RELATIVISTIC MASS OF THE ELECTRON

INTRODUCTION

In this experiment the momentum and velocity of high energy electrons are measured to obtain the dependence of m/e on velocity for electrons of energies ranging from 200 keV to 800 keV. Assuming that the electron charge is independent of velocity, we wish to measure the mass of the electron, which appears to change with velocity.

In a book written for a general audience, Einstein describes the effect in terms of inertial mass:

> A body at rest has mass but no kinetic energy, that is, energy of motion. A moving body has both mass and kinetic energy. It resists change of velocity more strongly than the resting body. It seems as though the kinetic energy of the moving body increases its resistance. If two bodies have the same rest mass, the one with the greater kinetic energy resists the action of an external force more strongly.[1]

In his commentary on this quotation, Carl Adler writes:

> … I wish to address the use of the word "seems" in the quoted passage. This word is well advised because the apparent increase in "resistance" with velocity of a moving particle is an illusion. It comes about simply because of time dilation. An applied force on a moving particle of a given rest mass apparently takes a longer time to accelerate it when the particle is moving faster than the same force applied to the same particle when moving slowly. Thus the particle appears to have more resistance to acceleration at high speeds. In reality, measured by a clock instantaneously traveling with the particle, the same force always produces the same effect in the same time interval.[2]

Instead of defining a "relativistic mass," we simply consider an electron of mass m and charge e moving perpendicular to a uniform magnetic field. The momentum of such an electron is

$$p = eBR$$

where R is the radius of curvature of the electron's trajectory. In our spectrometer, a slit is used to define R which selects electrons of a given momentum for fixed B.

Electrons which pass perpendicularly through the slit enter a region between two parallel plates. For a certain magnitude and sign of the electric field, the electric and magnetic forces on the particle are equal and opposite:

$$e\vec{E} = e\vec{v} \times \vec{B}.$$

---

[1] Einstein, A., The Evolution of Physics, Simon and Schuster, New York, 1938, p 207.
[2] Adler, C., "Does mass really depend on velocity, dad?", Am. J. Phys., 55 (8), August 1987.

Therefore,

$$v = \frac{E}{B} .$$

Measuring p and v independently, one can verify the relativistic expression

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} .$$

INSTRUMENTATION

electron source and detector:

The electron source, ^{210}Bi (Ra E), has a half life of only 4.85 days. The parent substance, ^{210}Pb (Ra D) with a $\tau_{1/2}$= 25 yr, has been electroplated on a platinum strip (about 1 mm wide by 6 mm long) to provide a stable source of electrons of energies up to about 1 Mev. (Why does the source emit a continuum of energies?) The source is not sealed. **It should not be touched with fingers or tools!** It is absolutely safe when the spectrometer is closed, but proper precautions must be taken when the spectrometer is disassembled for viewing and geometric measurements. Consult the faculty or staff before opening the spectrometer. Two geometric parameters must be measured: the radius of curvature of the electrons' path and the plate separation. The radius is fixed by the positions of the source and the entrance slit. The separation of the plates has been set to 1.099 ± 0.006 mm. Please do not touch the plates or insert feeler gauges or other measuring tools into the gap. The plates are made of soft copper and scratch easily. Scratches cause arcing between the plates which limits the range of electron energies accessible to the apparatus.

The electrons leave the evacuated spectrometer through a thin aluminized mylar window (take care not to damage it). The detector is a Geiger-Mueller tube (GM detector) with a very thin end window.[3] **Be careful not to accidentally puncture this window; both windows are very fragile**. Read about the care and feeding of GM detectors elsewhere (see Knoll or Moore/Davis/Coplan, for example). The general operating procedure is as follows: With the high voltage at zero, connect the GM detector to the MHV connector on the rear panel of the HV supply/scaler[4]. Place a radioactive source (preferably a ß emitter) near the GM tube. Depress RESET and then COUNT. Slowly increase the high voltage until counting just begins (threshold voltage). Then determine the plateau region where the GM tube's efficiency is "flat" with respect to high voltage. **Caution: maximum voltage should be about +400V -- do not destroy the GM tube by application of excessive voltage.** Note also that the GM tube,

[3] LND type 712.
[4] The Nucleus, model 500 Nuclear Scaler.

