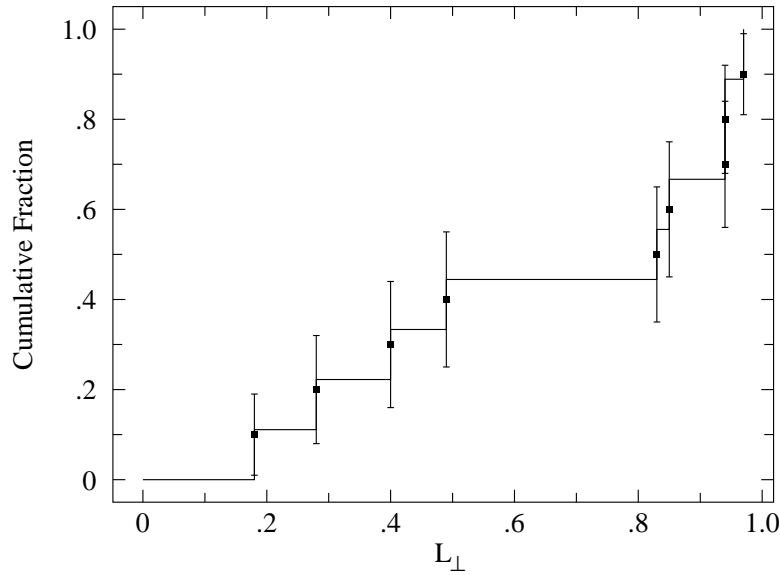


Figure 1.4: The cumulative fraction function for the example data set is plotted along with the data points for the percentile estimate of the distribution function $\hat{c}(x)$.

x_N is the maximum. Thus our example data set:

$$\{.94, .83, .85, .49, .40, .18, .97, .94, .28\} \quad (1.33)$$

becomes:

$$\{.18, .28, .40, .49, .83, .85, .94, .94, .97\} \quad (1.34)$$

As defined above, $c(x)$ is given by:

$$c(x) = \frac{i}{N} \quad \text{where } i \text{ is such that: } x_i \leq x < x_{i+1} \quad (1.35)$$

That is to determine $c(x)$ for some x , we see how far down the sorted list we must travel to find the spot where x fits between two adjacent data points: $x_i \leq x < x_{i+1}$. Clearly there are a total of i data points less than or equal to x (out of a total of N), so $c(x) = i/N$. If x happens to equal one of the data points, things are a bit undefined because $c(x)$ has a jump discontinuity at each x_i . It turns out that the best estimate for \hat{c} at these discontinuities is:

$$\hat{c}(x_i) = \frac{i}{N+1} \equiv \alpha_i \quad (1.36)$$

Of course this estimate can be wrong; it has an uncertainty of

$$\sigma = \sqrt{\frac{\alpha_i(1-\alpha_i)}{N+2}} \quad (1.37)$$

See Figure 1.4 for a comparison of the estimated $\hat{c}(x_i)$ (called the percentile) and the cumulative fraction function. (The mathematics of these results is covered under the topic “order statistics” or “nonparametric methods” in advanced statistics books.)

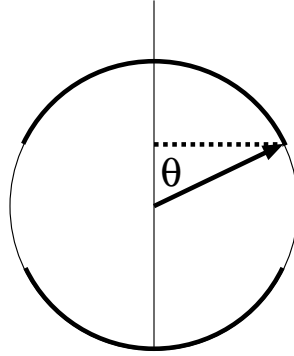


Figure 1.5: A particular line segment is displayed along with the measured L_{\perp} (dotted line). What fraction of randomly oriented segments would have a L_{\perp} smaller than this particular segment? The darkly shaded part of the circle shows possible locations for these small L_{\perp} segments. The fraction of such small L_{\perp} segments should be the same as the dark fraction of the circle: $4\theta/2\pi$.

