

# PHY5846C: INTRODUCTION TO EXPERIMENTAL TECHNIQUES

## ACCELERATORS

\*\*\*\*\* Homework due 5 November 1998 \*\*\*\*\*

1. The luminosity  $\mathcal{L}$  of a colliding beam machine is defined by

$$N = \mathcal{L}\sigma ,$$

where  $N$  interactions occur per second for cross section  $\sigma$ . For a machine with  $n_1, n_2$  particles per bunch in the two beams with  $n_B$  bunches per beam, and a bunch revolution frequency  $f$ , show that

- (a) the luminosity is given by

$$\mathcal{L} = \frac{n_1 n_2}{A} f n_B ,$$

for uniform cylindrical bunches of cross sectional area  $A$ ,

- (b) while for bunches with gaussian profiles perpendicular to the beam direction, with rms widths  $\sigma_x, \sigma_y$ , the luminosity becomes

$$\mathcal{L} = \frac{n_1 n_2}{4\pi\sigma_x\sigma_y} f n_B .$$

- (c) Calculate the luminosity for an  $e^+e^-$  collider of circumference 2.4km with two bunches in each beam and beam currents 10mA each, for  $\sigma_x = 500\mu\text{m}$ ,  $\sigma_y = 50\mu\text{m}$ . How many events are accumulated per day for a cross section of  $1\text{nb}$ ?

- (d) Calculate the luminosity for a proton synchrotron with a cycle time of 50s delivering  $10^{11}$  particles per burst to a fixed liquid hydrogen target of length 50cm.

(See refs. [1], [2a], [2b]).

2. In a collider, the particle beams are stored in the machine for many hours. Throughout this time, for each turn, the two beams must pass close to each other at the focal points corresponding to the interaction regions where detectors are located. Furthermore, the energy loss due to synchrotron radiation must be compensated by supplying power to the accelerating RF cavities.

- (a) Calculate the angular accuracy required to maintain the relative alignment of the two beams to within  $100\mu\text{m}$  for ten hours, and compare it with the angular accuracy required to direct a space craft to within 20km at the orbit of Uranus (at  $\approx 3 \cdot 10^9\text{km}$  from the Sun).

- (b) Calculate:

- the energy lost due to synchrotron radiation by one particle per revolution
  - the average power that the RF cavities must transfer to the beams in order to compensate for the energy loss by synchrotron radiation
- (b1) in PETRA (circumference 2.4km, bending radius 192m) at 15 GeV, for 10mA current in each beam;
- (b2) in LEP (circumference 26.66km, bending radius 3096m) at 45 GeV, for 3mA current in each beam;
- (b3) in LEP at 80 GeV, for 3mA current;
- (b4) in the TeVatron collider (circumference 6.28km, bending radius 754m) at 800 GeV, for 3mA proton and 1.5mA antiproton current.

(See refs. [3], [2a], [2d], [2e]).

### References

1. Landau-Lifshitz: Theory of classical fields, chapter 2  
(for def. of cross section and relativistically invariant formulation of flux factor)
2. Particle Data Group: Review of particle properties, European Physical Journal C **3** (1998) 1 - 794.
  - (a) p. 69 for physical constants,
  - (b) p. 76 for properties of materials used in detectors,
  - (c) p. 138 for physics of colliders
  - (d) p. 141 for properties of colliders,
  - (e) p. 78 for useful electromagnetic relations,
  - (f) p. 186 for useful formulae of relativistic kinematics.
3. J.D. Jackson: Classical Electrodynamics (sect. 14.2 for synchrotron radiation).