approaching the end of its life, may become erratic and change its rate drastically up or down under otherwise constant operating conditions. The sensitivity may vary with count rate even for a good GM tube. The GM counter output (BNC connector on rear panel) is a TTL pulse and this signal goes to the PCI-6602 Counter/Timer card input.

high voltage sweep and measurement:

A fixed magnetic field is used to select electrons of a given momentum (actually, a small range of momenta). By varying the electric field over some appropriate range, the electron detector will see a maximum signal when the electric and magnetic forces are balanced. The electric field is varied by "sweeping" the high voltage on the deflection plates in the velocity selector. In this case, sweeping means changing the voltage on the plates linearly and symmetrically[5] over some predetermined range as a function of time (a sawtooth function).

Equal and opposite voltages are applied to the two capacitor plates. Two voltage-programmable Bertan power supplies provide the high voltage. Each supply has an output range of 0 to 10 kV, one positive and the other negative. The program input voltage to the HV supplies should be set to 0 to 5 volts. It is provided by a Wavetek function generator[6]. **Adjust the amplitude, dc offset, mode, frequency range and frequency appropriately *before* plugging the generator into the HV program input.** It is not desirable to sweep the full 20 kV[7]. Two ten-turn potentiometers are installed on a panel in front of the HV supplies. The dial labeled "ramp offset" adds a dc level to the sawtooth input and the "ramp voltage" dial attenuates it, so any desired voltage range can be swept. **The dials read directly in kilovolts and the sum of the two numbers should never exceed 10!** A high voltage monitor output is provided for convenience but is of limited accuracy (0.1% of reading + 0.1% of maximum).

The high voltage measurements are performed with a voltage divider which, in turn, must be calibrated using the high-accuracy (0.03%) 100 kV divider[8]. Faculty or staff should be consulted first concerning proper safety precautions and procedures, and the following pointers are given here as a reminder. First, before disconnecting any high voltage leads, **turn off the high voltage power supplies!** Next, **disconnect only one** of the two spectrometer capacitor plates from the home-made voltage divider. **Connect the donut-shaped dome** (via the special high-voltage test lead and SHV connector) of the high-accuracy divider to the point where you disconnected the capacitor plate. **Make very sure that the high-accuracy divider is**

---

[5] with respect to ground.
[6] Wavetek model 21, 11 MHz Stabilized Function Generator.
[7] Field emission and secondary electron emission from the plates become much more pronounced at high voltages resulting in a marked increase in background counts. Electrical discharges are also a problem if the vacuum is not good enough and/or the cable connector(s) are "dirty", the insulation has a nick out of it, the ground shield has not been pulled back sufficiently, or the plates have been scratched.
[8] Hallmark Standards model PVD, 100kV Potential divider.

properly grounded. The jack labeled "LOW" should be attached to a reliable ground such as the brass bar connected to a water pipe. Make sure that you do not touch or accidentally brush against the donut-shaped dome — it will be at high voltage when the power supplies are on! Use a digital voltmeter to measure the voltage across the resistor at the low-end of the divider chain; that is, measure the voltage between ground and the jack labeled "1V". Simultaneously monitor the low voltage output of the home-made voltage divider and calibrate that voltage against the high-accuracy divider. **Turn off the high voltage power supplies before repeating the measurement process for the other polarity (the other capacitor plate).**

data acquisition and control:

A Multichannel Scaler (MCS) stores a histogram of counts versus time. The MCS consecutively advances through a given number of channels, "dwelling" in each one for a preset amount of time. It stores the number of counts received during the dwell time in that particular channel. Advancing through all the channels once is referred to as a "sweep." The MCS can be set to execute many sweeps, adding to data taken during previous sweeps. By synchronously sweeping the voltage on the plates (in the spectrometer) with the MCS cycle, one obtains a histogram of detected electrons vs time. By noting the plate voltages at the beginning and end of the ramp and assuming a linear ramp (how good an assumption is this?), one can convert the counts vs. time histogram to counts vs. electric field and ultimately, electron velocity. Be aware of limitations in high voltage slew rates when setting dwell times (about 1 second/channel is reasonable).