Your experimental estimate of \hat{c} should be compared to the theoretically expected \hat{c} . The example data set was generated from randomly oriented line segments in a plane. As shown in Figure 1.5, it is expected that the fraction of a data set less than some particular value of L_{\perp} is:

$$\hat{c}(L_{\perp}) = \frac{4\theta}{2\pi} \quad \text{where: } \theta = \arcsin(L_{\perp}/L) \quad (1.38)$$

$$= \frac{2}{\pi} \arcsin(L_{\perp}/L) \quad (1.39)$$

Our formula for \hat{c} involves the unknown parameter L ; we adjust this parameter to achieve the best possible fit to our experimental estimate for \hat{c} . Using the program `fit`:

```
tkirkman@bardeen 7% fit
* set f(x)=2*asin(x/k1)/pi k1=1.
* read file cf.L.dat
* fit
Enter list of Ks to vary, e.g. K1-K3,K5 k1
FIT finished with change in chi-square= 5.4810762E-02
3 iterations used
REDUCED chi-squared= 0.2289333 chi-squared= 1.831467
K1= 0.9922597
```

Using the covariance matrix to determine errors⁶, we conclude $k1 = 0.992 \pm .025$. This reported random error is about $\frac{1}{6}$ that obtained above using $\langle L_{\perp} \rangle$.

SHOW: Derive yourself the theoretical function $c(L_{\perp})$ for line segments in space. Hint: Begin by noting that if the segments shared a common origin, the segment ends would uniformly populate the surface of a sphere of radius L . Segments with measured L_{\perp} less than some particular value would lie on a spherical cap, the three dimensional version of

⁶Reference 2, Press et al., says usually error estimates should be based on the square root of the diagonal elements of the covariance matrix

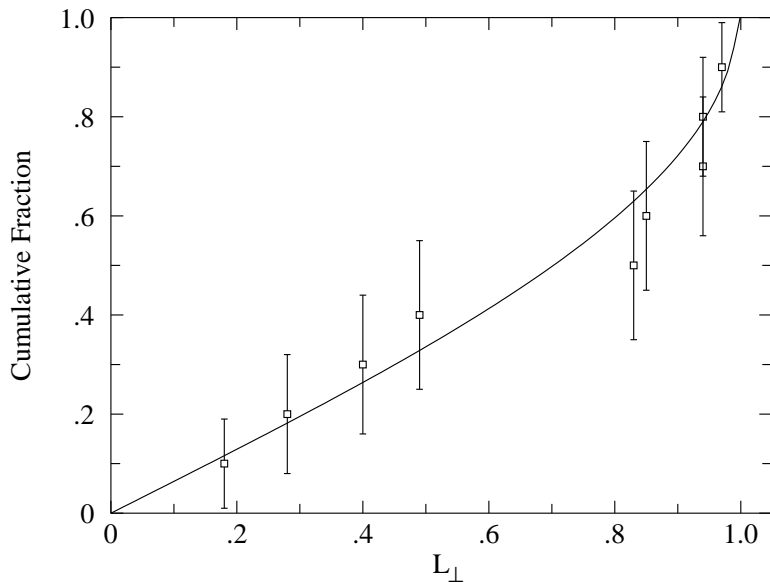


Figure 1.6: The theoretical distribution function (Equation 1.39) fit to the “experimental” data points derived (Equations 1.36 & 1.37) from the example data set. As a result of the fit we estimate: $L = 0.992 \pm .025$.

arc caps displayed in Figure 1.5. The ratio of the area of these caps to the total surface area of the sphere gives the expected value for \hat{c} . You will need to calculate the area of a spherical cap by integration.

