PHY 4222 : Advanced Mechanics, Spring 2001

January 12^{th}, 2001

Assignment # 1

(due Friday January 19^{th}, 2001, at the beginning of class)

Notes For each problem of this homework, even if not explicitly requested, I would like you to justify your solution step by step. This will help you understanding the Lagrangian formalism without applying it in very pedestrian way. You are welcome to solve any problem using *Maple*. However, also in this case, you will have to explain your procedure.

1. What generalized coordinates can be used to completely specify the motion of each of the following? Explain what the constraints are in each case and how they determine the number (and the choice) of the generalized coordinates you need case by case.

   (a) A particle constrained to move on a sphere.

   (b) A circular cylinder rolling down an inclined plane.

   (c) A disk rolling without slipping across a horizontal plane. The plane of the disk remains vertical, but it is free to rotate about a vertical axis.

2. Consider a particle constrained to move in a plane under the action of a central force field $\mathbf{F_r} = F_r \hat{\mathbf{e}}_\mathbf{r}$.

   (a) Discuss the constraint(s) and your choice of generalized coordinates.

   (b) Write the Lagrangian.

   (c) Derive the equations of motion and explain them. How could you tell (without having to calculate the equations of the motion) that the angular momentum of the particle is conserved: (*i*) from the physics of the problem and (*ii*) from the form of the Lagrangian?

3. Problem 7.4 of Marion and Thornton's book.

4. Problem 7.5 of Marion and Thornton's book.

5. Problem 7.6 of Marion and Thornton's book. (*Hints.* (*i*) You need one generalized coordinate for the incline and one for the hoop. A good choice is: the horizontal position of the inclined plane and the position of the contact point of the hoop along the incline, respectively. To avoid confusion, do not call them x or y! (*ii*) In order to calculate the potential and kinetic energy of the hoop, you will need to express the cartesian coordinates of its center of mass in terms of the generalized coordinates.)