A LabVIEW virtual instrument called mcs.vi communicates with a National Instruments PCI-6602 Counter/Timer card. The card accepts pulses from the instrument labeled "The Nucleus" and sends an output pulse to the Wavetek function generator to trigger the sweep. The duration of the high voltage sweep should be chosen to be slightly shorter than the time required for one MCS sweep.

magnetic field:

The electromagnet is powered by a Hewlett Packard 6114A Precision Power Supply, wired for external constant-current resistance programming. The programming resistor, (in a small aluminum box) determines the output current. The front-panel controls can override the programming resistor, so they should be set well above the expected operating parameters. A 1Ω resistor is wired in series with the magnet. The voltage across this resistor serves as a current monitor.

A digital teslameter[9] is used to determine magnetic field strengths. It is a microprocessor based Hall-effect instrument. The probe is very expensive, so please handle the instrument with care and follow the manual instructions to achieve high precision measurements.

vacuum system:

Electrons don't travel very far through the air. For this reason the spectrometer needs to be evacuated. In addition to the plumbing, the vacuum system consists of a mechanical pump, vapor diffusion pump, and vacuum gauges (thermocouple and cold cathode). Consult the references below for background information on how these pieces of apparatus work. The mechanical pump is used for "roughing out" the system to bring it down to the tens of microns (one micron = 10^{-3} mm Hg = millitorr) pressure range and is referred to as a "roughing pump" in this context. It also serves as a backing pump for the diffusion pump and is then called a "fore pump" or "backing pump". The thermocouple gauge is good for measuring pressures down to micron range (typical mechanical pump pressures) while the cold cathode gauge is used for lower pressures achieved with the diffusion pump, typically 10^{-5} to 10^{-6} mm Hg. Consult the faculty or staff concerning procedures in "pumping down" and "breaking vacuum" as well as the do's and don'ts. A summary of procedures is appended to this write-up.

One further important note: If you look at the bottom of the vacuum chamber you will notice a thin brass bar screwed onto it. This bar serves as a "stop" -- that is, the vacuum chamber should be pushed into the magnet as far as it will go; the "stop" prevents you from pushing it in too far and properly positions the chamber within the magnet.

EXPERIMENTAL PROCEDURE

While the spectrometer is at atmosphere, open it and make whatever geometrical measurements are needed. Pump down using first the roughing pump, then the diffusion pump. Measure the geiger tube plateau using a test source. Calibrate the high voltage. Set up the high voltage sweep. Familiarize yourself with the LabVIEW vi. Decide how many channels to use and the dwell time per channel. Begin taking data. On the order of one day is needed to accumulate adequate statistics for each electron energy.

[9] Group 3 model DTM-141 Digital Teslameter.

"About once every year or so, Desmond sets some time aside to give Einstein's relativity theories another go . . ."

REFERENCES

for the physics:

G. Holton, Am J Phys **30**, 462 (1962). Resource Letter SRT-1 on Special Relativity Theory is a great place to start.

A.H. Compton and S.K. Allison, *X-rays in Theory and Experiment,* (D. Van Nostrand, New York, 1936). Appendices I and II...here is one place where *all* of the transformation relations are derived on the basis of the special theory of relativity.

H.A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, *The Principle of Relativity,* (Dover, New York, 1952). Original papers are translated with notes by A. Sommerfeld. Worth reading pp 35-50 for content, clarity of thought and expression, and simplicity. Also pp 69-71 - does the inertia of a body depend upon its energy content?

R.T. Weidner and R.L. Sells, *Elementary Modern Physics*, 3rd ed., (Allyn & Bacon, Boston, 1980). Most good modern physics textbooks will help you out - see Holton above for suggestions. This one happens to have a good comparison of classical, relativistic, and extreme relativistic curves of energy vs momentum.

E. Taylor and J.A. Wheeler, *Spacetime Physics*, (W.H. Freeman, San Francisco, 1966).

W. Rindler, *Essential Relativity - Special, General, and Cosmological*, (Van Nostrand Reinhold, New York, 1969).

C.G. Adler, Am J Phys **55**, 739 (1987). "Does Mass Really Depend on Velocity, Dad?"