The above discussion has focused on path lengths as that is the quantity measured in pion decay. In muon decay, the radius of positron orbits in the applied magnetic field is measured. Weinberg-Salam theory provides a complete description of the decay process, including the distribution of positron momentum (which in turn determines the radius of positron orbits R). Kirkman has shown that the the R s should be distributed according to

$$c(R) = \left(\frac{3}{2} x^2 - 1 \right) \sqrt{1 - x^2} + 1 - \frac{1}{2} x^4 \log \left(\left(1 + \sqrt{1 - x^2} \right) / x \right) \quad (1.40)$$

where $x = R/R_m$, and R_m , the maximum value of R , is the value of R that corresponds to $p_{\perp}c = m_{\mu}c^2/2$. The adjustable parameter R_m (of course called `k1` in `fit`), can be adjusted to give the best possible fit to the experimental distribution. From R_m , m_{μ} can be determined.

Biases and Robust Estimation

The bane of every particle physics experiment is bias. Biases are data collection techniques that produce nonrepresentative data. For example, short L_{\perp} are harder to notice than long L_{\perp} , and thus long L_{\perp} tend to be over represented in the data sample, producing a high $\langle L_{\perp} \rangle$. Use of the cumulative fraction function allows this biases to be detected. In addition

to biases, the cumulative fraction function allows you to detect likely mistakes: for example, particle path lengths that are extraordinary given the entire data set.

The detection of a likely mistake suggests corrective actions like removing the “bad” point. You should almost never do this! (You will find a chapter in Taylor on this “awkward” and “controversial” problem.) A better option is to use analysis methods that are “robust”, i.e., that are insensitive to individual “bad” points.⁷ Imagine we modify our example data set by adding a “bad” point: $L_{\perp} = 2$:

$$\{.18, .28, .40, .49, .83, .85, .94, .94, .97, 2.00\} \quad (1.41)$$

Adding this outlying⁸ data point totally messes up the $\max(\{L_{\perp}\})$ method (the least robust method). Since it increases both the mean and the standard deviation, the estimated L based on the $\langle L_{\perp} \rangle$ method shifts from $1.03 \pm .16$ to $1.24 \pm .26$.

Changing the number of data points requires recalculating the estimated distribution function for every point (because the value of \hat{c} depends on the set size N). If we carry through the total analysis with our enlarged data set we find the fit L shifts from $0.992 \pm .025$ to $1.05 \pm .05$. We can conclude that the cumulative fraction method is less sensitive to bad data than the average method.⁹

Unconscious (uncontrolled) biases produce tainted data which can be rescued in part by robust estimation. Once bias is recognized the experiment can be rearranged to adjust for its effects. This requires that the bias be exactly reproducible. For example, short apparent muon paths (paths mostly towards or away from the camera) are inconspicuous and hence more likely to be missed on some occasions. One solution is formalize this bias and intentionally ignore all photographic paths shorter than say .3 cm long (about 5% of the data). This cut (formalized noncollection of data), can be included in the theoretical distribution function so it will not affect parameter estimation.

Experimental Arrangement

Our bubble chamber photographs were taken using the 385 MeV proton accelerator at Nevis Lab which is a part of Columbia University. A pion beam was produced by colliding accelerated protons with a copper target. The pion beam was directed through an absorber to slow the pions so that a sizeable fraction of the pions would come to rest in the adjacent bubble chamber. See Figure 1.7. The path of charged particles from the pion decay ($\pi \rightarrow \mu \rightarrow e$) as recorded by a nearby camera, encodes the information needed to calculate m_{π}

⁷Removing a data point is a lie. A more subtle sort of lie comes from the existence of choice of methods. Clearly, you can analyze the data several different ways, and then present only the method that produces the answer you want. Darrell Huff’s book *How to Lie with Statistics* (Norton, 1954) can help you if that is your goal. I probably don’t need to remind you that schools with “Saint” in their name do not recommend this course of action. Choice and ethics are interlocking concepts.

⁸While not exactly relevant, this data point is $2.34 \times \sigma$ above the mean and hence an outlier by Chauvenet’s criterion (see Reference 4, Taylor). It is also an outlier by Tukey’s criterion (see Reference 3, Hogg & Tanis).

⁹Do note that both results remain consistent with the intended value of 1.00. Also note that the median could have provided a more robust alternative to the average. However, that would have required a discussion of the uncertainty in the median, which is beyond the intended aims of this lab. In this lab—and in most any experiment—there are many possible ways to analyze the data. Choice of method often involves art and ethics.

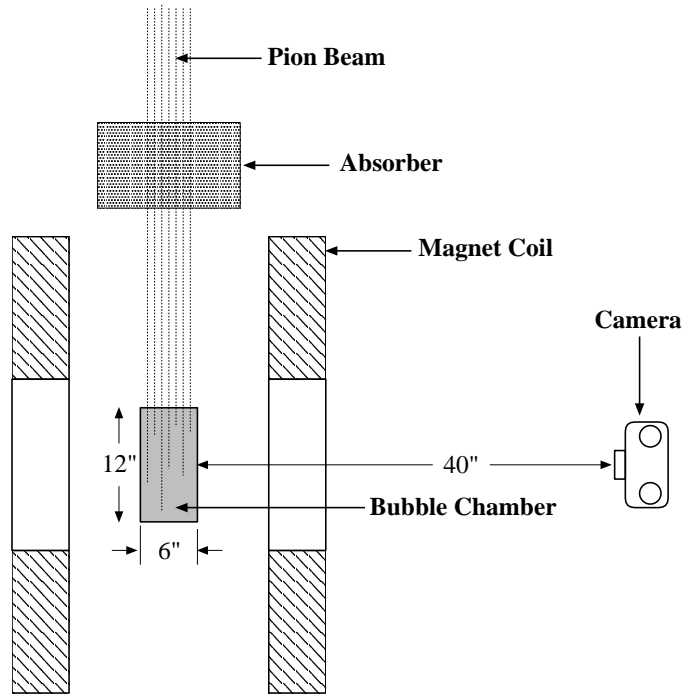


Figure 1.7: A pion beam is slowed by an absorber so the pions are likely to come to rest inside a liquid hydrogen bubble chamber. The path of charged particles from the pion decay ($\pi \rightarrow \mu \rightarrow e$) is recorded by a nearby camera. (Of course, the uncharged neutrinos from the decay leave no ion trail, and hence no bubbles grow to mark their path.) A magnetic field ($B = .88$ T, directed toward the camera) produced by the current in the coil, bends the path of all charged particles into helices, but the effect is most visible with the low mass electrons. The radius of the helix, as recorded by the camera, can be related to the particle's p_{\perp} . Since the muon decays into three particles, allowing varying distribution of energy, the electron's momentum can vary from 0 up to a maximum of $m_{\mu}c/2$. Since the pion decays into just two particles, there is only one way to distribute the released energy so the muon's initial kinetic energy is determined uniquely which produces a fixed stopping distance L . (Of course, the apparent muon path length, L_{\perp} , recorded by the camera will vary.)