R.W. King, Rev. of Modern Phys. **26**, 327 (1954). "Table of Total Beta-Disintegration Energies"

L. Marton, C. Marton, and W.G. Hall, *Electron Physics Tables*, National Bureau of Standards Circular 571, March 1956.

A.T. Nelms, *Energy Loss and Range of Electrons and Positrons*, Supplement to NBS Circular 577, July 1958. Tabulations of the mean energy loss due to ionization and excitation and the range derived from this quantity are given for electrons and positrons in several materials.

for radiation measurement and protection:

W. Peter Trower, Am J Phys **38**, 795 (1970). Resource Letter PD-1 on Particle Detectors.

Glenn F. Knoll, *Radiation Detection and Measurement*, 2nd ed., (John Wiley, N.Y., 1989). A great comprehensive book on the subject!

K. Siegbahn, *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, (North Holland Publ., Amsterdam, 1965). Especially useful for the operation of β-ray spectrometers.

J.H. Moore, C.C. Davis, and M.A. Coplan, *Building Scientific Apparatus*, 2nd ed, (Addison-Wesley, Reading MA, 1989).

Jacob Shapiro, *Radiation Protection - A Guide for Scientists and Physicians*, 2nd. ed., (Harvard Univ Press, Cambridge MA, 1981).

vacuum techniques

S. Dushman, *Scientific Foundations of Vacuum Technique*, 2nd ed., J.M. Lafferty, editor, (John Wiley & Sons, New York, 1962). This is the bible on the subject and will explain everything to you.

J. Moore, C. Davis, and M. Coplan, *Building Scientific Apparatus - A Practical Guide to Design and Construction*, 2nd ed., (Addison-Wesley, Reading MA, 1989). An excellent book to start with. Many additional current references are contained herein.

APPENDIX: Getting started on the vacuum system

The m/m_o vacuum system is a basic high vacuum system incorporating elements often found in research apparatus. These elements are:

l) The mechanical pump is located on the floor. This removes most of the air from the system, a process called "roughing it out." It also serves as a backing pump for the diffusion pump.

2) The oil diffusion pump, which is placed between the mechanical pump and the chamber. It improves the vacuum by several orders of magnitude by using streams or jets of oil vapor to push molecules down from the top to the bottom of the pump. It has a heater in the bottom to boil the oil and is cooled at the top by a fan.

3) The vacuum chamber which contains the electron source and spectrometer.

4) The associated valves and vacuum gauges.

Vacuum systems in general are highly intolerant of misuse; care must be used at all times. With a diffusion pump system, an error in use will not destroy the apparatus, but it will spread oil over all the internal parts including the chamber and will take many hours of cleaning with solvents to rectify. When in doubt, do nothing; ask a staff member or professor for assistance.

<u>How to use the vacuum system</u>

1) CONDITION: system is off and cold and you want to pump down the chamber

a) Close all valves.

b) Turn on the mechanical pump. (This will also turn on the fan on the side of the diff. pump, but not the heater on the bottom.) Open the roughing and foreline valves and wait for the pressure to drop below 100 mtorr (within 1 min.)

c) Close the roughing valve. Open the high vacuum valve. Turn on the diffusion pump. The pumpdown time depends on several variables including when the system was last used, if it was opened to air, if the inside surfaces are covered with fresh fingerprints, etc.

As the diff. pump starts to work, the pressure on the roughing gauge will fall. After about 20 min. typical pressures are less than 10 mTorr on the roughing gauge and 50 mTorr on the backing gauge. When the chamber pressure, as read by the roughing gauge, falls below 10 microns, turn on the cold cathode gauge. It may take a few moments for it to start. This gives the most accurate measure for the pressure in the chamber. After an hour typical pressures are 9×10^{-5} Torr for the chamber and 45 microns for the backing pressure. The system is normally left on continuously for the duration of the experiment.

2) CONDITION: system is running and you want to shut down and open the chamber.

a) Turn off the cold cathode gauge. Close the high vacuum valve. Turn off the diffusion pump. Bleed air in through the toggle valve with a yellow handle located next to the roughing valve. Be certain that both valves on each side of the toggle valve (i e. the high vacuum and roughing valves) are closed. Open the KF25 fitting near the chamber and remove the bellows. Take care not to kink the bellows by letting it hang unsupported.

b) After about 20 minutes, feel the bottom of the diffusion pump. When it is cool to the touch, close the foreline valve and shut off the mechanical pump. Bleed air back into the hose leading from the mechanical pump by lifting the black toggle valve located on the tee just above the mechanical pump inlet (beneath the bench).

Many other conditions occur in normal usage, but it is not possible to consider them all here; when in doubt, consult a staff member.