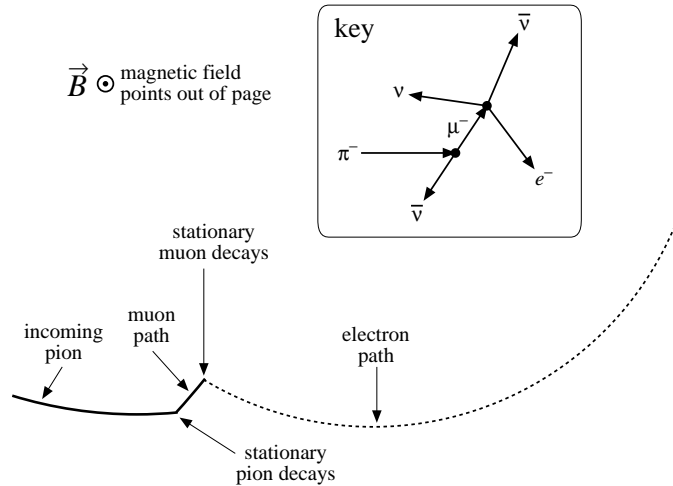


Figure 1.8: A typical decay process as recorded in a bubble chamber photograph. A pion (π^-) slows and comes to rest inside the bubble chamber. A short time later it decays into a muon and an antineutrino ($\pi^- \rightarrow \mu^- + \bar{\nu}$). A kink in the path marks the decay location. The muon is in turn slowed and comes to rest after traveling a short distance. A short time later the muon decays into an electron and two neutrinos ($\mu^- \rightarrow e^- + \bar{\nu} + \nu$). The high speed electron leaves a sparse track in accord with the Bethe-Block Equation (1.14) (reduced energy loss due to large β means fewer ions produced and hence fewer bubbles). You will be measuring apparent muon path lengths, L_{\perp} , and electron helix radii R .

and m_{μ} . Apparent muon path length, L_{\perp} , will allow you to determine the actual muon path length L , from which in initial muon kinetic energy T_0 can be determined. A magnetic field ($B = .88$ T, directed toward the camera) bends the path of all charged particles into helices; but the effect is most visible with the low mass electrons. The radius of the electron helix determines (Equation 1.13) the electron's $p_{\perp}c$. From the distribution of electron $p_{\perp}c$, you can determine both $\langle pc \rangle$ and the maximum pc , from which m_{μ} can be determined.

Figure 1.8 shows an idealized decay sequence as might be recorded in a bubble chamber photo. The paths of interest start on the left (pions from the accelerator), have a short (~ 1 cm) kink (muon), connecting to a sparse loop (electron).

Procedure

Measurement of R s and L_{\perp} s is computerized. (Indeed almost all of your data collection this semester will be computerized.) As I'm sure you know, while computers can be useful devices, they seemingly have a knack for unintended/unexpected disasters, which are called 'user error'. Thus the most important lesson of computer use is: GIGO ('Garbage In; Garbage Out'¹⁰) — a computer's output must be considered unreliable until, at the very least, you know the limitations/uncertainties in the input data producing that output. In this lab you will be using a CalComp 2500 digitizing tablet to measure distances. The process seems simple (aligning a point between crosshairs and clicking to take the data-

¹⁰Sometimes this acronym is reported as 'Garbage In, Gospel Out' stressing many people's (mistaken) faith in computer output.

point) but involves problem of definition errors (including systematic biases, see page 13) in addition to more familiar device limitations (random and calibration errors). To have some justified confidence in this process, you must measure a known and see what the computer reports ('trust, but verify'). I have provided you with a simulated bubble chamber photo in which all the path lengths are 1 cm and all the curvatures are 20 cm. (If you don't trust this fiducial—and you might not since it depends on the dimensional stability of printers and paper—you can measure the 'tracks' with an instrument you do trust.) Begin by logging into your UNIX account using the Visual 603 terminal with attached CalComp 2500 digitizer, and running the program bubbleCAL:

```
tkirkman@linphys8 1% bubbleCAL
```

The following directions are displayed:

```
The general procedure will be to place the cursor crosshair at
the needed place and press a cursor button to digitize. Press
cursor button "0" when obtaining muon path lengths (digitize
beginning and end of track); press cursor button "1" when obtaining
electron radii of curvature (digitize three points on curve, from
which the computer can figure R); press cursor button "2" to cancel
an in-progress data point or clear error; press cursor button "3" to
remove the last data point of the presently selected type. Additional
data points may be removed by number when done. Digitizer will beep
between data points. Hit ^D (control D) on keyboard when done.
Files containing your data (unsorted) will be created: Lcal.DAT & Rcal.DAT.
THIS PROGRAM IS FOR CALIBRATION/TESTING NOT DATA COLLECTION
```

With the simulated bubble chamber photo taped in place on the digitizer, check some long distances (> 10 cm) on the scales. Then measure 16 path lengths and 16 curvatures. Record the means and standard deviations reported by the program. Does the probable range for each mean include the known value? (If not discuss the problem with Dr. Kirkman.) Note that if you took more data points, the probable range for the mean would become increasingly small and eventually you would detect a systematic error limiting the ultimate accuracy of the device.

The program `bubble` works very much like `bubbleCAL`, except in the end it will produce files containing the cumulative fraction of L_{\perp} (`cf.L.dat`) and R (`cf.R.dat`). Please do not deface the bubble chamber photos! Lightly tape each bubble chamber photograph to the digitizer to keep the photo from moving as you collect data. Collect ~ 64 L_{\perp} s and ~ 64 R s. (This will require scanning about 20 bubble chamber photos.) The program will make four files: `cf.R.dat` contains the usual three columns: R , the percentile estimate for $c(R)$ calculated from your data, and the error in the estimate, `cf.L.dat` similarly contains the sorted L_{\perp} , estimated $c(L_{\perp})$ and error, `L.DAT` and `R.DAT` contain the (sorted) raw data.

Use the web¹¹ or `spj` to calculate $\langle L_{\perp} \rangle$ and $\langle R \rangle$ and their standard deviations.

Use `fit` to fit each dataset (`cf.R.dat` & `cf.L.dat`) to the appropriate theoretical distribution function. Produce a hardcopy of your `fit` results to include in your notebook. Use

¹¹http://www.physics.csbsju.edu/stats/cstats/paste_form.html

`plot` to produce a hardcopy plot of your data with fitted curve to include in your notebook. The formula for $c(R)$ (Eq. 1.40) is a bit tricky to type in, so I've provided you with a shortcut: $c(R)$ will be automatically entered into `fit` (with `K1` as the adjustable parameter R_m) if, in `fit`, you:

```
* @/usr/local/physics/help/crfit.fun
```

and $c(R)$ will be automatically entered into `plot` (with `K1` as R_m) if, in `plot`, you

```
* @/usr/local/physics/help/crplot.fun
```

Copy and paste is the easiest way to transfer `K1` between `fit` and `plot`.

Calculation of m_μ

Since energy is conserved, the rest energy of the muon must end up in its decay products:

$$E_\mu = m_\mu c^2 = E_e + E_\nu + E_{\bar{\nu}} \quad (1.42)$$

where E_μ , E_e , E_ν , $E_{\bar{\nu}}$ are respectively the total energy of the muon, electron, neutrino, and antineutrino.

You should expect that the rest energy of the muon would, on average, be evenly divided between the three particles. Thus

$$\langle E_e \rangle \approx \langle E_\nu \rangle \approx \langle E_{\bar{\nu}} \rangle \approx \frac{1}{3} m_\mu c^2 \quad (1.43)$$

In fact, the Weinberg-Salam theory of weak decays predicts

$$\langle E_e \rangle = .35 m_\mu c^2 \quad (1.44)$$

Since electrons with that much energy have $\beta = .9996$, to a good approximation $E_e = p_e c$. Thus

$$.35 m_\mu c^2 = \langle E_e \rangle = \langle p_e c \rangle = \frac{4}{\pi} \langle p_{e\perp} c \rangle \quad (1.45)$$

so you will find $m_\mu c^2$ from $\langle p_{e\perp} c \rangle$ (which, in turn, may be found from $\langle R \rangle$).

Additionally, you will `fit` the unknown parameter R_m in the theoretical expression for $\hat{c}(R)$ (Equation 1.40) to the experimental data. From the above discussion you already know

$$3BR_m = p_\perp c = m_\mu c^2 / 2 \quad (1.46)$$

allowing you to determine $m_\mu c^2$ from B and R_m .

Recall: Standard deviation of the mean is used for errors in averages (see Taylor). The square root of the diagonal elements of the covariance matrix determine the errors in `fit` parameters (See Press, , *et al.*).

Calculation of m_π

The pion mass can be determined from the muon path length L : From L you can find the initial kinetic energy of the muon (T_0); adding the rest energy of the muon (use a high accuracy book-value for $m_\mu c^2$) gives you the total muon energy, E_μ .

$$E_\mu = T_0 + m_\mu c^2 \quad (1.47)$$

From momentum conservation and the fact the neutrinos have essentially no mass and hence travel at the speed of light, you can show

$$E_\nu = p_\nu c = p_\mu c = [E_\mu^2 - (m_\mu c^2)^2]^{1/2} \quad (1.48)$$

Finally, energy conservation of the decaying pion requires

$$m_\pi c^2 = E_\mu + E_\nu = E_\mu + [E_\mu^2 - (m_\mu c^2)^2]^{1/2} \quad (1.49)$$

from which $m_\pi c^2$ can be calculated.

We have discussed two ways of determining L (using Equation 1.30 with a measured value for $\langle L_\perp \rangle$ and by fitting the theoretical $\hat{c}(L_\perp)$ curve to your experimental data), so you will produce two values for m_π .

Note: to calculate the error in $m_\pi c^2$ ($\delta m_\pi c^2$), you will need to calculate $\frac{\partial m_\pi c^2}{\partial E_\mu}$ and δE_μ . The error in the book value of m_μ used to calculate m_π should be small enough to ignore.

Some Words About Errors

We noted above that different analysis methods yield different statistical error estimates. That is, statistical errors are important because there is something you can do about them. (Of course, robust methods that produce smaller uncertainties are preferred.) Because statistical errors *can* be reduced, they are usually *are* reduced until systematic errors dominate. (For example, statistical errors can almost always be reduced simply by collecting more data. But no amount of statistical averaging can erase a systematic error in the original measurements.) In this lab it is quite possible to reduce¹² the statistical error in L to nearly .001 cm. Yet many human factors (positioning the crosshairs, determining where to locate the path ends given finite path thickness [problem of definition]), digitizer factors (basic digitizer accuracy, width of crosshairs), and apparatus factors (the bubble chamber's 6" thickness means paths closer to the camera are enlarged in photographs compared to those further from the camera, varying magnetic field within the bubble chamber, expansion/contraction of the photographs due to varying humidity) systematically limit accuracy of the measurements. This is an example of GIGO: it is unlikely the real uncertainty in L is below .01 cm (my guess at the magnitude of the systematic error). In addition, the basic parameters of the apparatus (B field strength, ρ of the liquid hydrogen, the "2.1" in the Bethe-Block Equation (1.14)) have been given to just one or two significant digits with no reported error. Thus the statistical errors you calculate in this lab are probably

¹²The reason too-small errors are found when fitting to cumulative fraction functions may be found in Reference 2, Press, *et al.*

lower bounds to the actual error. The actual experimental error is available to us (since m_μ and m_π have since been measured to high accuracy), but how could the original investigators estimated these errors? A short answer to this question is calibration: “placing” known pathlengths and radii into the bubble chamber, and comparing the known values to the measured results. Indeed, experiments exactly like this lab are used to calibrate the detection systems in modern experiments.

Histograms

The cumulative fraction function described above is generally used when fewer than a thousand data points are available. Histograms are a more familiar way of displaying distributions. Histograms are made by dividing the range of the data set $\{x_i\}$ into several (usually equal-sized) sub intervals call bins. The data are sorted into these bins, and the number of data points in each bin is recorded as the y value for the average x value of the bin. The resulting plot is closely related to dc/dx (also known as the probability density function). Histograms are valued because they immediately show which bins are highly populated, i.e., what values of x occur frequently.

Report Checklist

1. Book values for muon (μ^\pm) and charged pion (π^\pm) mass. Find (and cite) a source that reports at least five significant digits.
2. Derivations:
 - (a) Equation 1.13
 - (b) Equation 1.30
 - (c) Area of a spherical cap
 - (d) Use above to derive $\hat{c}(L_\perp)$
3. Read: <http://www.physics.csbsju.edu/stats/display.distribution.html>
Let's call the last two digits of your CSB/SJU ID number: XY .
 - (a) Go to the web site: <http://www.physics.csbsju.edu/370/data/> and select the file: `4_X.dat`. This file contains 4 random data points. *By hand* draw in your notebook the cumulative fraction step-curve for this dataset along with the percentile values with errors. The result should be similar to Figure 1.4. Show your work!
 - (b) From the same web site, select the file: `1000_Y.dat` (or `1000_Yw.dat` with the same data in multiple columns). Select reasonable bin values and *by hand* draw the histogram for this dataset including error bars. Show your work! Copy & paste the 1000 data points into the web¹³ site and produce a hardcopy percentile plot. Comment on the relationship between features in your histogram and features in the percentile plot.

¹³http://www.physics.csbsju.edu/stats/cstats/paste_form.html

(Note: I'm requesting individual work—rather than partnered work—to check that everyone understands these plots. Do feel free to help your partner succeed, but don't just do the work for him/her.)

4. Results from the program `bubbleCAL`.
5. Two values (with errors) for m_μ : one based on $\langle R \rangle$ and the other on a `fit` to Equation 1.40. Hardcopies of the `fit` results and a `plot` of the best-fit curve with your data points. (The plot is analogous to Figure 1.6.)
6. Two values (with errors) for m_π : one based on $\langle L_\perp \rangle$ and the other on a `fit` to the equation for $\hat{c}(L_\perp)$ you derived in 2(d) above. Hardcopies of the `fit` results and a `plot` of the best-fit curve with your data points.
7. A comparison of the values you reported above (5 & 6) and the “known” values cited above in 1.
8. Typically in this lab, the error estimates based on `fits` that are much smaller than the errors based on averages. Are these error estimates accurate? (If your masses (with errors) are inconsistent with book values, it is quite likely that something is wrong with your error estimates.) Explain how your error estimates could be improved. (For example, sometimes error estimates can be improved (and reduced) by simply taking more data. In other situations, improved errors come from proper consideration of systematic errors and result in larger estimates of error.) A few words about statistical and systematic errors are probably required; you might consult Chapter 0 if nothing occurs to you.

References

1. Lederman, Leon & Teresi, Dick *The God Particle* (1993)— In this general-audience book Nobel Laureate Lederman describes his work as an experimental particle physicist. Of particular interest: his work with pions in a cloud chamber using the Nevis accelerator (pp. 219–224); building cloud and bubble chambers (pp. 244–247); parity violation detected in the $\pi \rightarrow \mu \rightarrow e$ decay sequence (pp. 256–273).
2. Press, Teukolsky, Vetterling & Flannery *Numerical Recipes* (1992)— Title says it all. Cumulative fraction is discussed in the section on the Kolmogorov-Smirnov Test; covariance matrix in chapter on modeling data.
3. Hogg & Tanis *Probability and Statistical Inference* (1993)— Order statistics are discussed in Chapter 10.
4. Taylor, J.R. *An Introduction to Error Analysis* (1997)— derivative rule error propagation, standard deviation of the mean, histograms, random and systematic errors
5. I know of no book that derives Equation 1.40. A starting point is the Kurie Plot for weak decays, for example, from Halzan & Martin *Quarks and Leptons* (1984) p. 263:

$$\frac{d\Gamma}{dE} = \frac{G^2}{12\pi^3} m^2 E^2 \left(3 - \frac{4E}{m} \right) \quad (1.50)$$

6. Enge, H.A. *Introduction to Nuclear Physics* (1966)— Chapter 7 has a nice discussion on stopping power, range, the Bethe-Block equation and particle detectors including bubble chambers.
7. Coughlan, G. D. & Dodd, J. E. *The Ideas of Particle Physics: An Introduction for Scientists* (2003)— An excellent introduction to particle physics at undergraduate physics major level.
8. This lab is based on “The Pi-Mu-e Experiment” 33-3908 (1966) by Ealing Corporation.